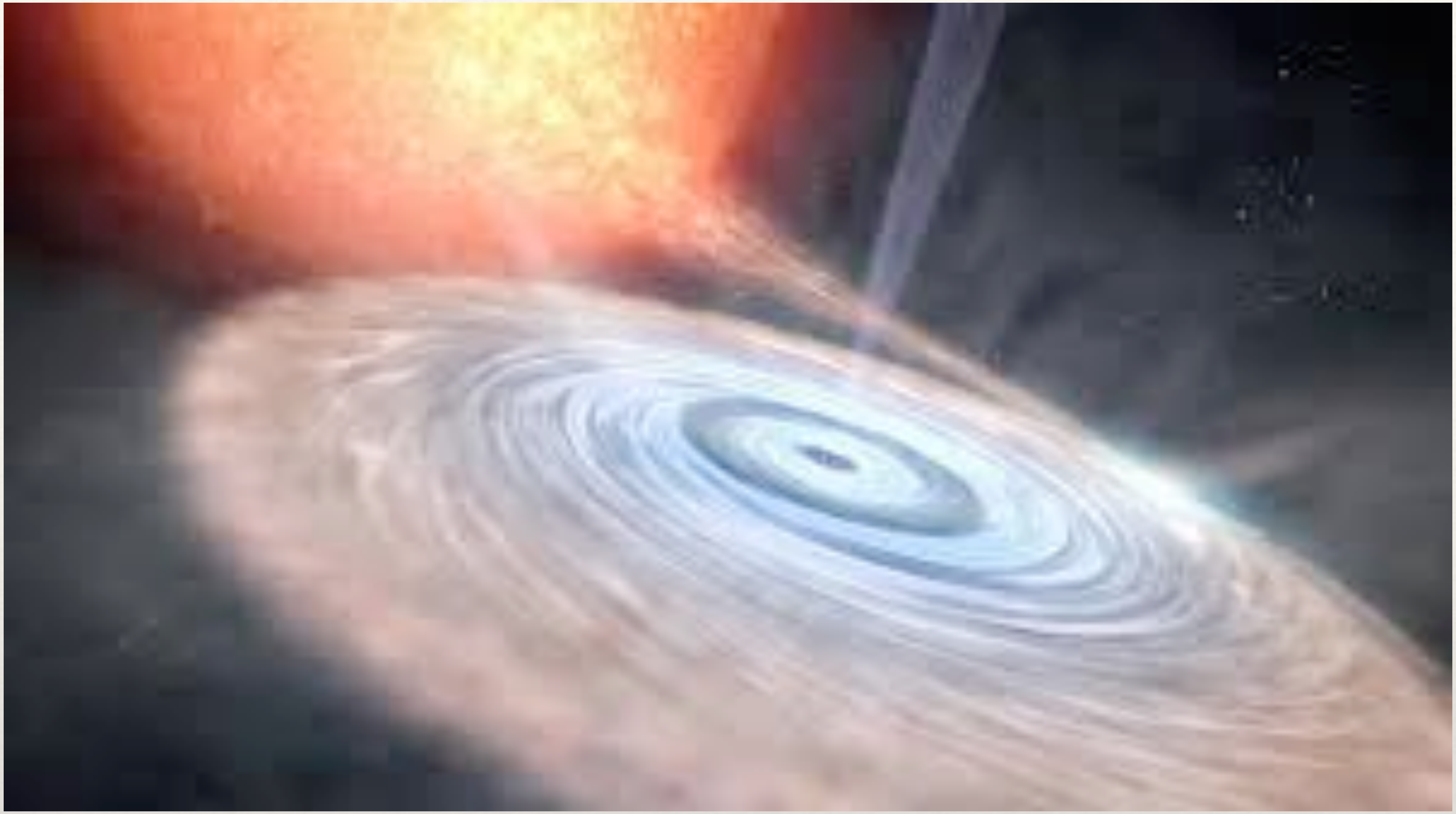
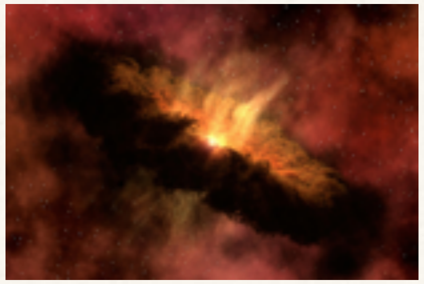


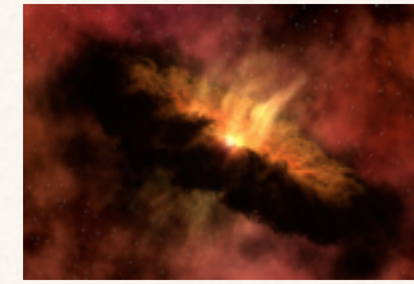
[Credit: Gabriel Pérez, SMM (IAC)]

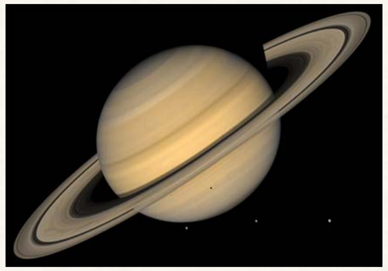
Astrophysical Accretion Disks

Henrik Latter
DAMTP, University of Cambridge

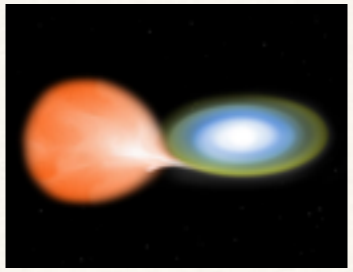


Contents





Introduction

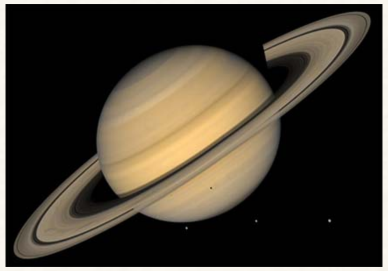


Why study accretion disks?

1. Well… they're everywhere

2. They occur on a vast range of scales (from 10^5 to greater than 10^11 km)

3. They can be composed of a diversity of stuff (dust, ice balls, poorly ionised gas to collisionless plasma), and involve a vast range of different physics



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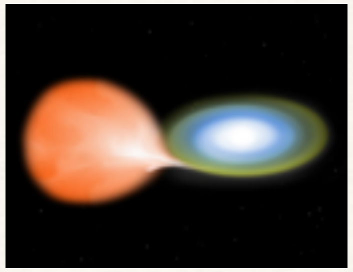

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3. They can be composed of a diversity of stuff (dust, ice balls, poorly ionised gas to collisionless plasma), and involve a vast range of different physics

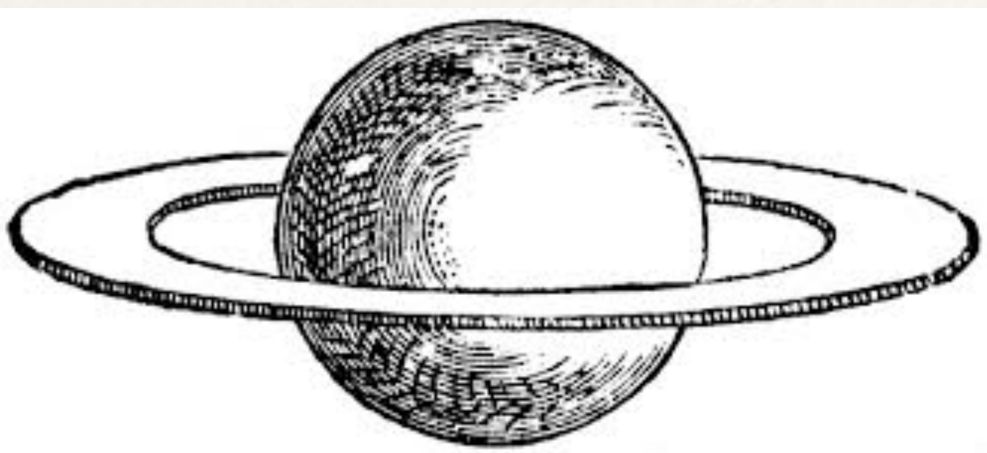
4. They're implicated in several **<u>essential</u>** astrophysical processes:
   - star formation
   - planet and moon formation
   - high energy phenomena
   - physics of galaxies, the ISM, and beyond

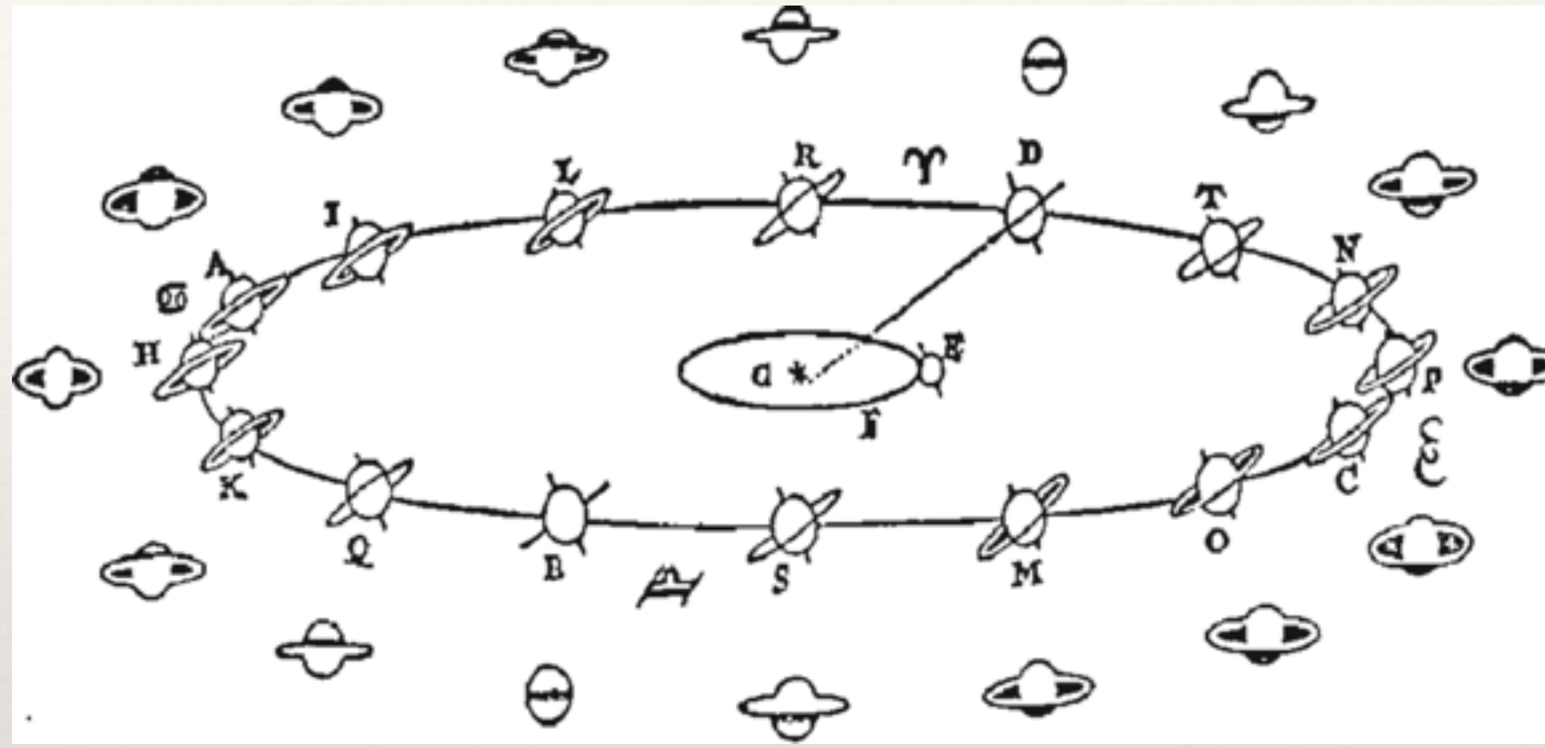
Survey of Astrophysical Accretion Disks

Planetary rings

Galileo (1610): “I have seen the most distant planet to have a triple form"

Planetary rings



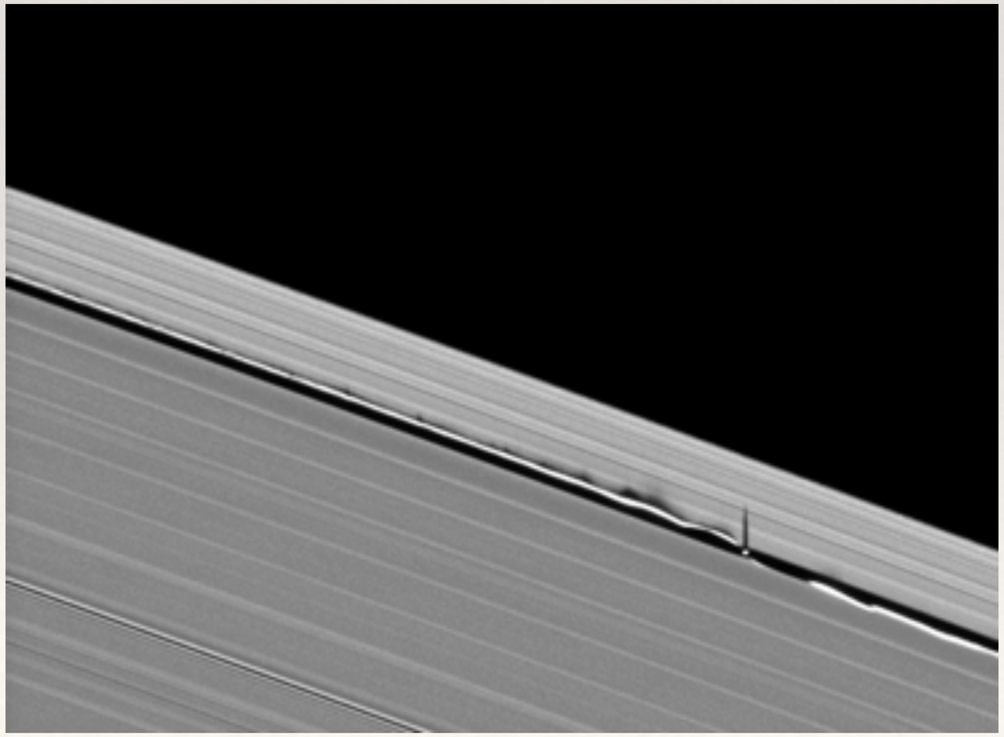


(Sketches by Huygens)

Huygens (1656): “It is surrounded by a thin flat ring, nowhere touching, and inclined to the ecliptic”

Planetary rings

Saturn's rings (*Cassini*)



Daphnis in the Keeler gap (*Cassini*)

Planetary rings

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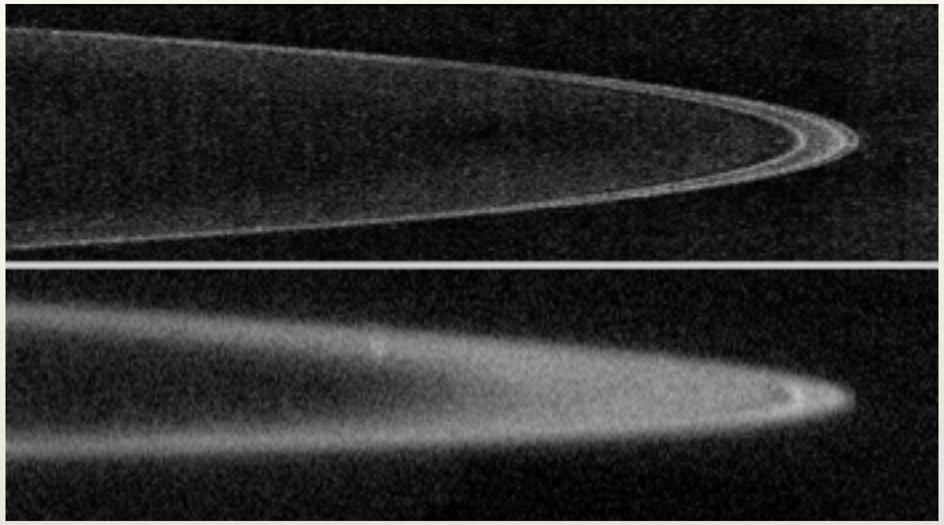
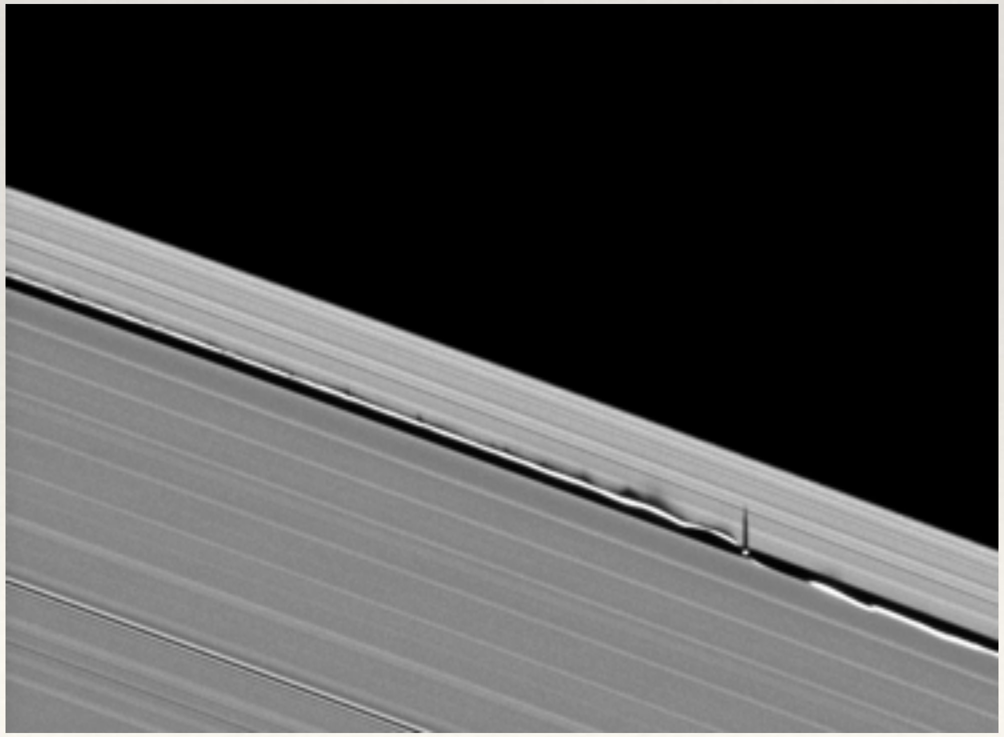


The Jovian main (dusty) ring *(New Horizons)*



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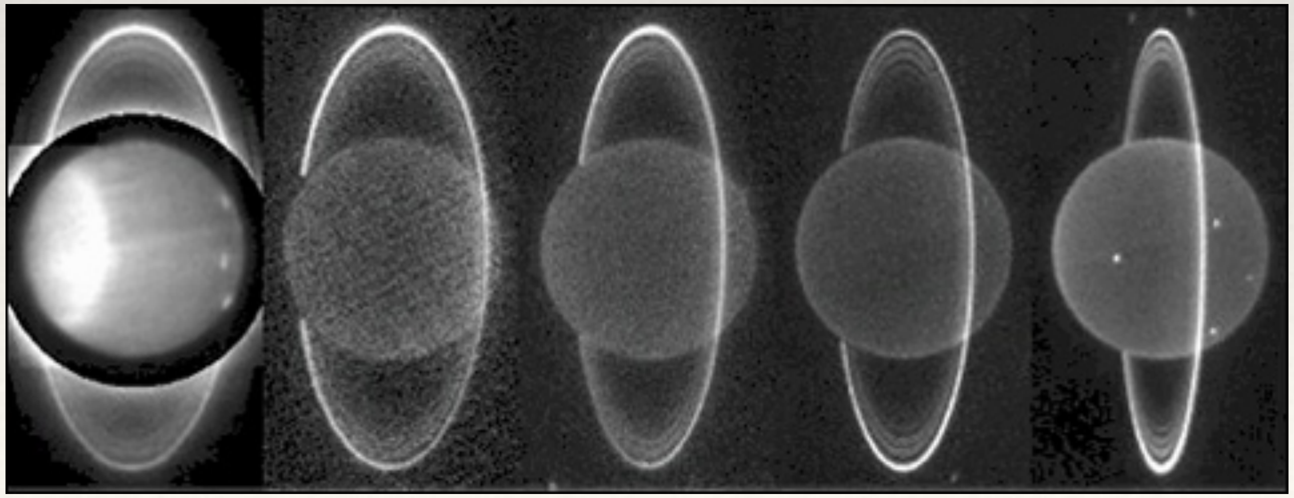


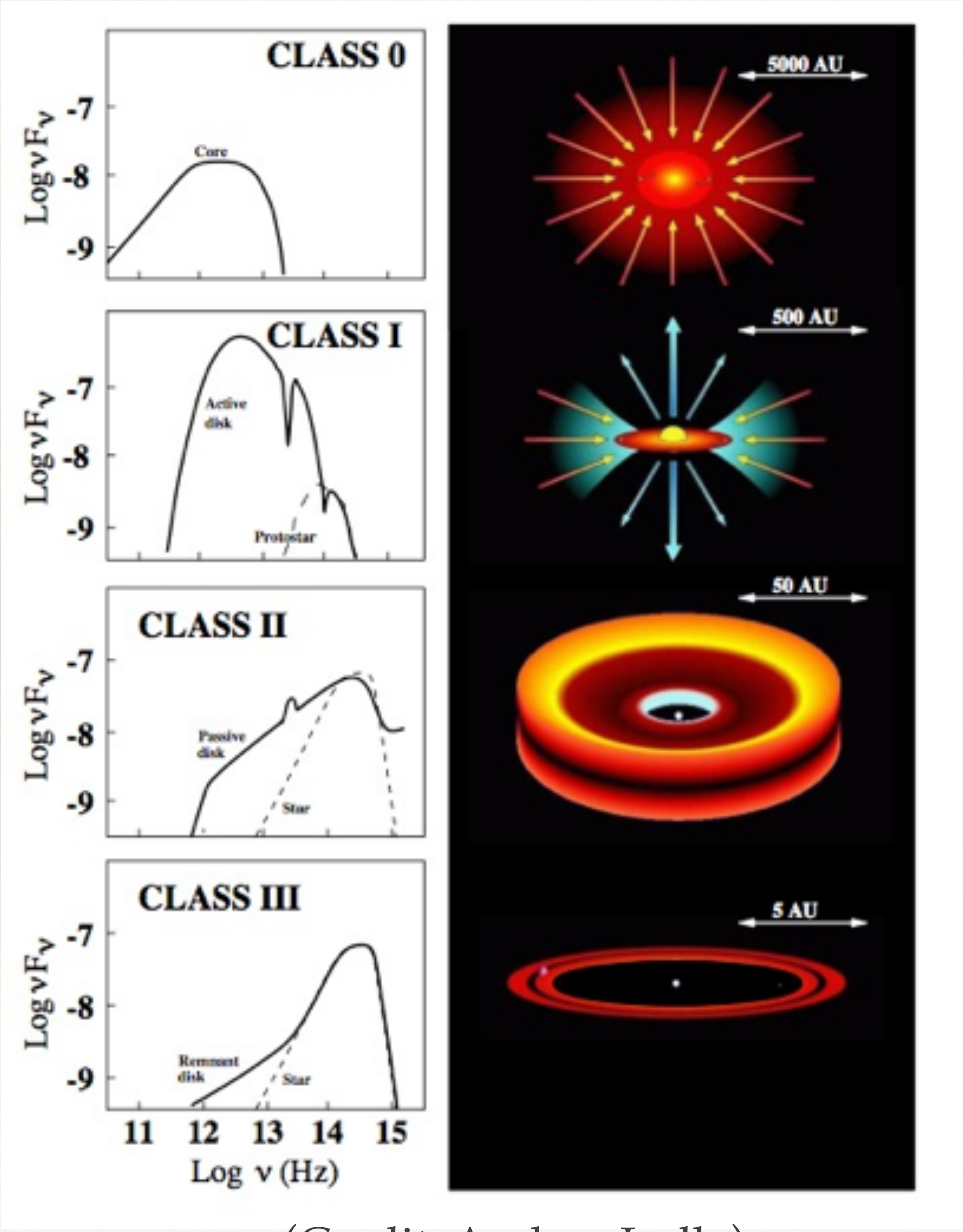
Uranus's rings (*Keck*)

Planetary rings: properties

- Radius: 140,000 km, for Saturn's rings
- Thickness: ~10 m
- Collide infrequently (a few times per orbit)
- Mean free path ~ disk thickness
- Composed of (water) ice balls mm to several m
- Theoretical approaches: dense gas (Enskog) kinetic theory, viscous hydro….

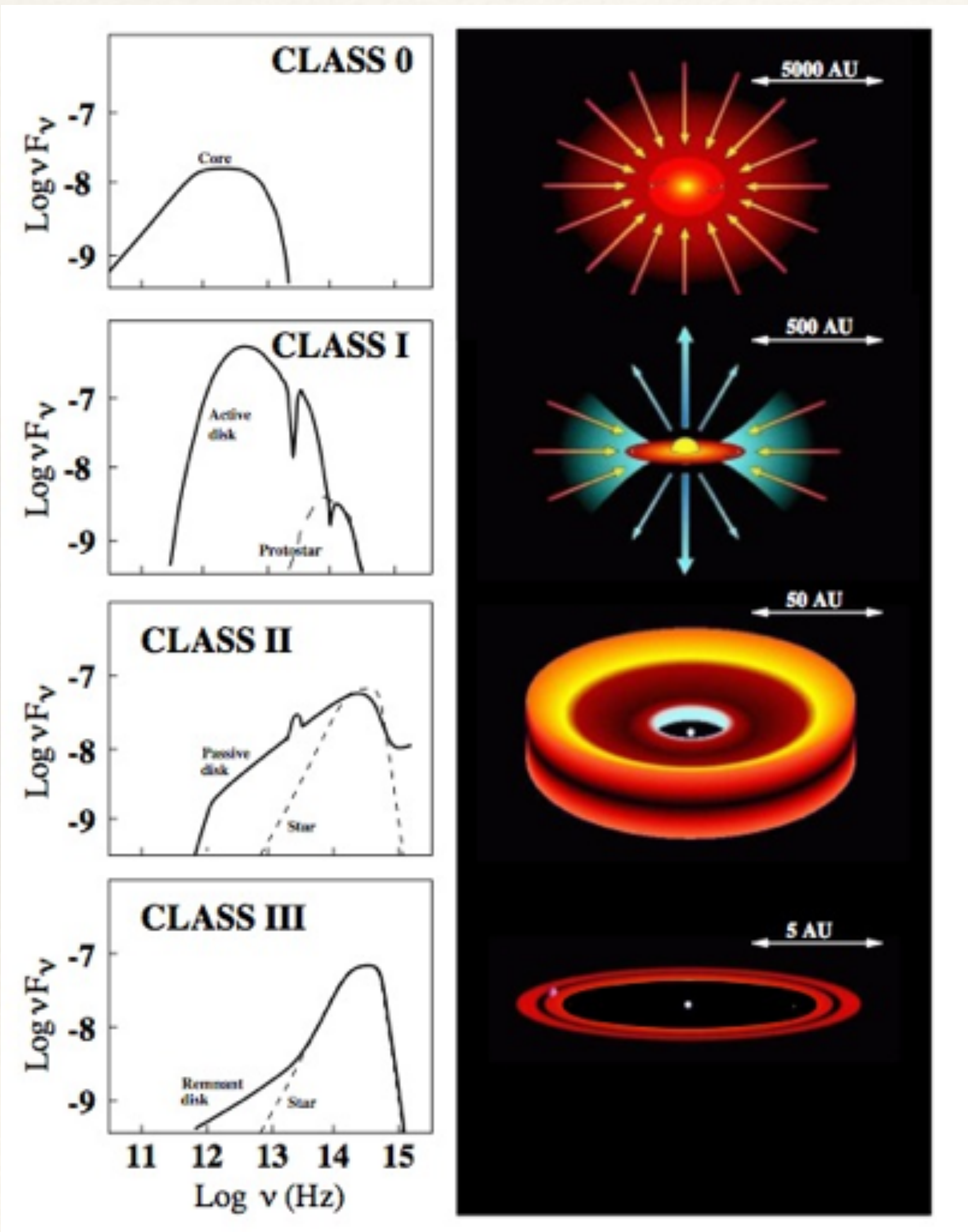
[Credit: Ron Williams, Black Cat Studios]

Protoplanetary disks



(Credit: Andrea Isella)

Protoplanetary disks



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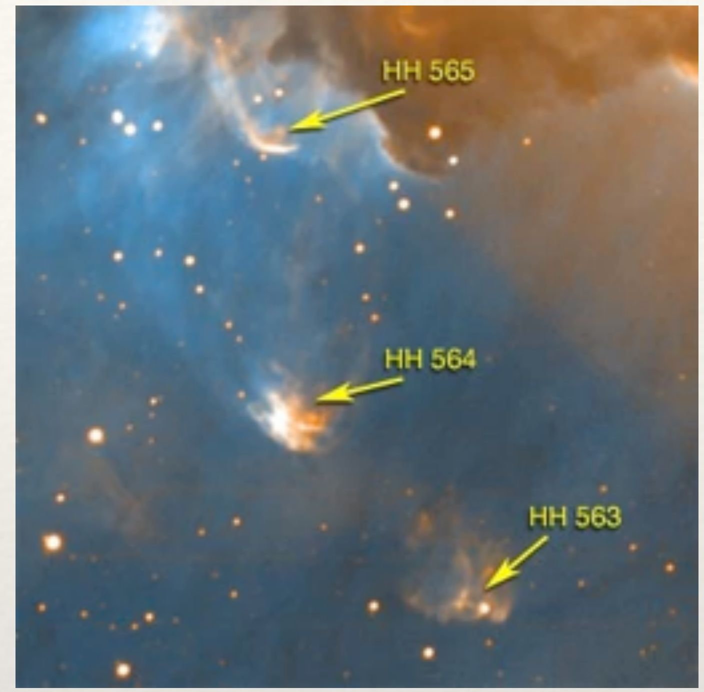
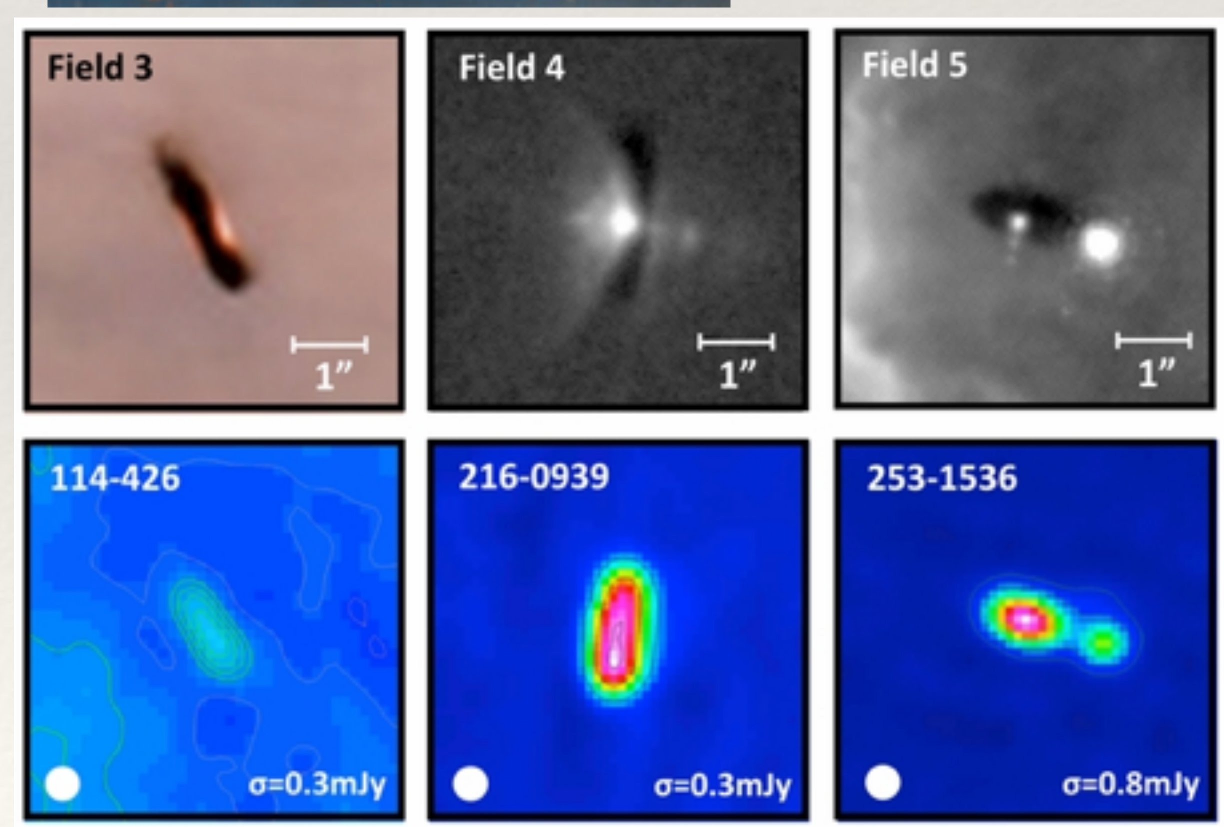


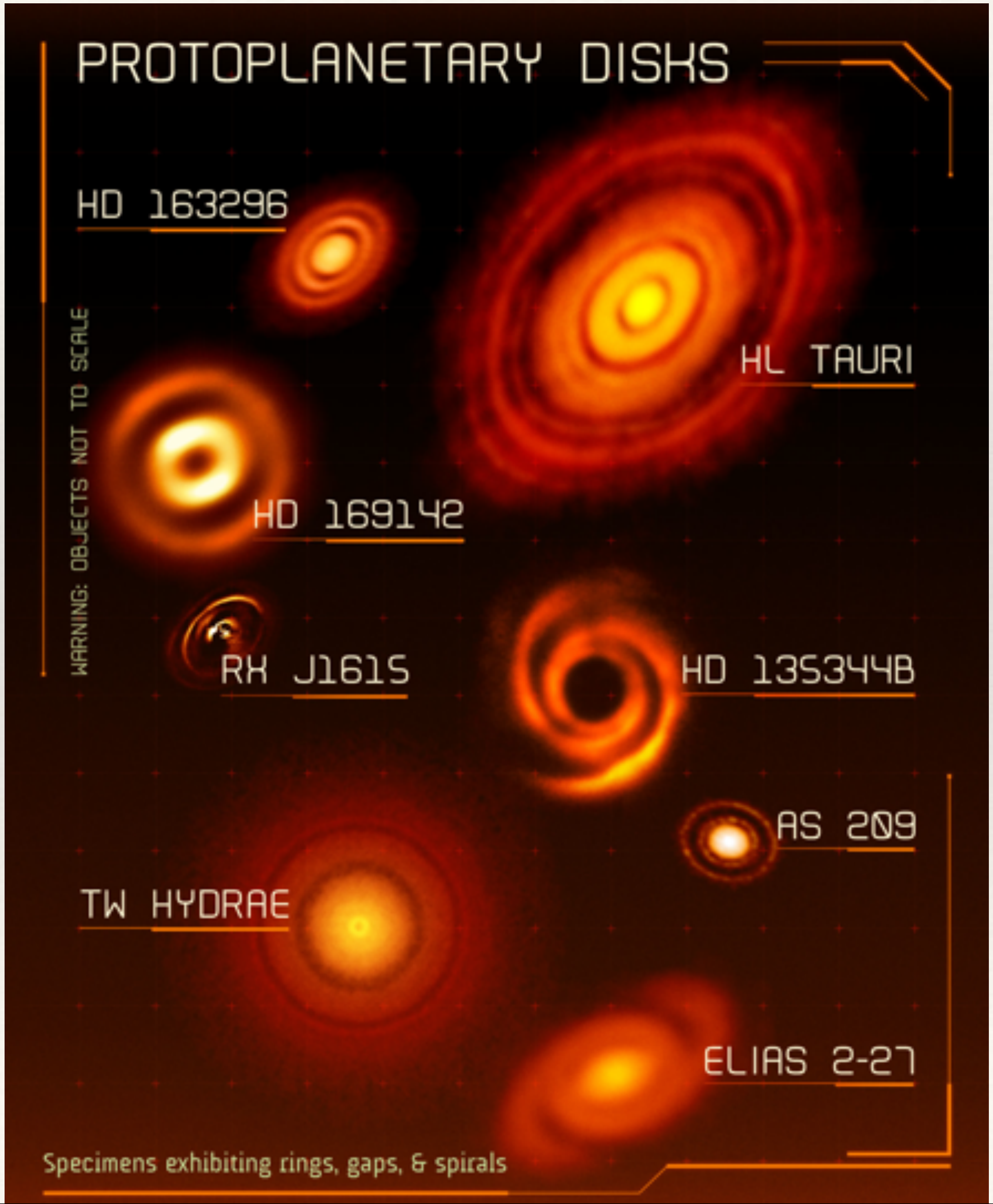
(*Hubble*)



(Mann+ 2014)

Protoplanetary disks

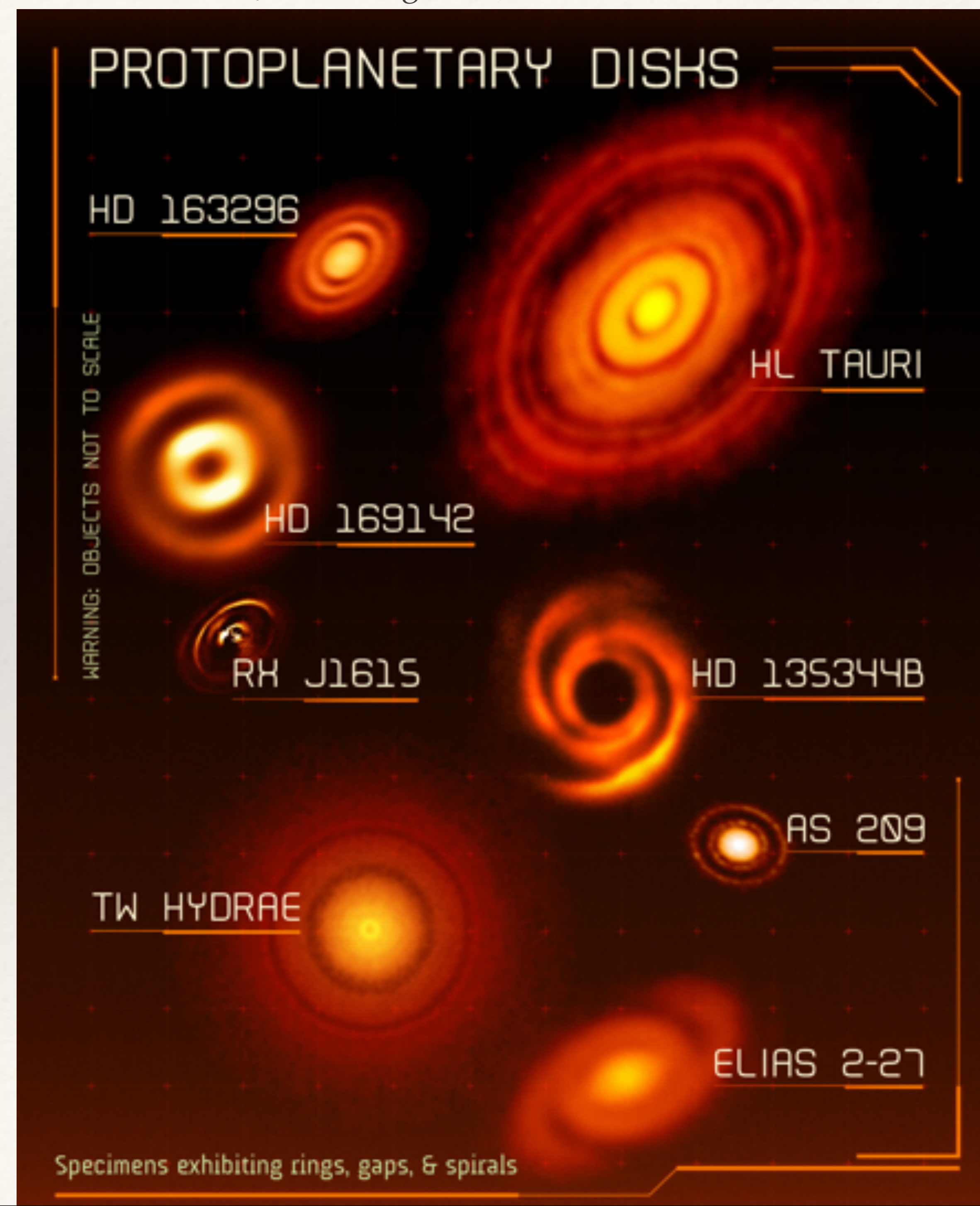
(Credit:
Quanta Magazine)



Protoplanetary disks

(Credit:
Quanta Magazine)

The jet of HH 47 *(Hubble)*



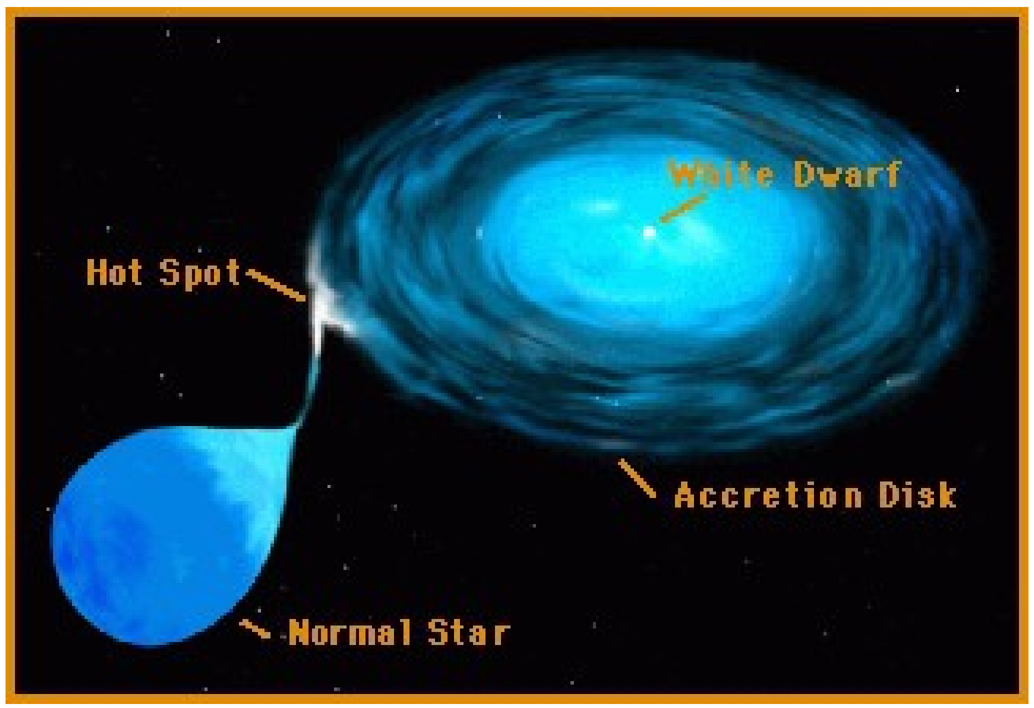
Protoplanetary disks: properties

- Radius: 10-1000 AU
- Thickness: ~0.05*radius
- Lifetime ~ 1-10 million years
- collision frequency >> local orbital frequency
- Temperatures: a few 1000K at inner radii to 50K further out
- Composed primarily of molecular hydrogen, but dust important
- Weakly ionised: coming from stellar X-rays and FUV, and cosmic rays
- Theoretical approaches: hydrodynamics, non-ideal MHD

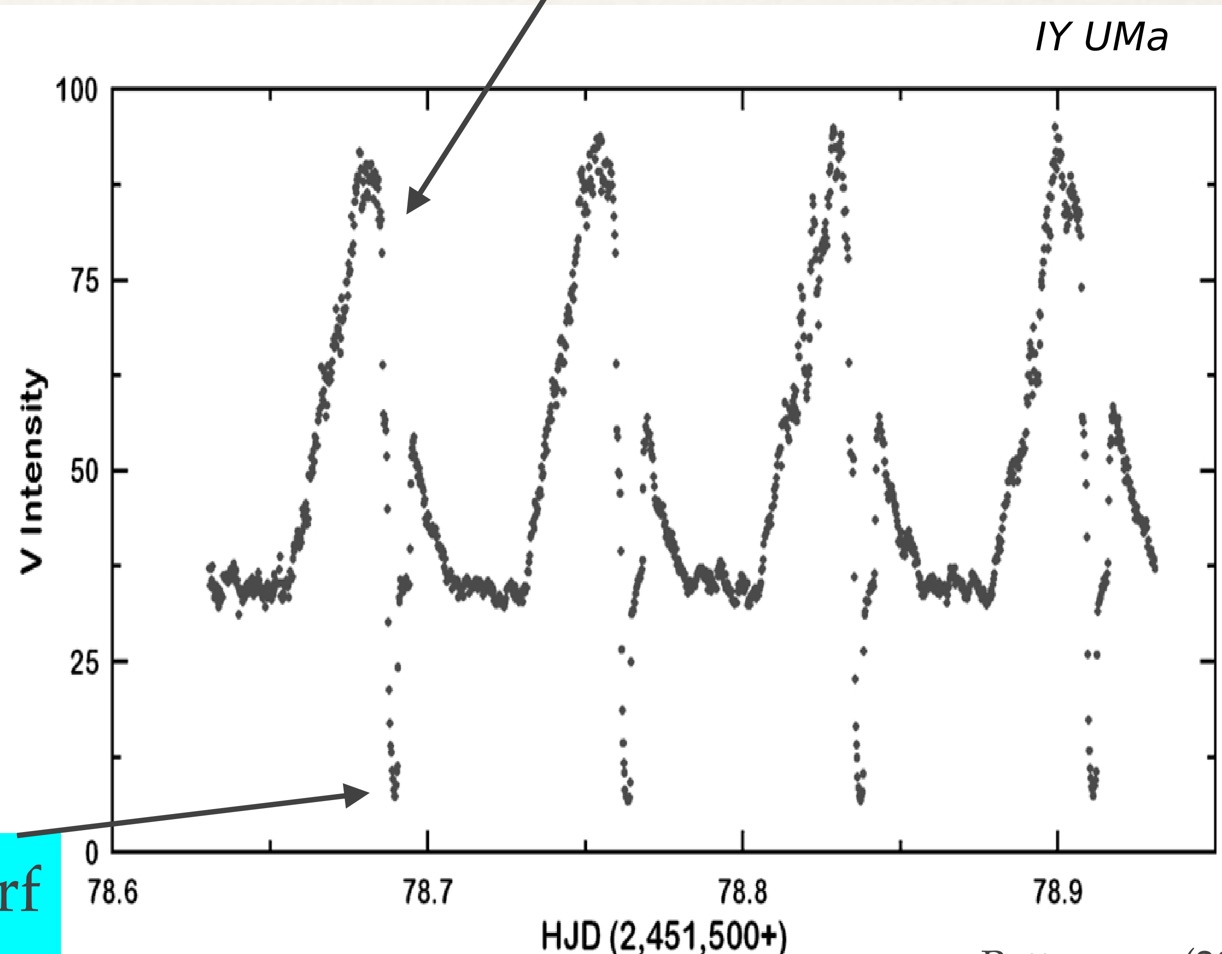
[Credit: Detlev van Ravenswaay]

Dwarf novae

Rotation of hot spot



[Credit: NASA]

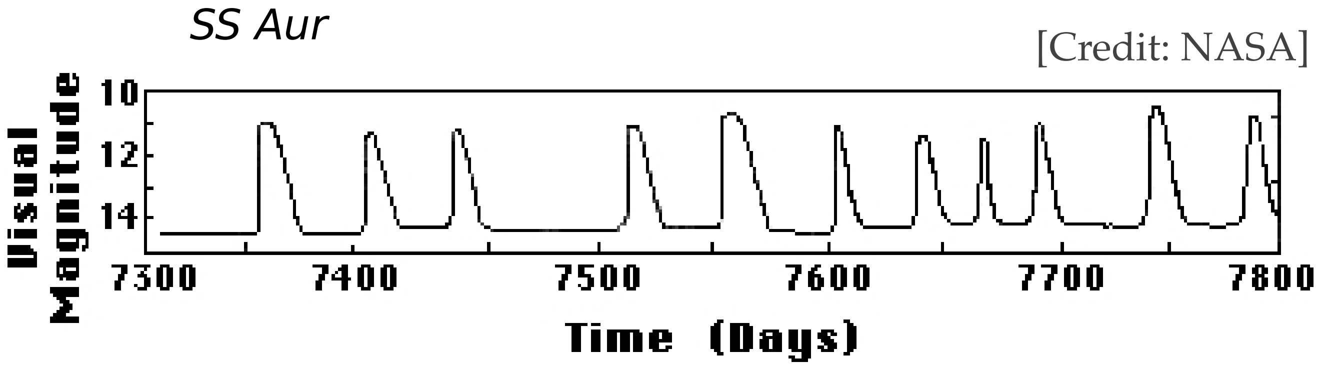


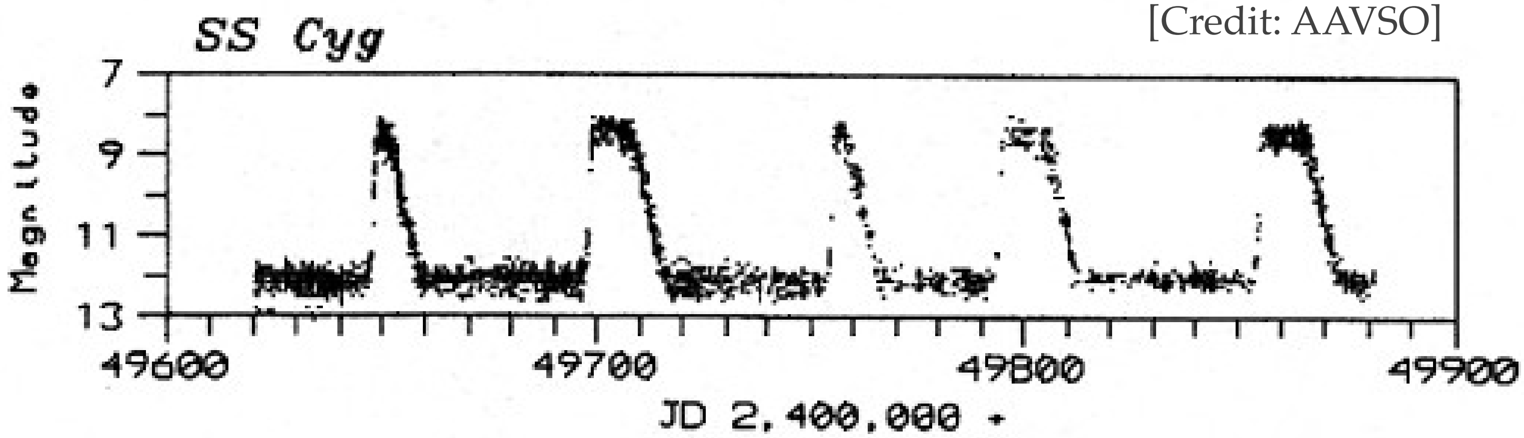
Eclipse of white dwarf by secondary

Patterson+ (2000).

Luminosity variability (orbital time)

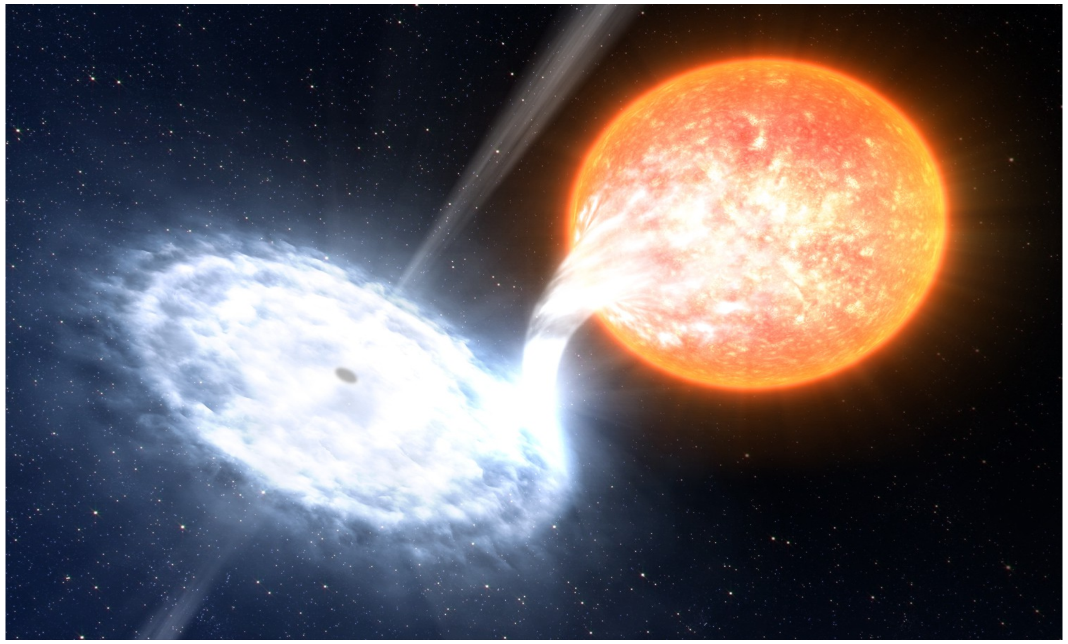
Dwarf novae



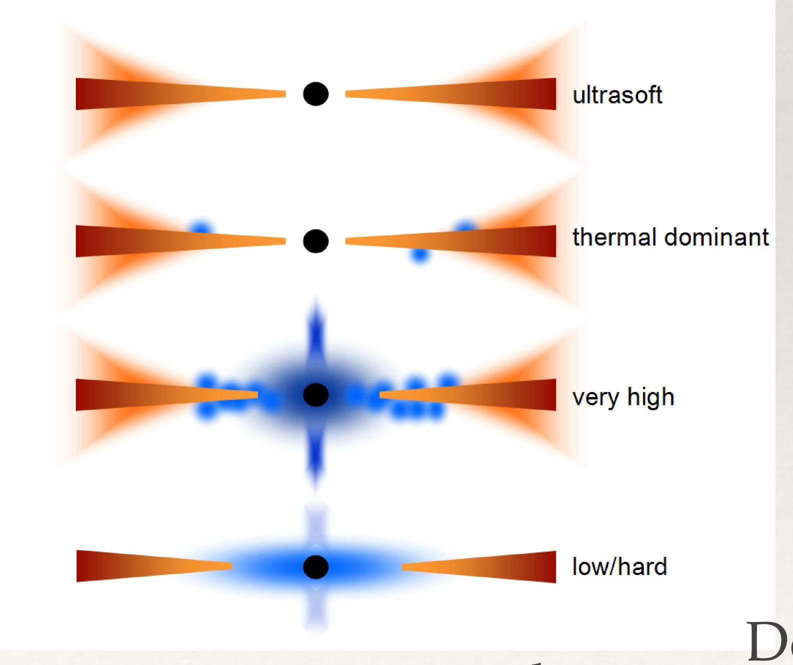


Outbursts (days-weeks)

(Low mass) X-ray binaries



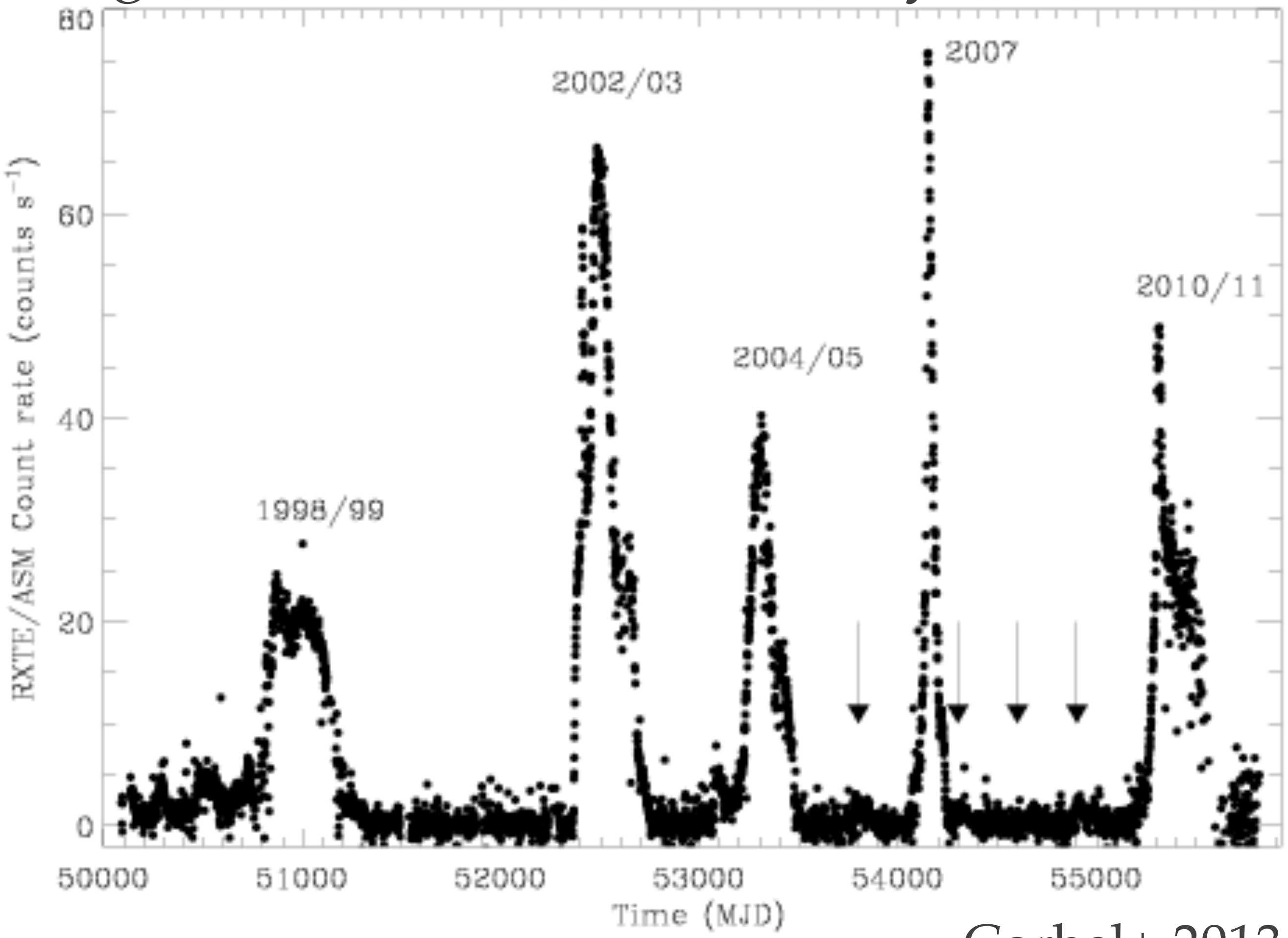
[Credit: ESO, L. Calcada]



Done+ 2007

Possible states of the inner accretion disk during the outburst cycle

Light curve of black hole binary GX 339-4



Corbel+ 2013

X-ray variability: Outbursts (years)

Close binaries: properties

- Outer radius ~ 500,000 km
- Inner radius ~ 10,000 km (dwarf nova), < 100 km (XRB)
- Thickness: ~0.01*radius
- collision frequency >> local orbital frequency *generally*
- Temperatures, a few 10^3 K (quiescence) to >10^6 K (outburst) *generally*
- Composed primarily of molecular hydrogen in quiescence, and ionised hydrogen in outburst
- Theoretical approaches: MHD (relativistic if needed)

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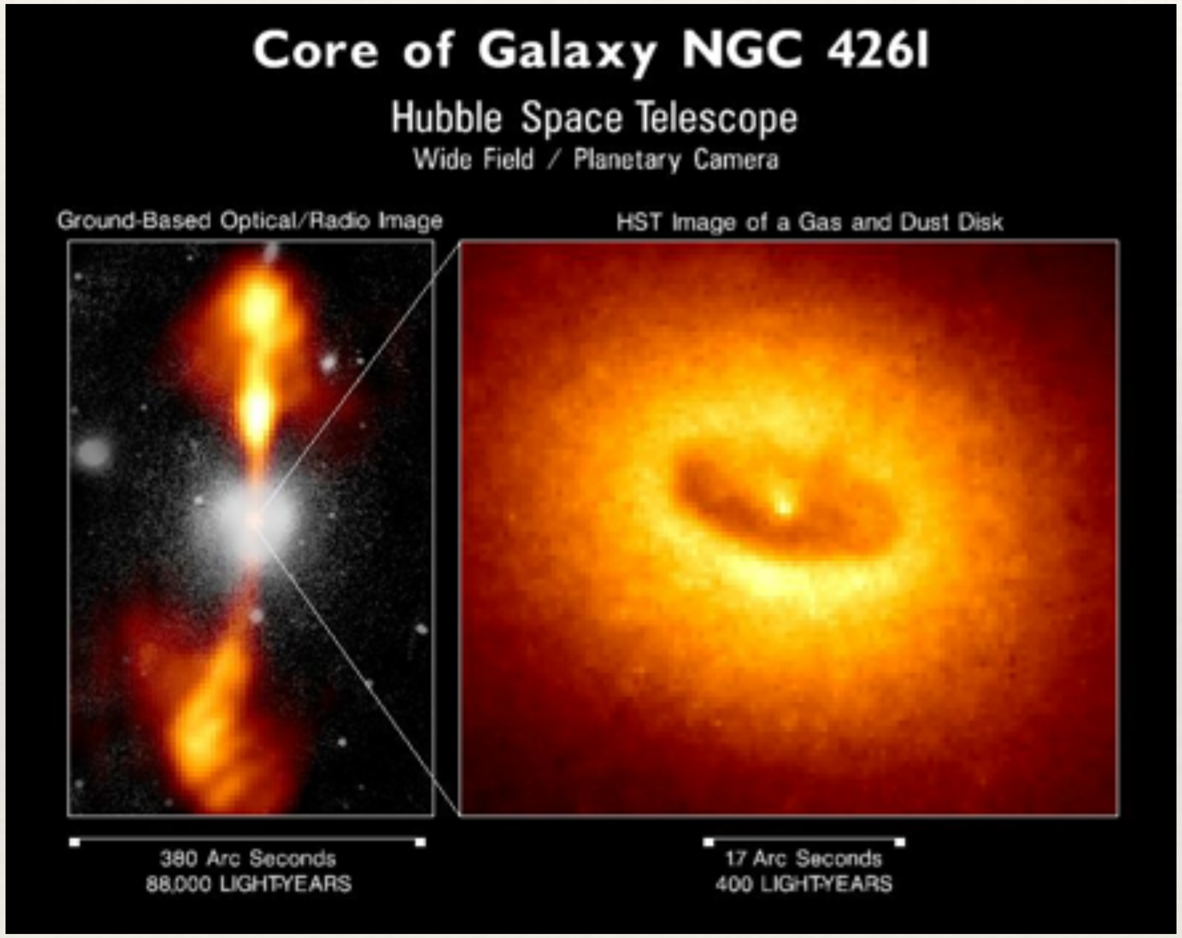
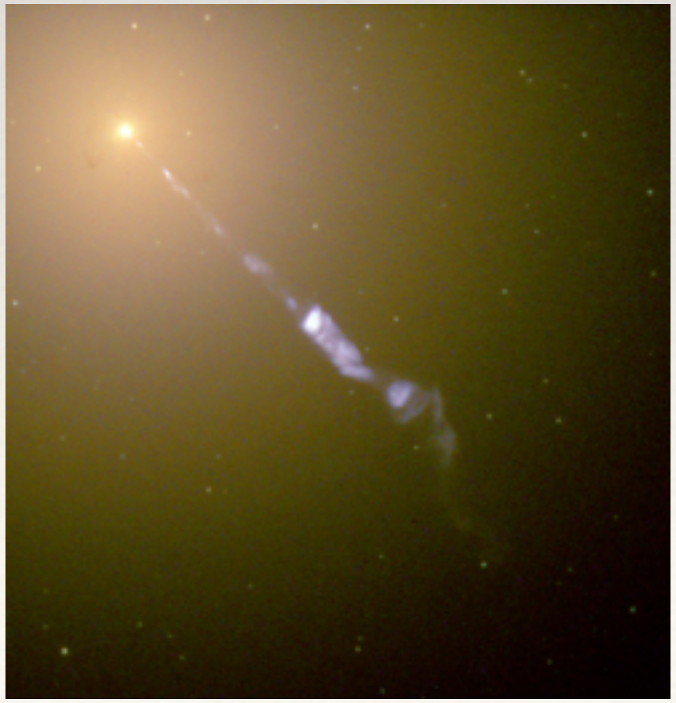
- **<u>However:</u>** X-ray binaries in the inner radii of XBs during quiescence (i.e. low/hard state) are radically different:
    - collision frequency << local orbital frequency
    - Larmor radius << mfp ~ radius
    - Electron temperature ~ 10^9 K …. ion temperature ~ 10^12 K !
    - Theoretical approach: extended MHD, plasma physics

Galactic Nuclei



M87 *(Event Horizon Telescope)*



M87's jet *(Hubble)*

Active Galactic Nuclei: properties

- Inner radius ~ innermost circular stable orbit ~ 1-100 AU
- Outer radius of disk ~ 1000 r_g ??? Beyond that the BLR…
- Thickness: ~0.001*radius
- collision frequency >> local orbital frequency *generally*
- Temperatures, a few 10^3 K to 10^5 K *generally*
- Well ionised thermally or by radiation
- Theoretical approaches: hydrodynamics, non-ideal MHD

- However, low luminosity nuclei (Sgr A*) may be similar to quiescent XBs:
  - extremely hot, and weakly collisional at best

Basic Disk Physics

Basic Physics

- The ubiquity of disks may come down to:
  - gravitational attraction
  - angular momentum conservation

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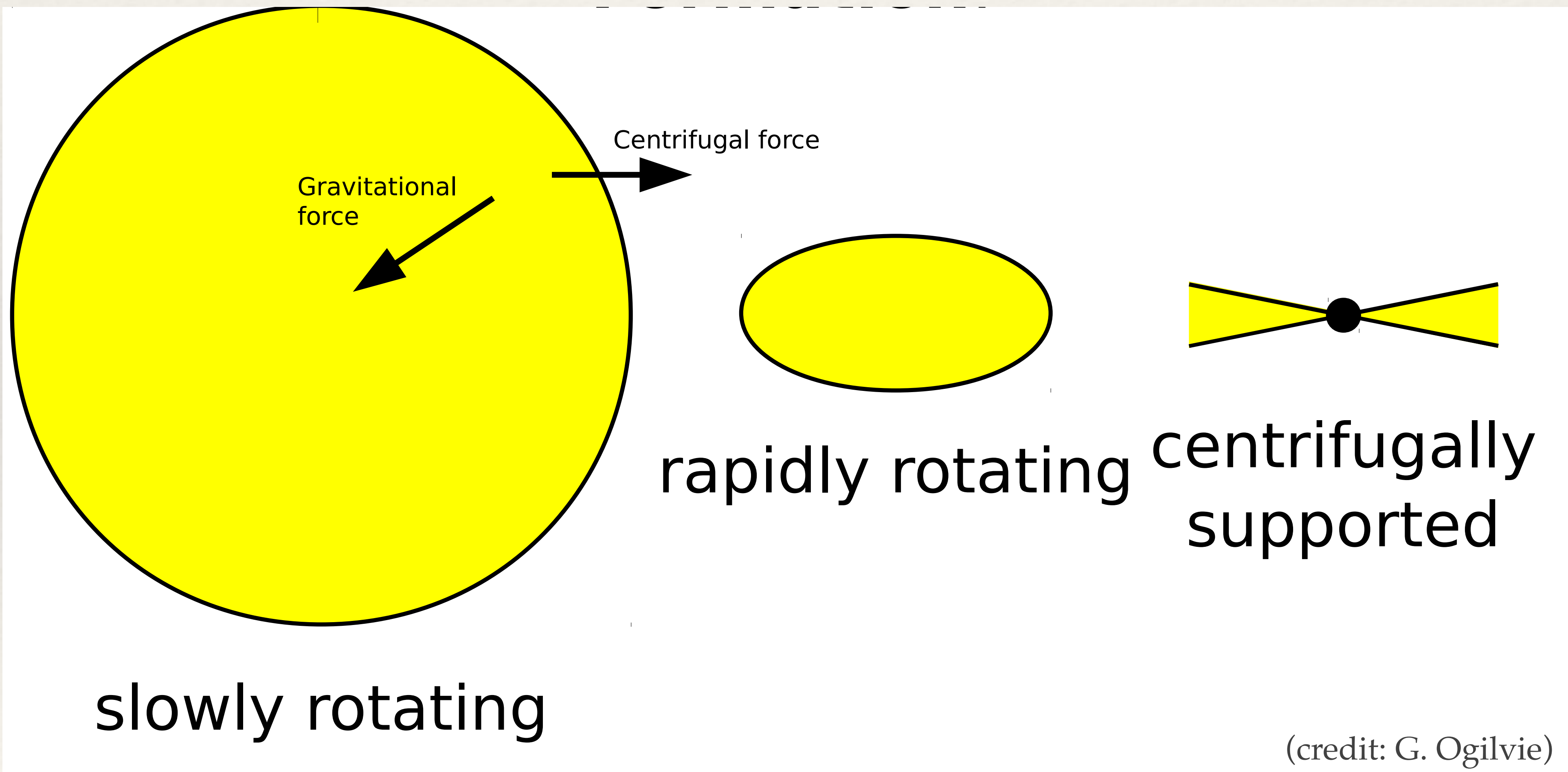

(credit: G. Ogilvie)

Basic Physics

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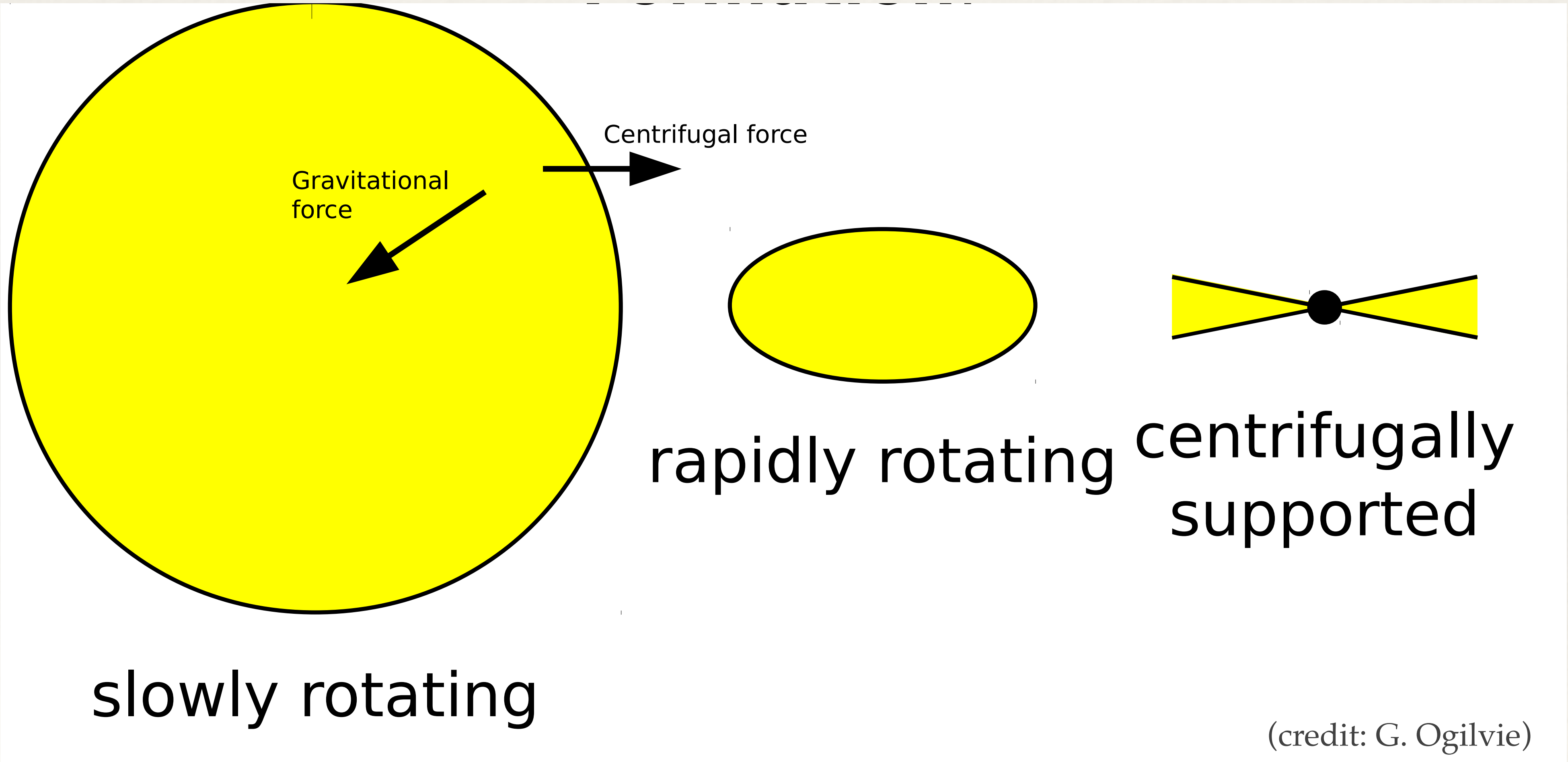
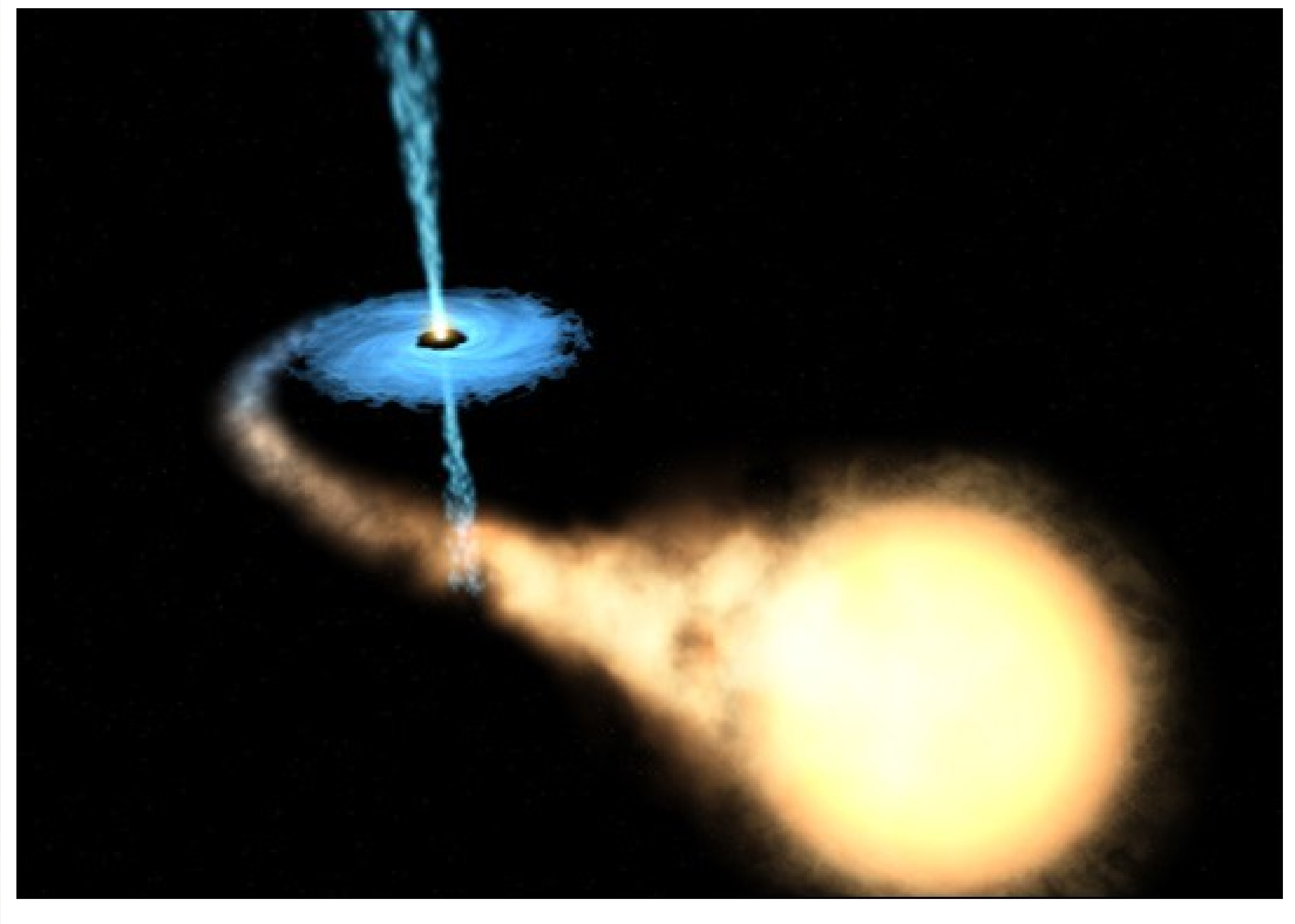


- The flattening tendency is resisted by pressure
- the colder/more dissipative the system, the 'flatter'

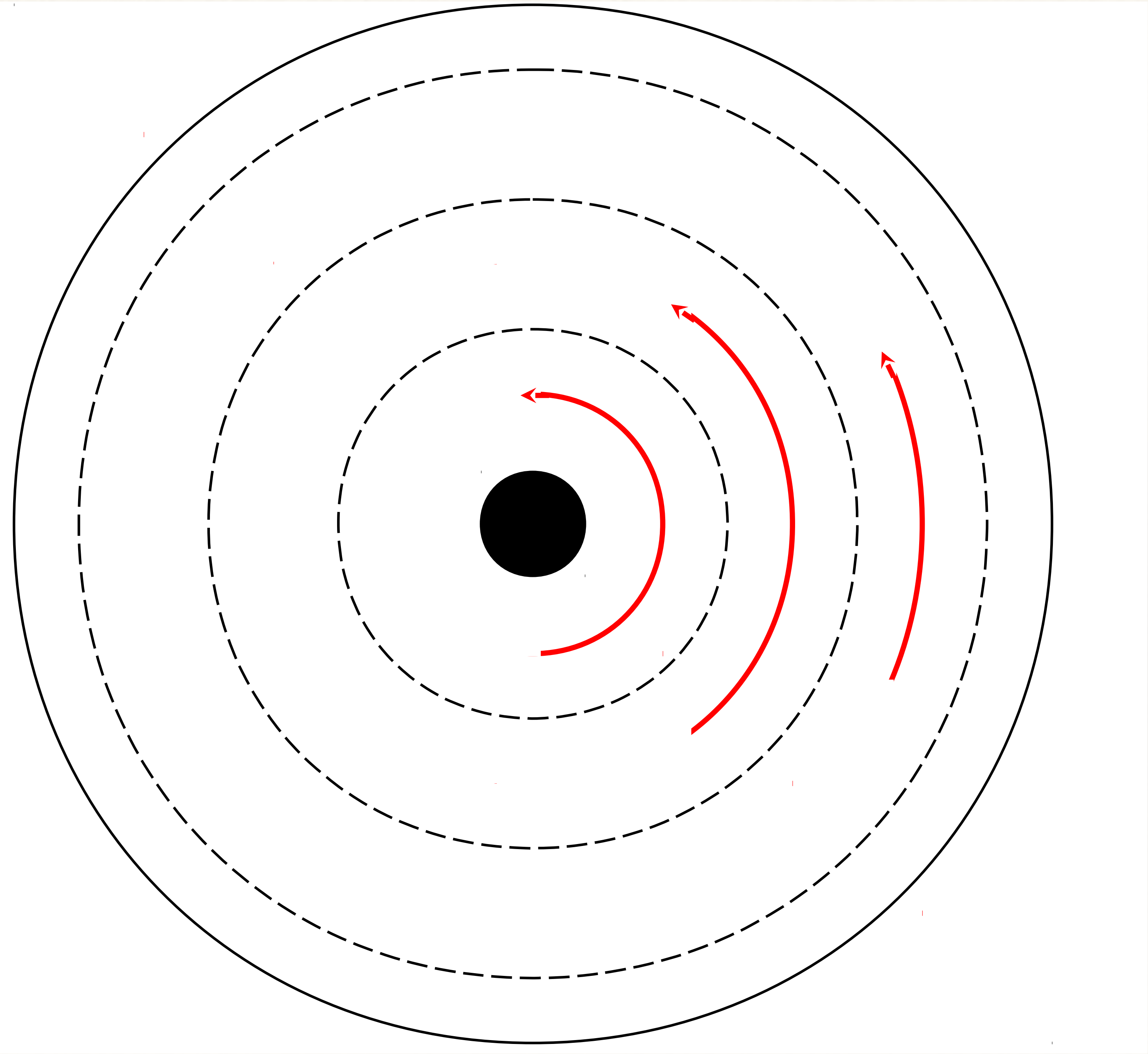
Basic Physics



[Credit: NASA]

- On the other hand, in binary systems, gas from the donor star has too much angular momentum to just fall straight on to the secondary

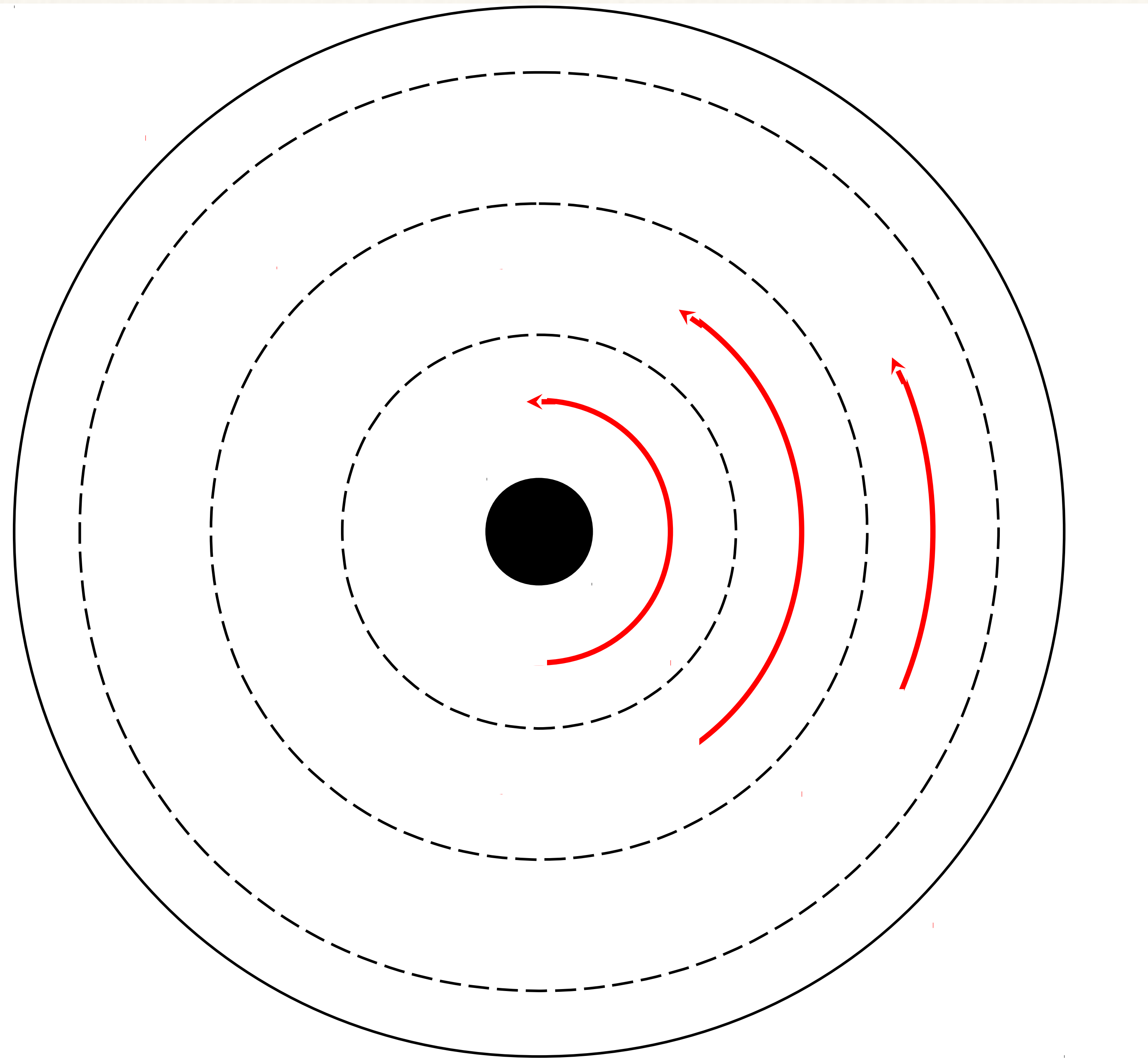
Basic Physics



(credit: G. Ogilvie)

- Gas settles down into differential rotation

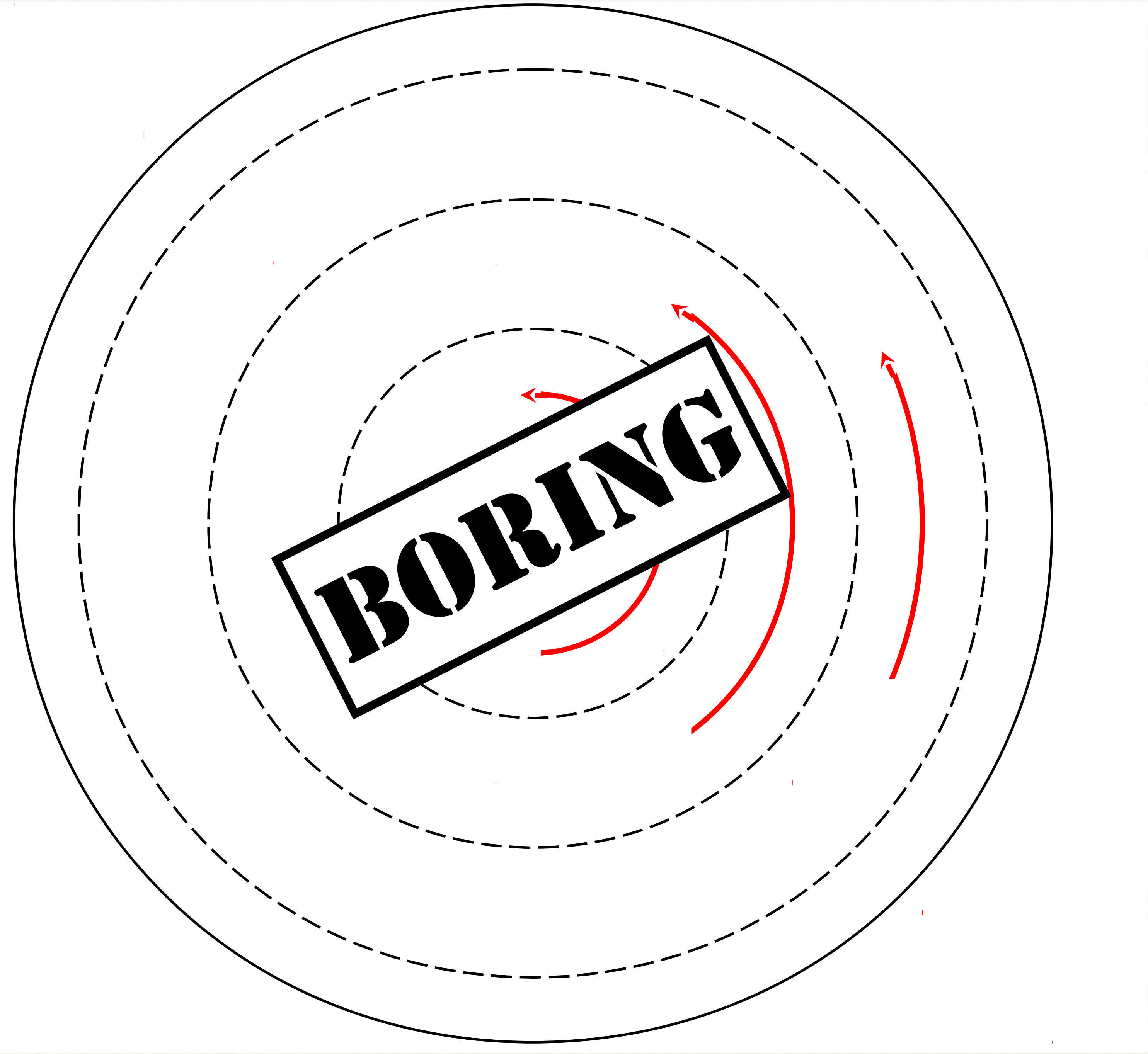
Basic Physics



(credit: G. Ogilvie)

- Gas settles down into differential rotation
- But if that were all, the gas will just orbit happily and nothing will happen:
  - No star formation
  - No black hole growth
  - No stunning luminosity
  - No jets

Basic Physics



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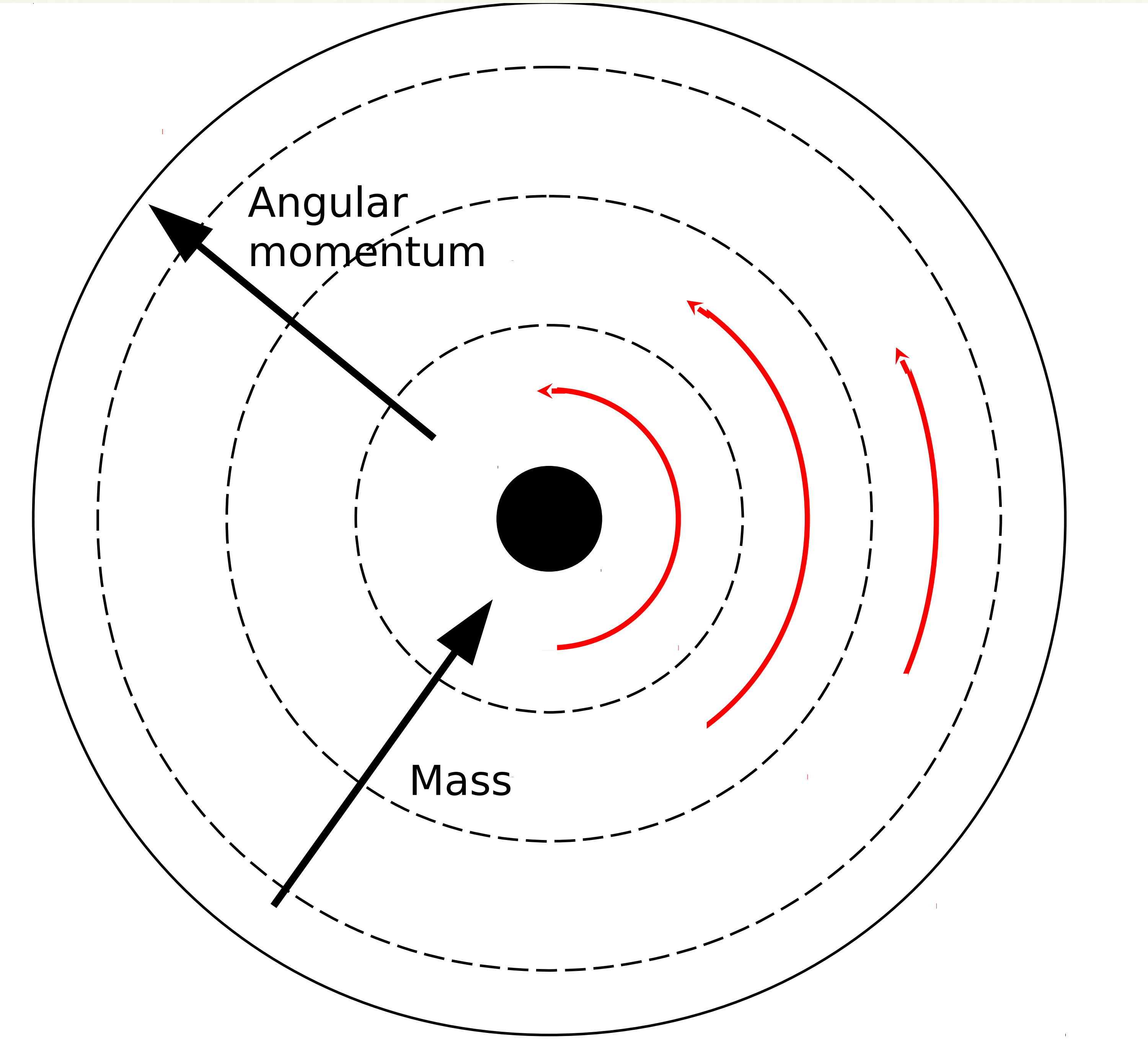
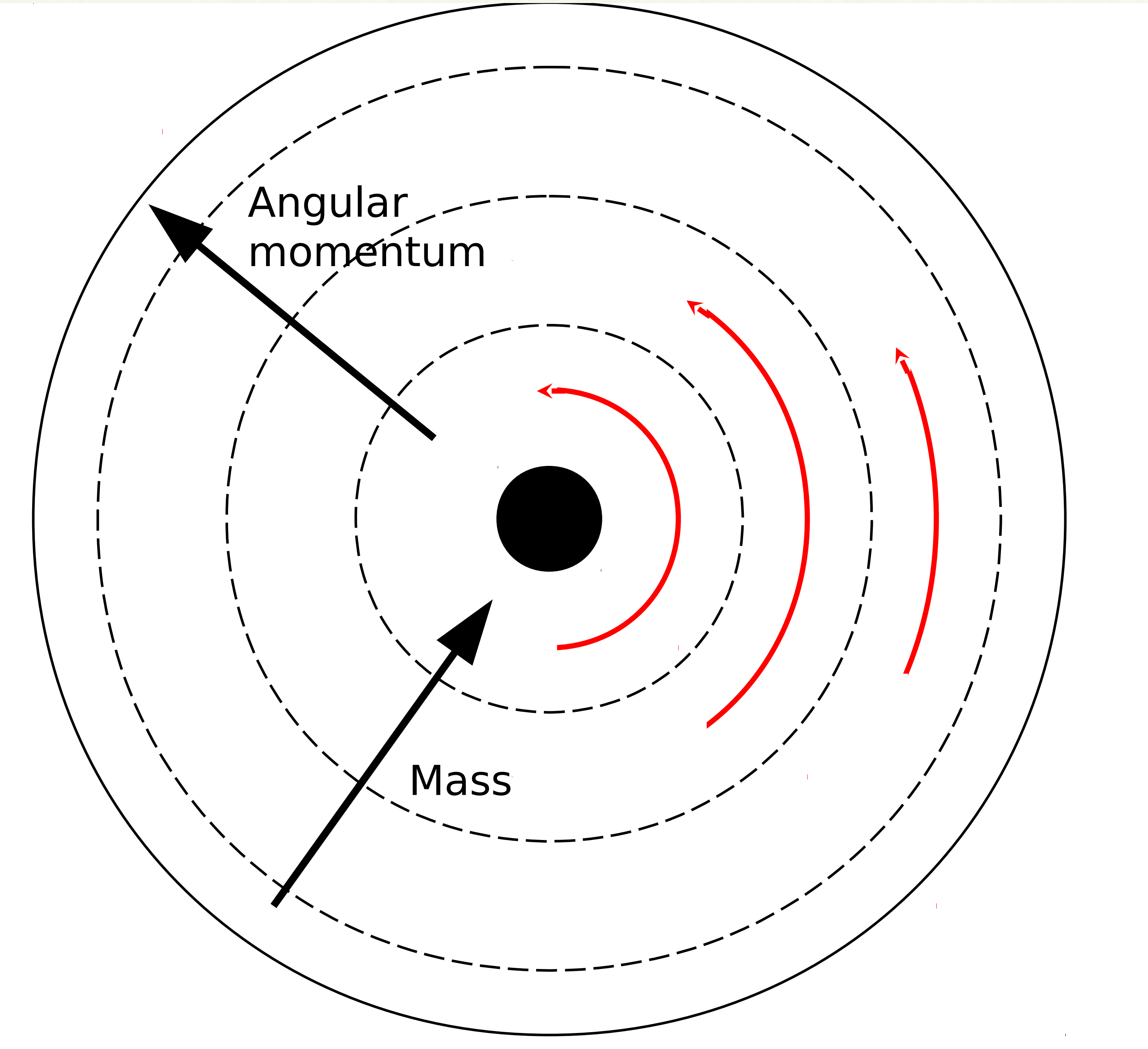


(credit: G. Ogilvie)

- We need **accretion**:

- This means there must be **angular moment transport**

Basic Physics: accretion



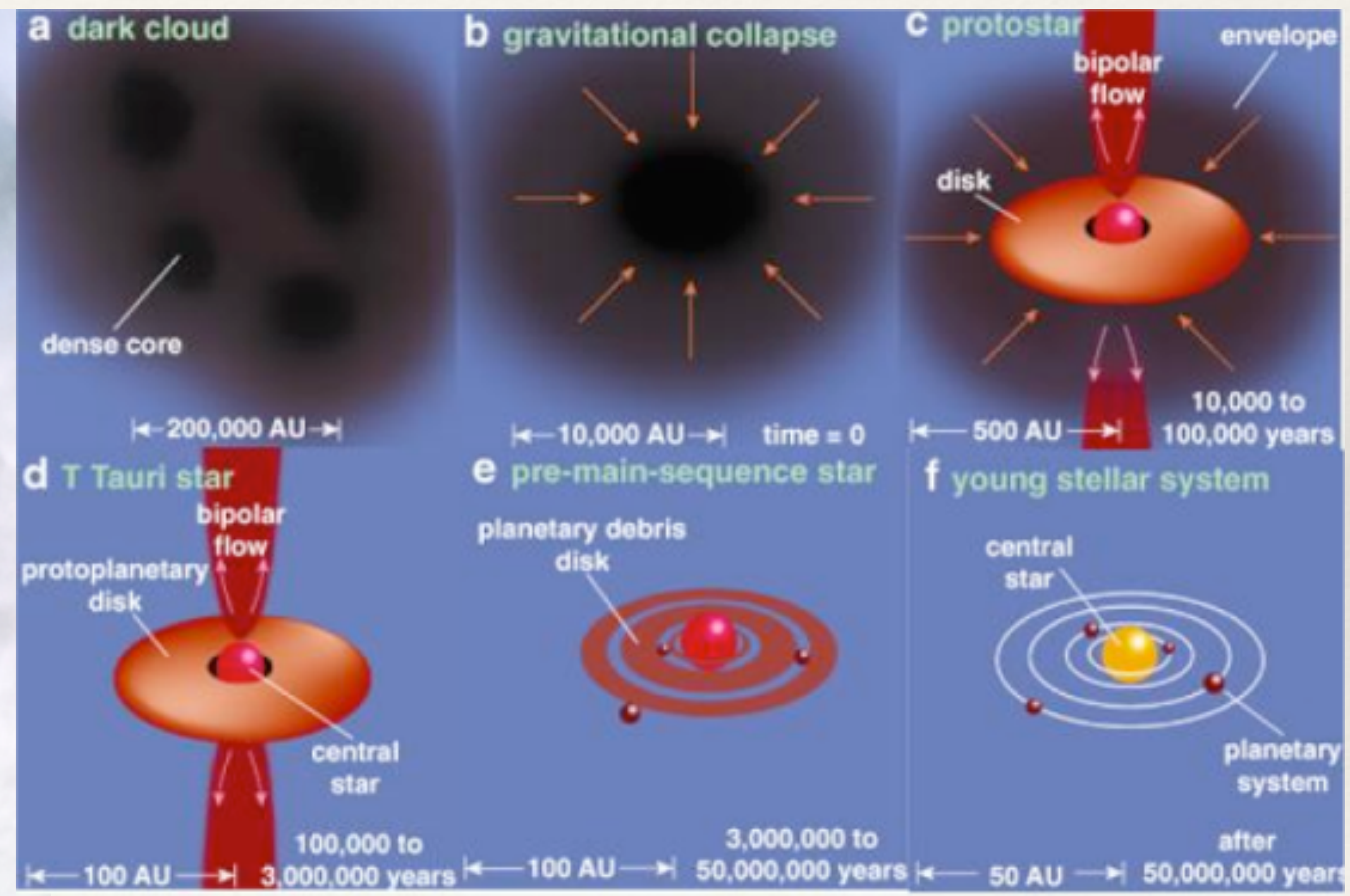
(credit: G. Ogilvie)

- We need **accretion**:
- This means there must be **angular moment transport**
- It also means that there must be **energy liberation**
- This energy goes first into heat and then possibly into light

Basic Physics: ang. momentum

- Consider the ‘angular momentum problem’ in star formation

The ‘Up Yours’ molecular cloud
in the Carina nebula *(Hubble)*



Cartoon of star formation
(Greene, American Scientist, Jul-Aug 2001)

Basic Physics: ang. momentum

- Consider the 'angular momentum problem' in star formation
- a dense core in the ISM collapses into a star.

$L_c = \text{ang. mom. of the core} \qquad L_s = \text{ang. mom. of the star}$

$$\frac{L_s}{L_c} = \left(\frac{R_s}{R_c}\right)^2 \left(\frac{\Omega_s}{\Omega_c}\right)$$

$R_s = \text{solar radius} \sim 10^6 \text{km}$

$R_c = \text{core radius} \sim 10^{13} \text{km}$

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$$\frac{L_s}{L_c} \sim 10^{-7}$$

A LOT of angular momentum has had to be redistributed for the star to form!

Basic Physics: energy

- Accretion of mass can liberate a lot of energy.

(Science fictional rendering of a luminous quasar: ESO, M. Kornmesser)

Basic Physics: energy

- Accretion of mass can liberate a lot of energy.
- Consider accretion of a fluid blob of mass $\boldsymbol{m}$ from infinity on to a massive body of mass $\boldsymbol{M}$ and radius $\boldsymbol{R}$:

$$E_{\rm acc} = \text{accretion energy} \sim \frac{mMG}{R} = \left(\frac{R_{\rm grav}}{R}\right) mc^2$$

$$R_{\rm grav} \sim \frac{GM}{c^2} \sim 10 \text{ km (for stellar masses)}$$

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- **Explains why accreting neutron stars and black holes are so luminous**

Basic Physics: accretion

- Okay, but what process is doing this angular momentum transport?
  - a frictional force, like viscosity?
  - magnetic fields…?

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- How about molecular viscosity?
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$$\nu \sim 10^4\ \mathrm{m^2 s^{-1}} \rightarrow \text{accretion timescale} \sim R_{\mathrm{disk}}^2/\nu \sim 10^{14}\ \text{years}$$

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$$\nu_{\text{turb}} \sim v_{\text{turb}} l_{\text{turb}} < c_s\, H \qquad \text{(where } H \text{ is disk thickness)}$$

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- So this gives us the "**alpha prescription**" (Shakura and Sunyaev 1973)

$$\nu_{\text{turb}} = \alpha c_s H$$

Observations indicate that alpha lies between 10^-4 and 0.1

Basic Physics: accretion

- So now the questions are:
    - what causes turbulence in accretion disks?
    - why does alpha take the value it does in different scenarios?

Instabilities

Instabilities: hydro

- Consider a very “vanilla” model of an accretion disk:
  - no thermodynamic gradients
  - incompressible
  - hydrodynamic

Instabilities: hydro

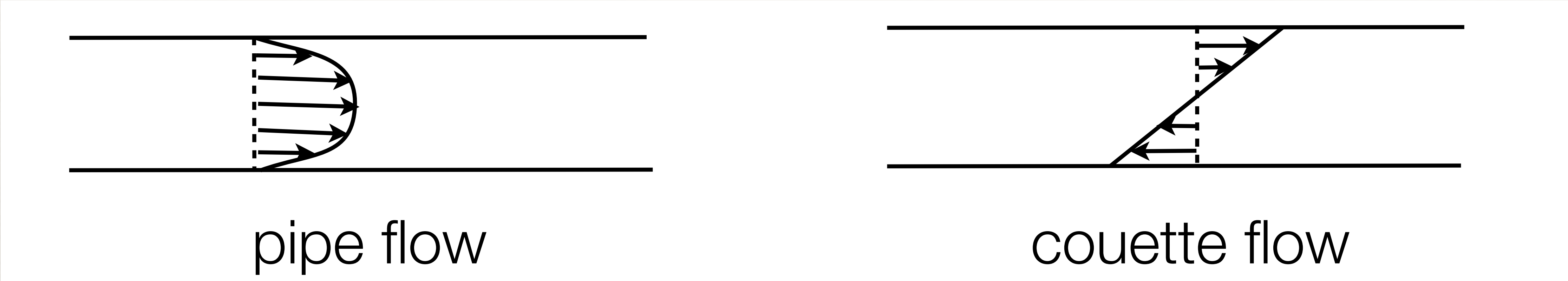
- Consider a very “vanilla” model of an accretion disk:
    - no thermodynamic gradients
    - incompressible
    - hydrodynamic
- Linear stability controlled by Rayleigh’s criterion:

$$\frac{d\left(\Omega R^2\right)}{dR} < 0$$

- Disks have rotation profiles that are (linearly) **STABLE**

Instabilities: hydro

- What about 'nonlinear instability', as in pipe and plane Couette flow?
- Reynolds numbers in disks ~ 10^10



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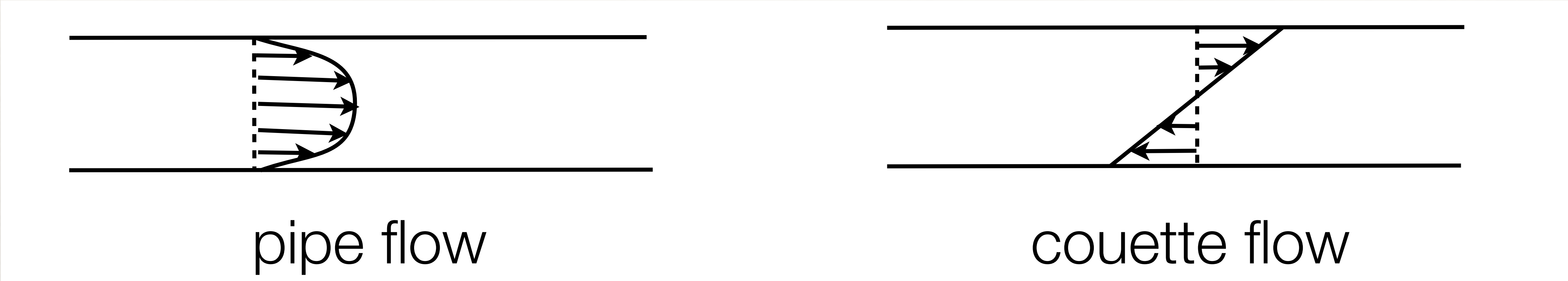

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- Also, phenomenology of non-rotating shear flow transition does not carry over to Keplerian type rotating flows (Rincon + 2007)

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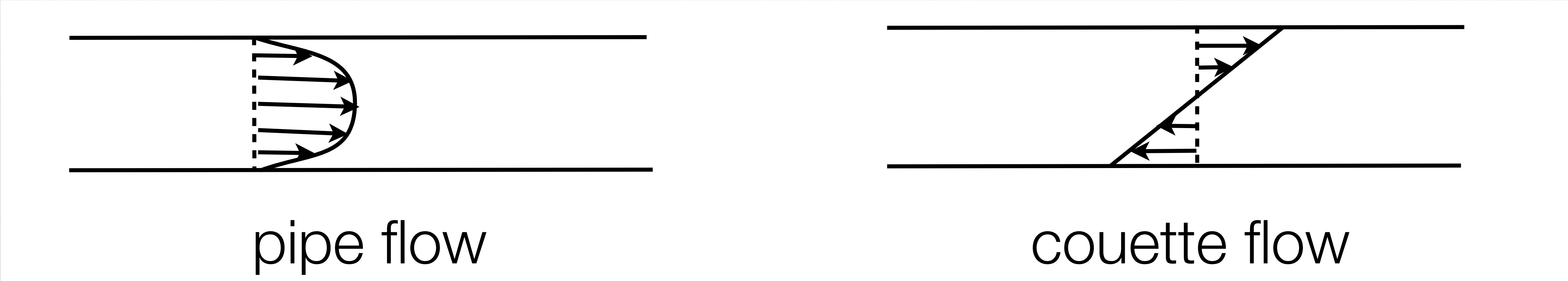

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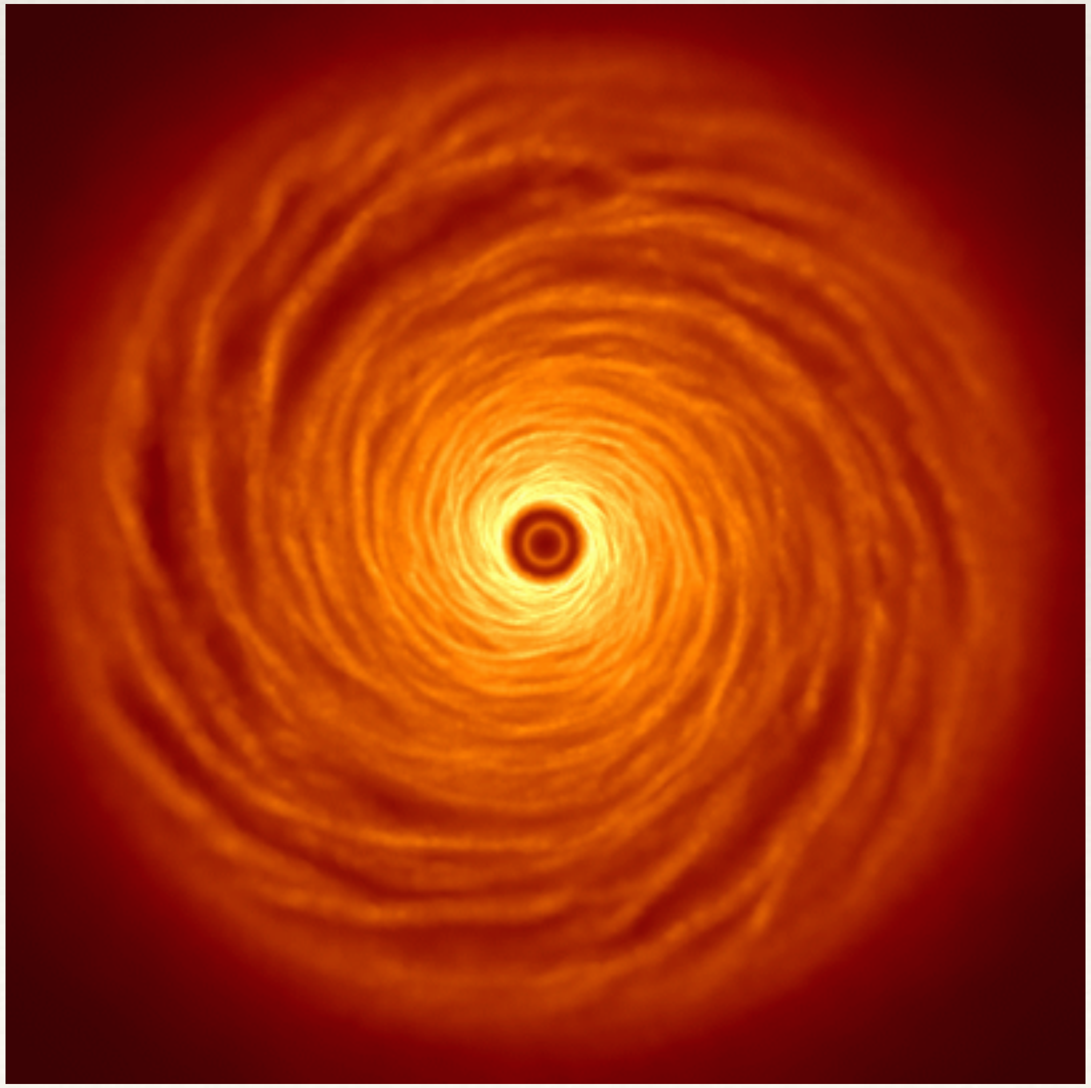
Astrophysical differential rotation is rather special

Instabilities: hydro

- But there other hydro instabilities:

Instabilities: hydro

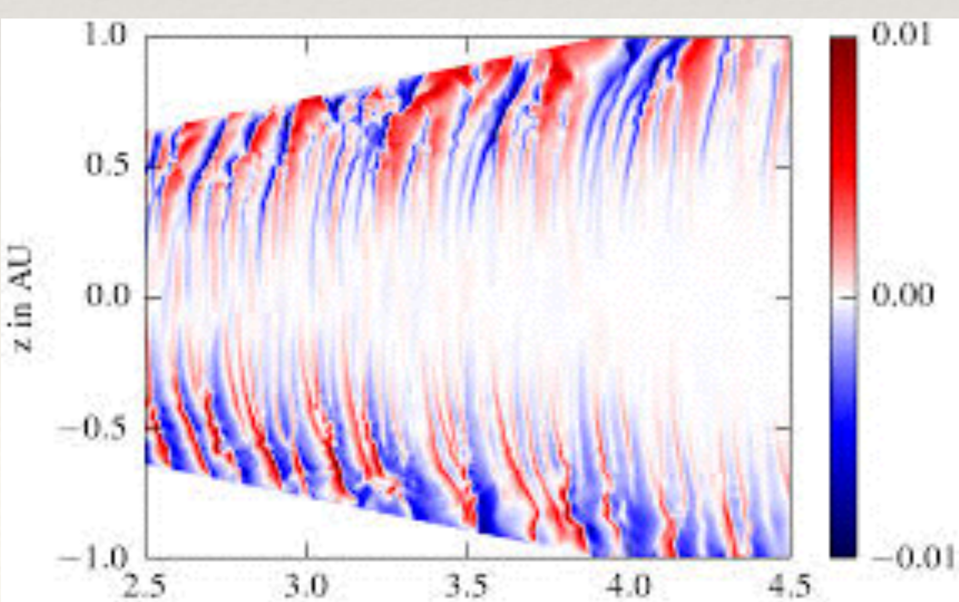
- But there other hydro instabilities:
- Gravitational instability
  - Toomre Q = Omega*c_s/(pi*G*M) < 1 (Toomre 1964)
  - Only attacks cold and massive disks (e.g. young PP disks)



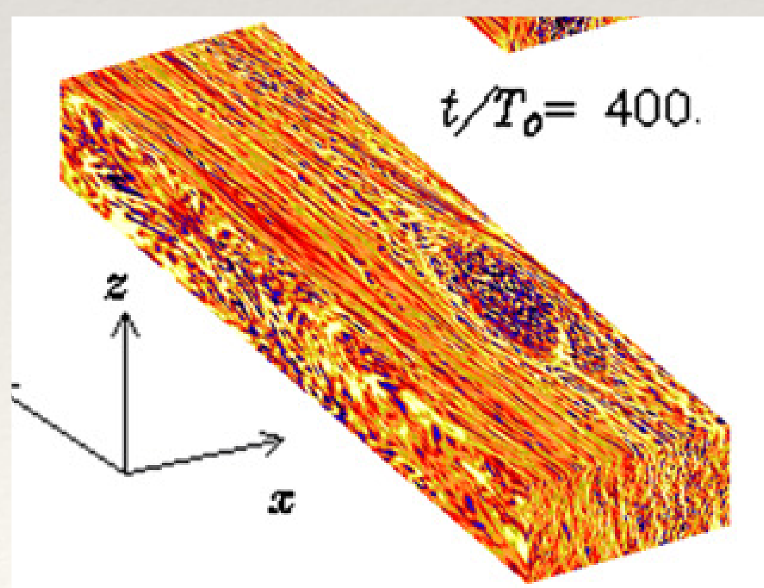
[Credit: Ken Rice]

Instabilities: hydro

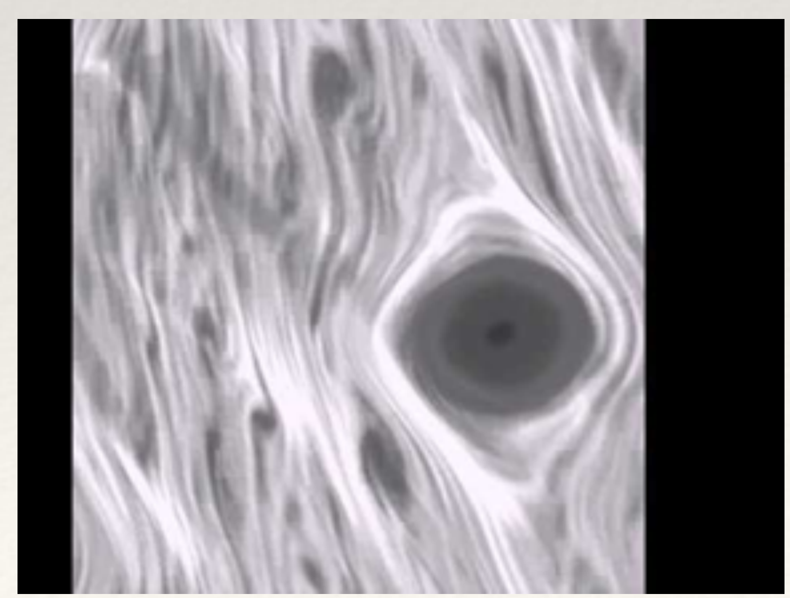
- But there other hydro instabilities:
- Gravitational instability
    - Q = Omega*c_s/(pi*G*M) < 1
    - Only attacks cold and massive disks (e.g. young PP disks)
- Various double-diffusive instabilities:
    - "Vertical shear instability" (GSF instability) (Nelson+ 2013, Latter+Papaloizou 2018)
    - "Convective overstability" (semi-convection) (Lyra 2014, Latter 2016)
    - Subcritical baroclinic instability (Lesur+Papaloizou 2010)



(Stoll & Kley 2014)



(Lyra 2014)


(Credit: Wlad Lyra)

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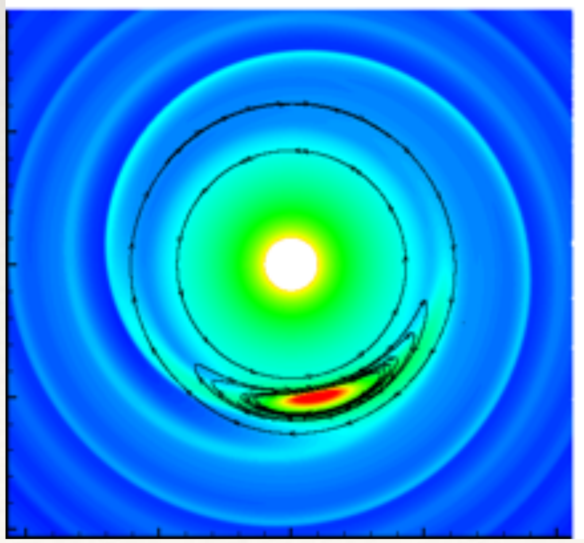
- "Rossby wave instability" (Lovelace+ 1999)
    - Requires a strong disk inhomogeneity (gap or pressure bump)



(Lovelace and Romanova 2014)

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    - Needs reflecting inner boundary
    - Exacerbated by GR

(Papalozou+Pringle 1984, Goodman+ 1985, Narayan+1987, Lai+Tsang 2009)

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- None sufficiently general, strong, efficient, etc.

Instabilities:MHD

- Okay, so what about magnetic fields?

Instabilities:MHD

- Okay, so what about magnetic fields?

- Um, things are really different. We get a strong linear instability when:

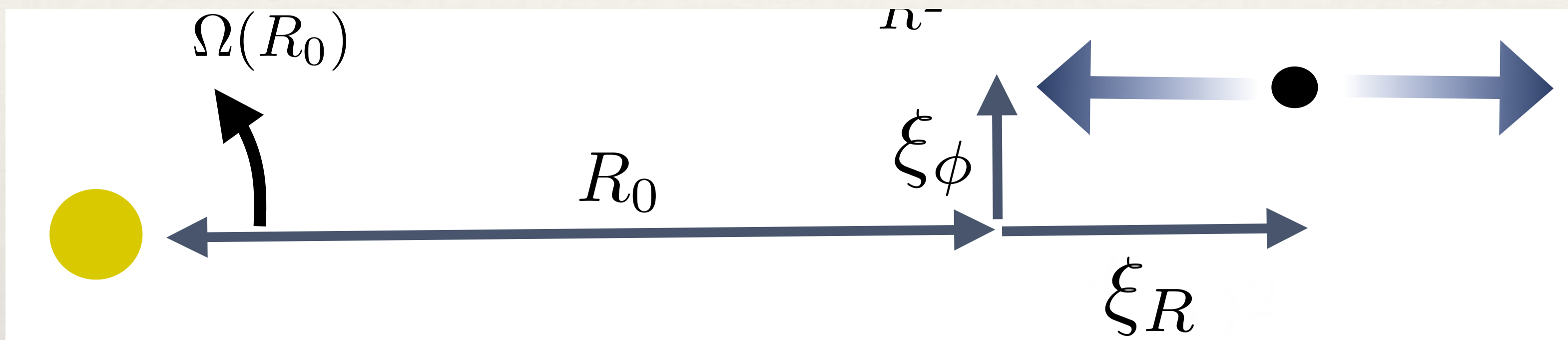
$$\frac{d\Omega}{dR} < 0$$

(compare with Rayleigh criterion)

- This is the magneto rotational instability (MRI) (Balbus & Hawley 1991)

Instabilities:MHD

- Okay, so what about magnetic fields?
- How does this work? Consider planar Lagrangian perturbations



Effective (tidal) radial acceleration: $-\xi_R \dfrac{d\Omega^2}{d\ln R}\boldsymbol{e_R}$

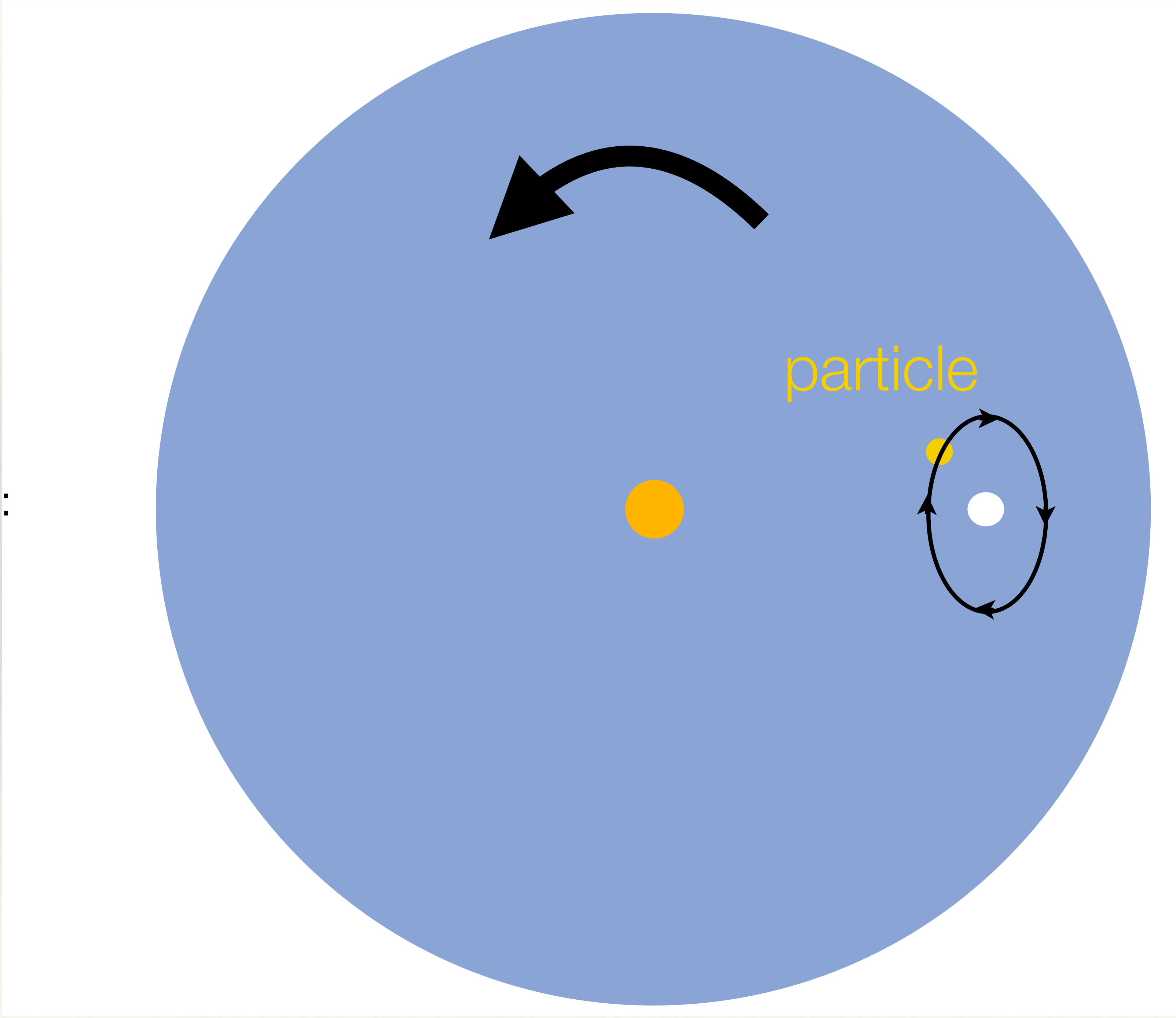
Resulting equation of motion for a fluid particle:

$$\begin{aligned} \ddot{\xi}_R - 2\Omega\dot{\xi}_\phi &= -\frac{d\Omega^2}{d\ln R}\xi_R \\ \ddot{\xi}_\phi + 2\Omega\dot{\xi}_R &= 0 \end{aligned}$$

$\Rightarrow$ Epicyclic oscillations at frequency $\kappa = \left(4\Omega^2 + \dfrac{d\Omega^2}{d\ln R}\right)^{1/2}$

(Credit: G Lesur)

Instabilities: MHD



Epicyclic circulations in co-rotating frame

(Credit: G Lesur)

Instabilities:MHD

Induction equation for a small displacement $\boldsymbol{\xi}$ and a spatial dependence $\propto \exp(ikz)$:

$$\delta \boldsymbol{B} = i(\boldsymbol{k} \cdot \boldsymbol{B}_{z_0})\boldsymbol{\xi}$$

The magnetic tension force is

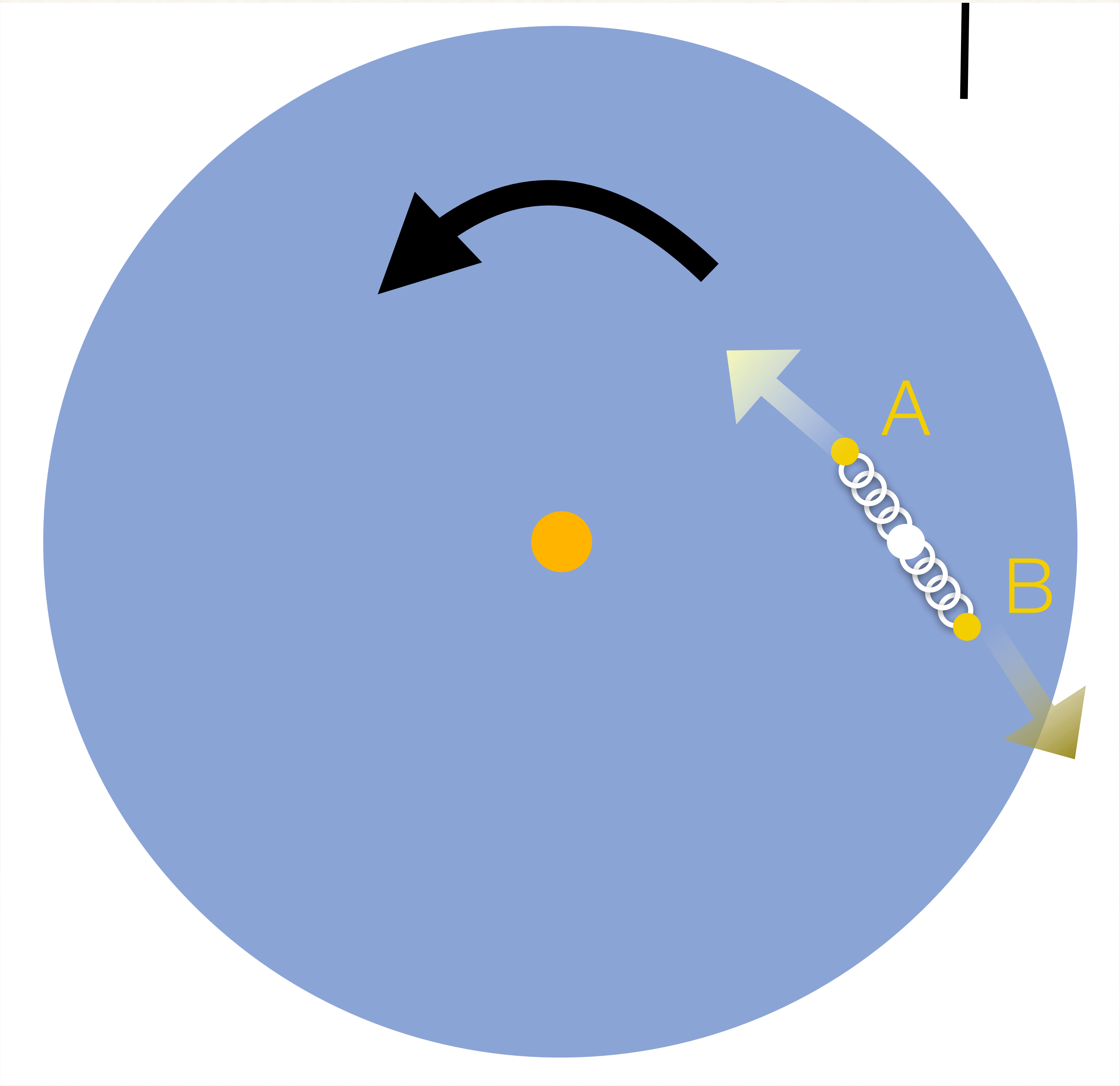
$$\frac{\boldsymbol{B}_{z_0} \cdot \boldsymbol{\nabla} \boldsymbol{B}}{\rho} = \frac{i(\boldsymbol{k} \cdot \boldsymbol{B}_{z_0})}{\rho} \delta \boldsymbol{B} = -(\boldsymbol{k} \cdot \boldsymbol{v_A})^2 \boldsymbol{\xi}$$

Resulting equation of motion for a fluid particle:

$$\begin{aligned} \ddot{\xi}_R - 2\Omega \dot{\xi}_\phi &= -\left(\frac{d\Omega^2}{d \ln R} + (\boldsymbol{k} \cdot \boldsymbol{v_A})^2\right)\xi_R \\ \ddot{\xi}_\phi + 2\Omega \dot{\xi}_R &= -(\boldsymbol{k} \cdot \boldsymbol{v_A})^2 \xi_\phi \end{aligned}$$

(Credit: G Lesur)

Instabilities:MHD



Forces on the right hand sides act a bit like a spring!

(Credit: G Lesur)

Instabilities: the MRI

Introduce: $\boldsymbol{\xi} = \boldsymbol{\xi}_0 \exp(i\omega t)$ → Dispersion relation

$$\omega^4 - \omega^2[\kappa^2 + 2(\boldsymbol{k}\cdot\boldsymbol{v_A})^2] + (\boldsymbol{k}\cdot\boldsymbol{v_A})^2\left[\underbrace{(\boldsymbol{k}\cdot\boldsymbol{v_A})^2}_{\text{Stabilizing}} + \underbrace{\frac{d\Omega^2}{d\ln R}}_{\text{Destabilizing}}\right] = 0$$

Instability when: $(\boldsymbol{k}\cdot\boldsymbol{v_A})^2 < -\dfrac{d\Omega^2}{d\ln R}$

So the MRI is a 'weak field' instability

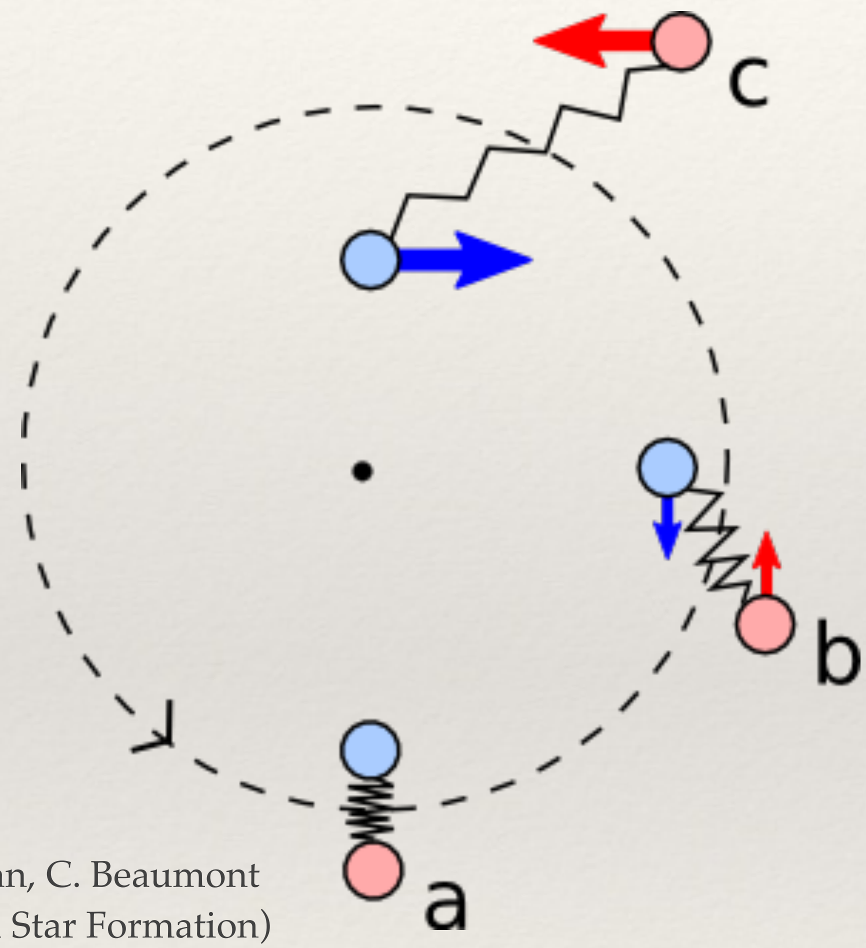
Fastest growth rate: $\Im(\omega) = \dfrac{1}{2}\dfrac{d\Omega}{d\ln R}$

Very fast!

(Credit: G Lesur)

Instabilities: the MRI

- Instability mechanism something like:



Credit: A. Goodman, C. Beaumont
(course on ISM and Star Formation)

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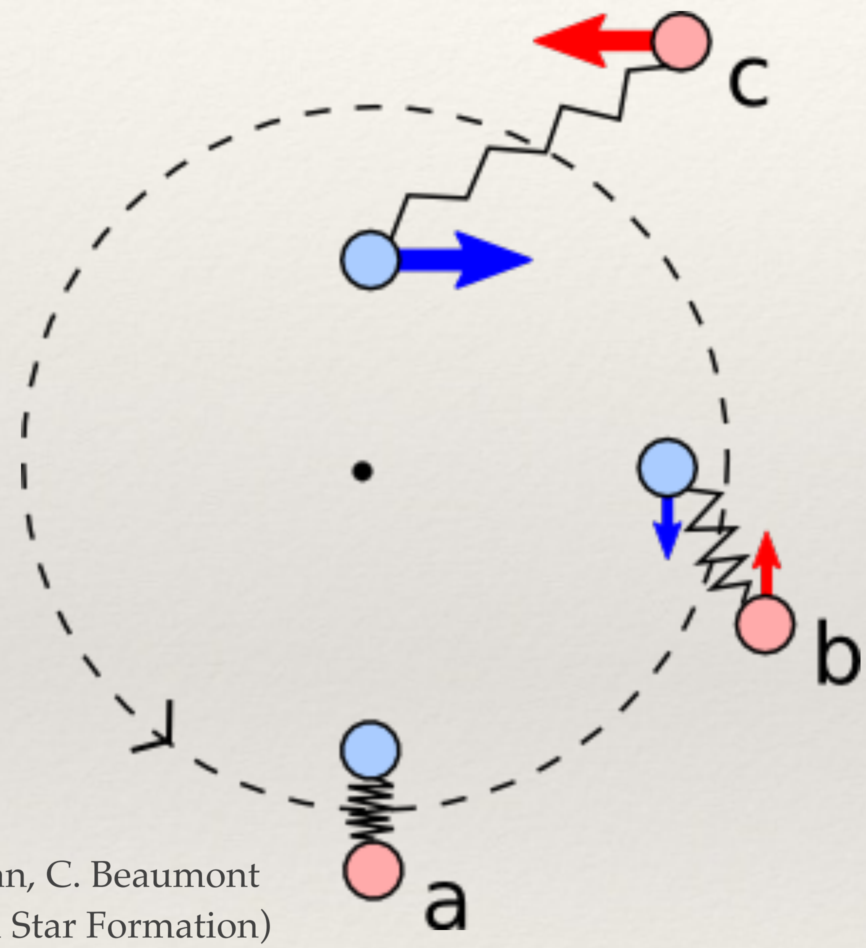

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Credit: spaghetti dude

Instabilities: the MRI

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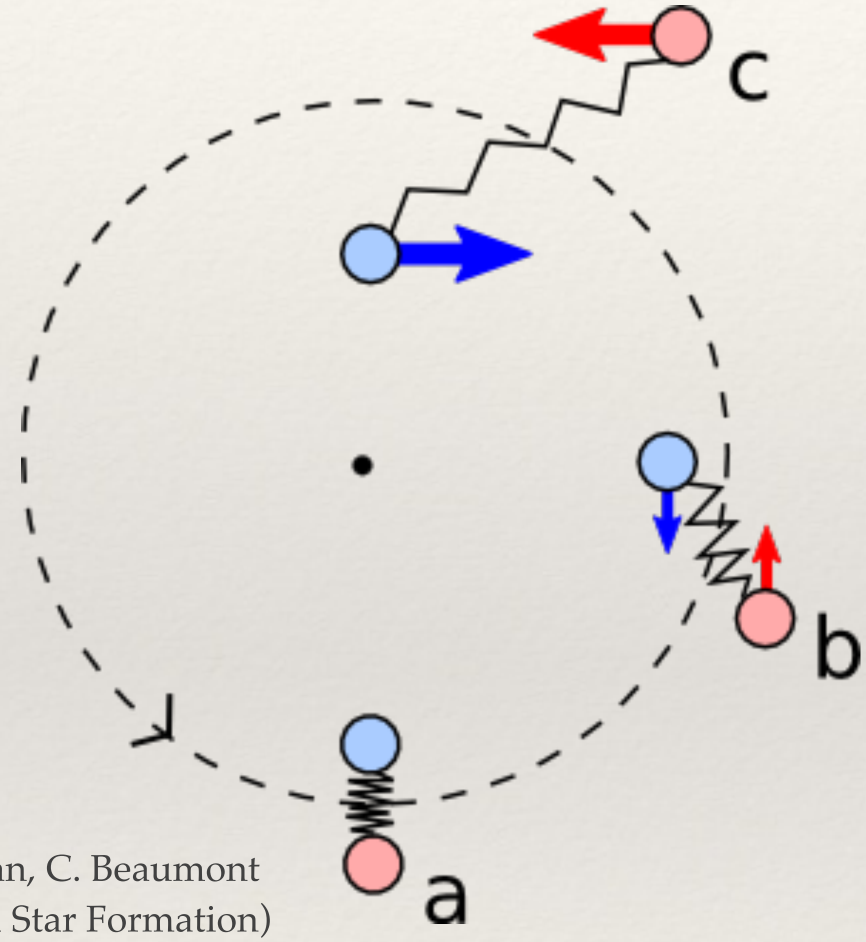

Credit: A. Goodman, C. Beaumont
(course on ISM and Star Formation)

Credit: spaghetti dude

- In more realistic disk models, some additional criteria:

$$\beta \gtrsim 1 \qquad \mathrm{Rm} \gtrsim \beta \quad \Lambda_{AD} > 1$$

Magnetohydrodynamic Turbulence

MHD turbulence

- Okay, so the disk goes MRI unstable; what happens next?

MHD turbulence

- Okay, so the disk goes MRI unstable; what happens next?
- Really, we need to perform numerical simulations to find out
- They come in various flavours:

MHD turbulence

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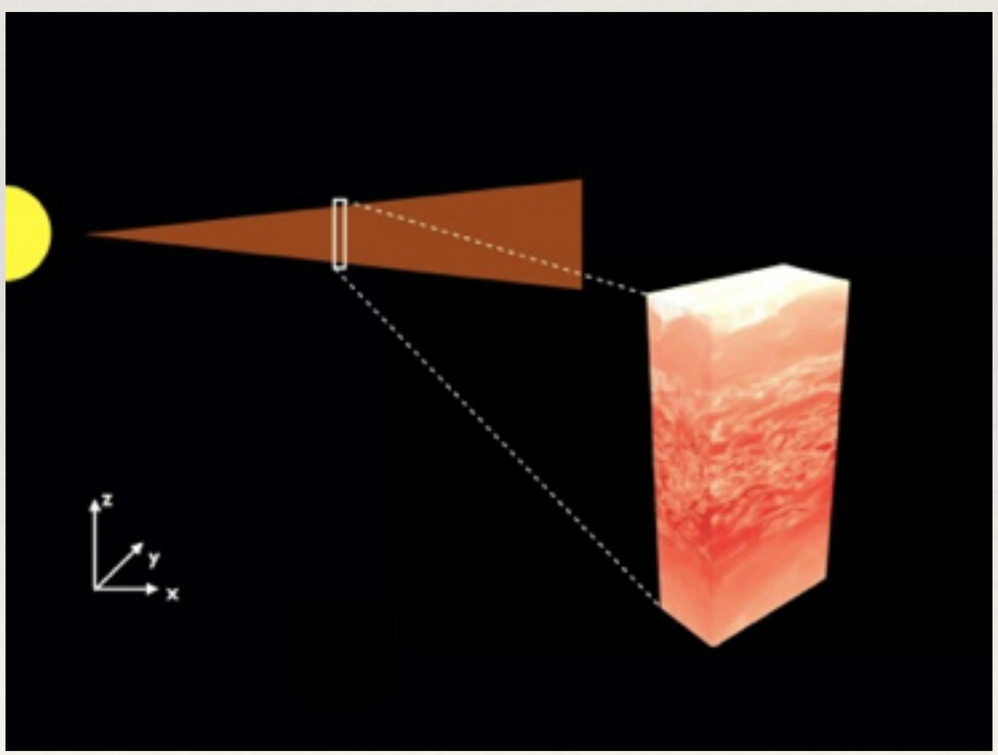
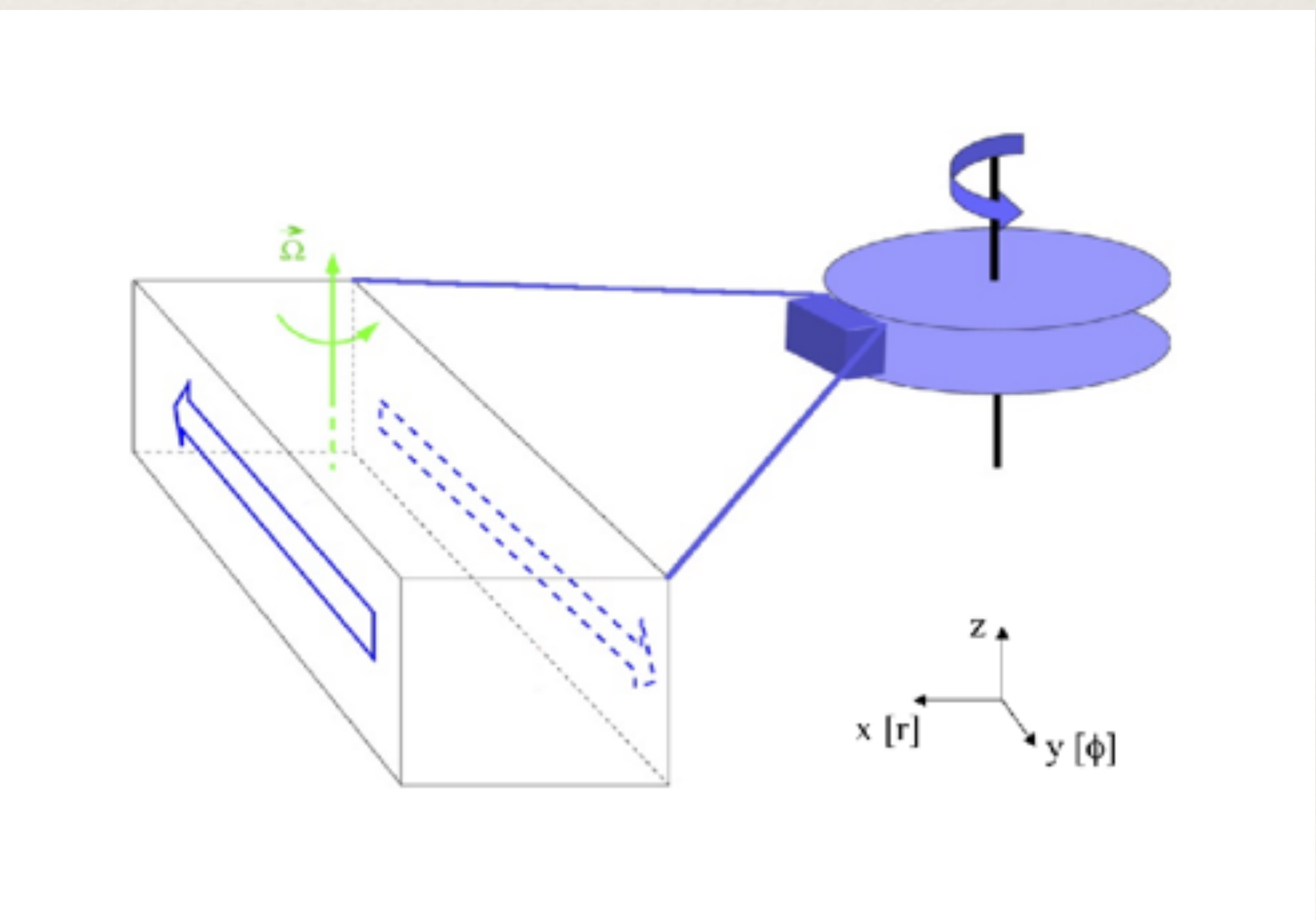
- Really, we need to perform numerical simulations to find out
- They come in various flavours:

- **<u>Local (shearing box) simulations</u>**



(credit: Jake Simon)



(credit: Jeff Lesur)

MHD turbulence

MHD in the shearing box

$$\partial_t \rho + \mathbf{u}\cdot\nabla\rho = -\rho\nabla\cdot\mathbf{u}$$

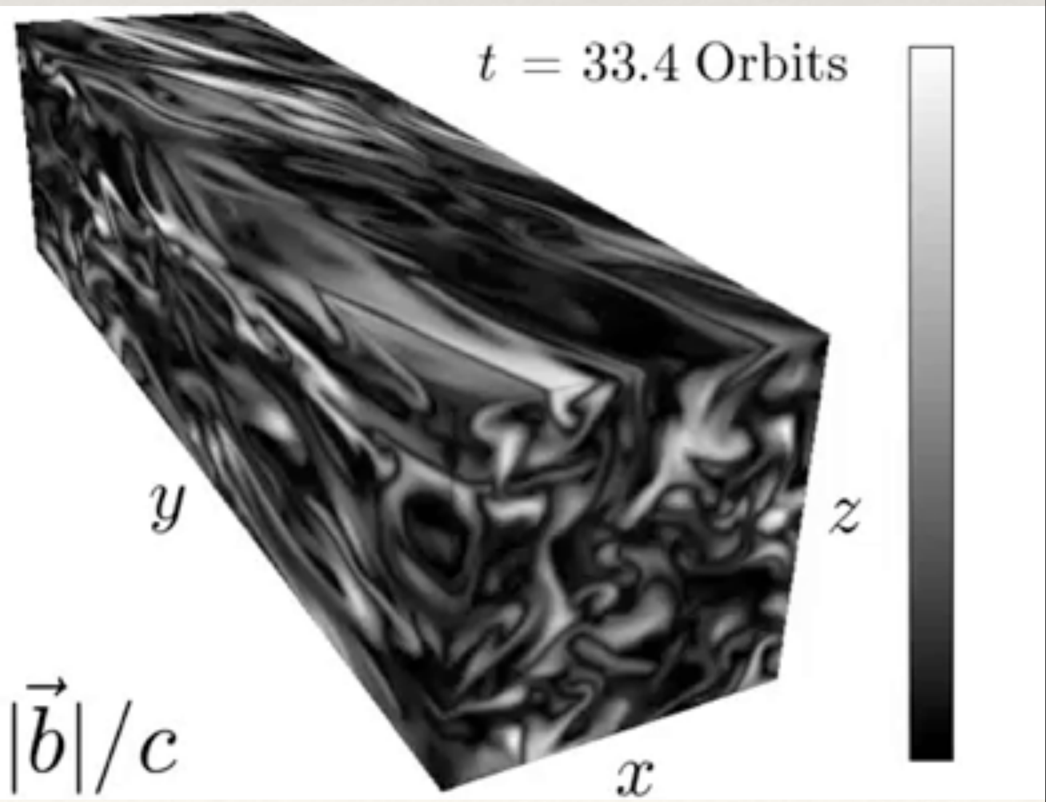
$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -2\boldsymbol{\Omega}\times\mathbf{u} - \frac{1}{\rho}\nabla P - \nabla\Phi_T - \nabla\Phi_D + \frac{1}{4\pi\rho}(\nabla\times\mathbf{B})\times\mathbf{B} + \nabla\cdot\mathcal{T}$$

$$\partial_t \mathbf{B} + \mathbf{u}\cdot\nabla\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{u} - \mathbf{B}\nabla\cdot\mathbf{u} + \eta\nabla^2\mathbf{B}$$

$$\partial_t \varepsilon + \mathbf{u}\cdot\nabla\varepsilon = -P\nabla\cdot\mathbf{u} + Q - \Lambda$$

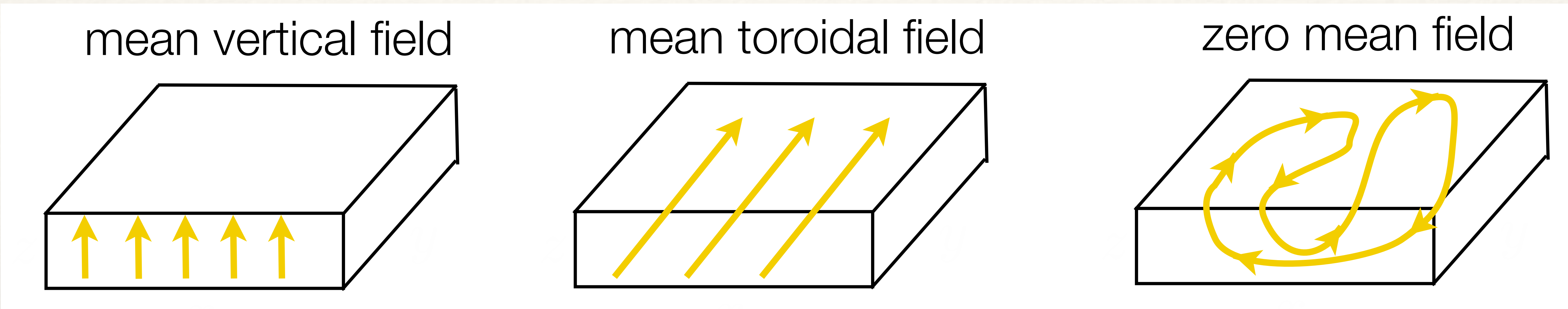
$$\nabla\Phi_D = 4\pi G\rho$$

$$\Phi_T = \tfrac{1}{2}\Omega^2 z^2 - \tfrac{3}{2}\Omega^2 x^2$$



(credit: T. Heinemann)

MHD turbulence

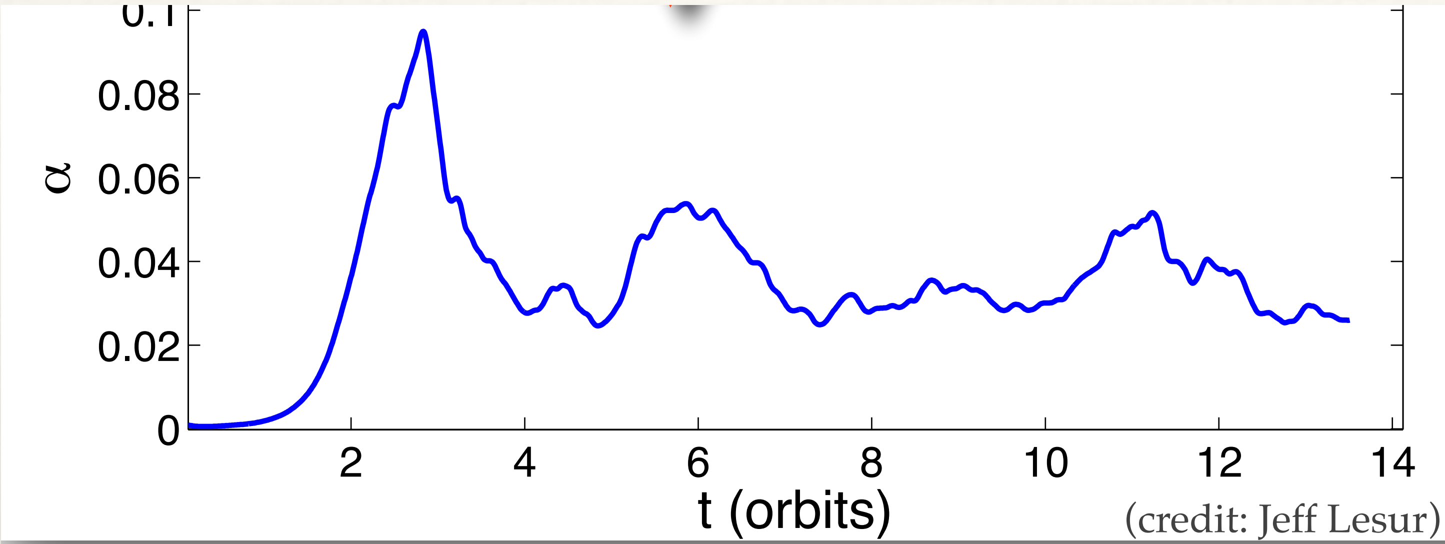


(credit: Jeff Lesur)

- Vertical case: linear channel modes are exact solutions. Bursty.
- Toroidal case: no linear axisym. instability; nonlinear non-axi instability
- Zero mean: bears similarities to small-scale turbulent dynamo
- Zero mean: turbulence dies out if magnetic Prandtl number below ~<2

MHD turbulence

- Generally:

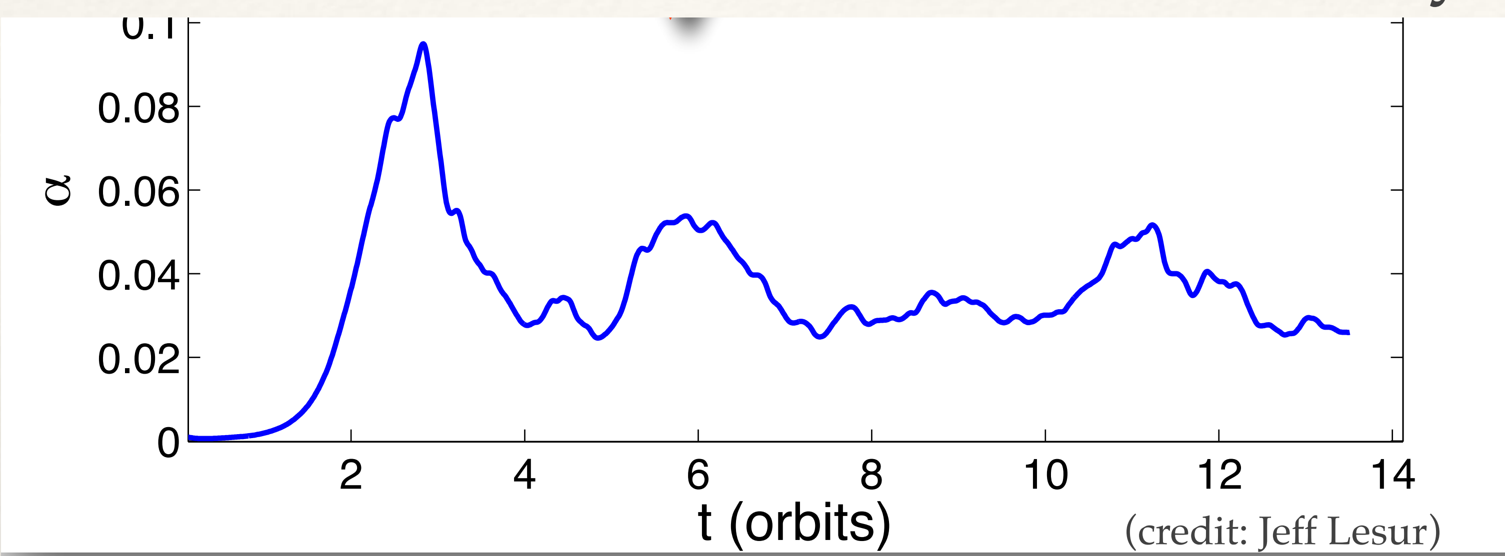


(credit: Jeff Lesur)

$$\alpha = \frac{\overline{\rho v_x v_y - B_x B_y}}{\overline{\rho}\Omega^2 H^2}$$

MHD turbulence

- Generally:



(credit: Jeff Lesur)

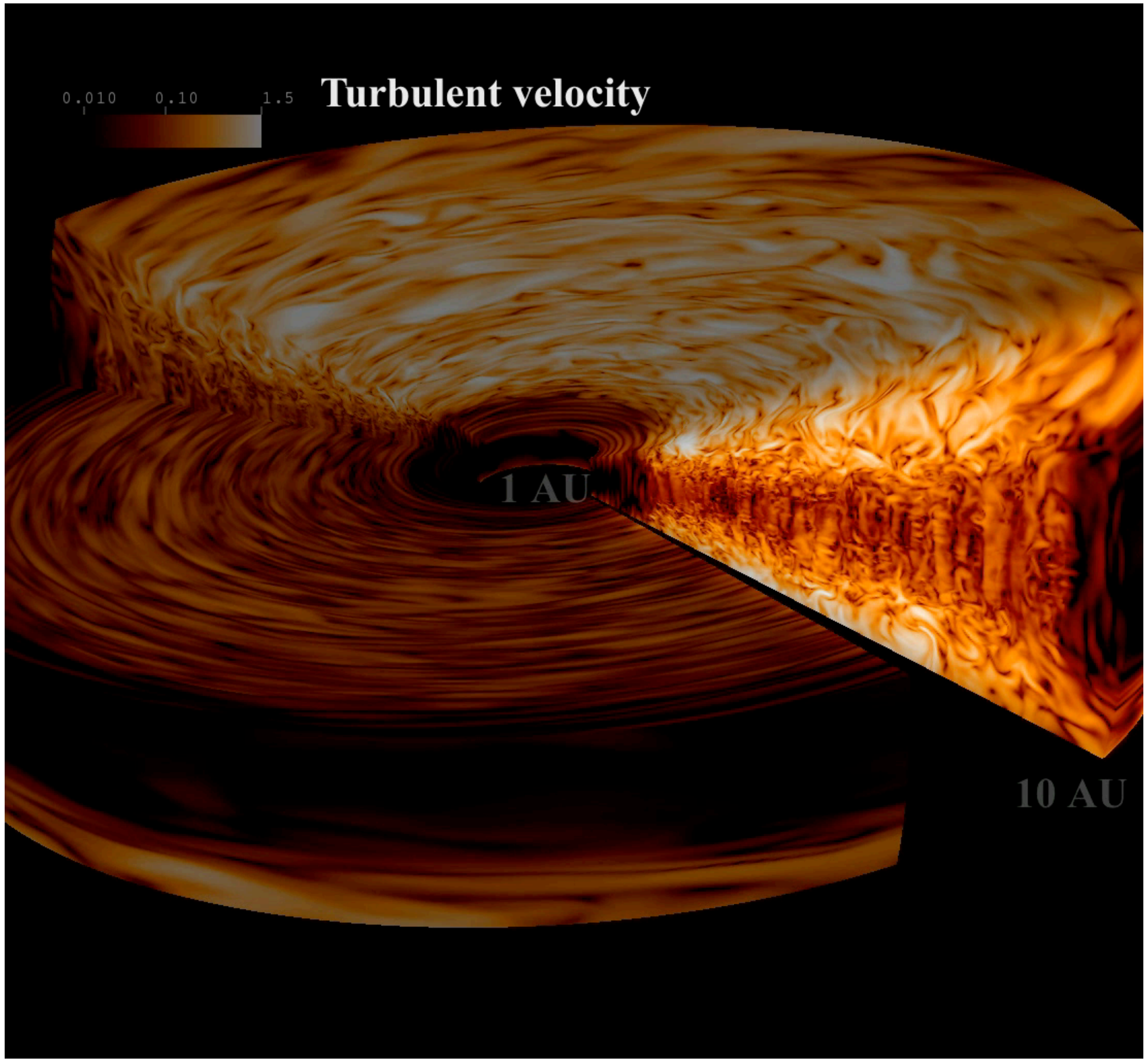
$$\alpha = \frac{\overline{\rho v_x v_y - B_x B_y}}{\overline{\rho}\Omega^2 H^2}$$

- Some issues:
  - alpha ~ 0.01 but observations require somewhat larger values
  - Zero mean case dies when Pm low. But that is precisely the case in most astrophysics
  - Sims are expensive: don't have great separation of scales, barely an inertial range:
    - overlapping injection scales, strong turbulence range, reconnection current sheet range
    - can barely reach the small scales where rotation and shear 'drop out'

MHD turbulence

<u>**Global sims**</u>

- diversity of scales between radius and thickness, makes thin disk very hard to simulate
- Resolution constraints mean turbulence doesn't 'exist' (i.e. injection scale too close to grid scale)

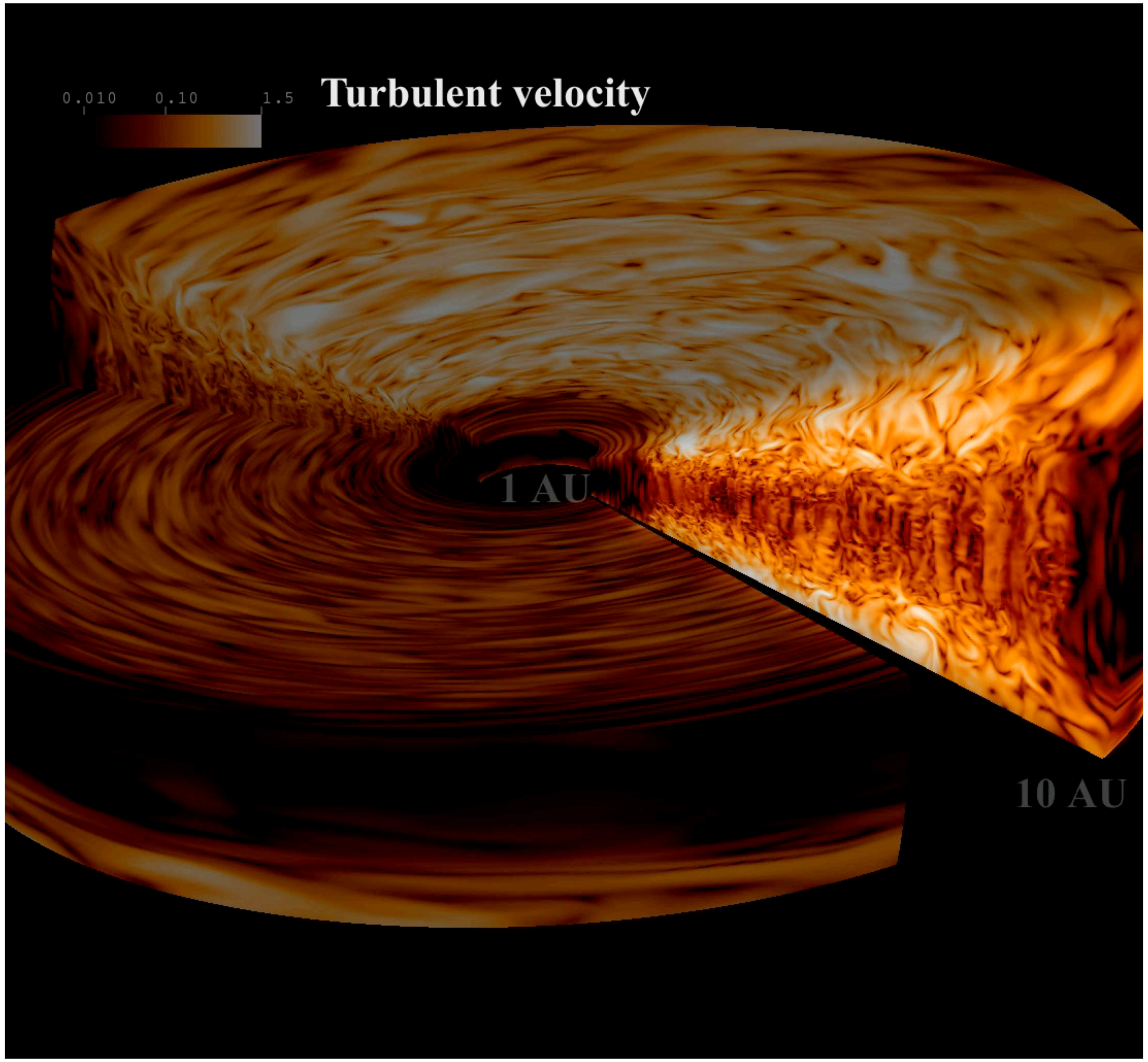


[Flock+ 2011]

MHD turbulence

Global sims

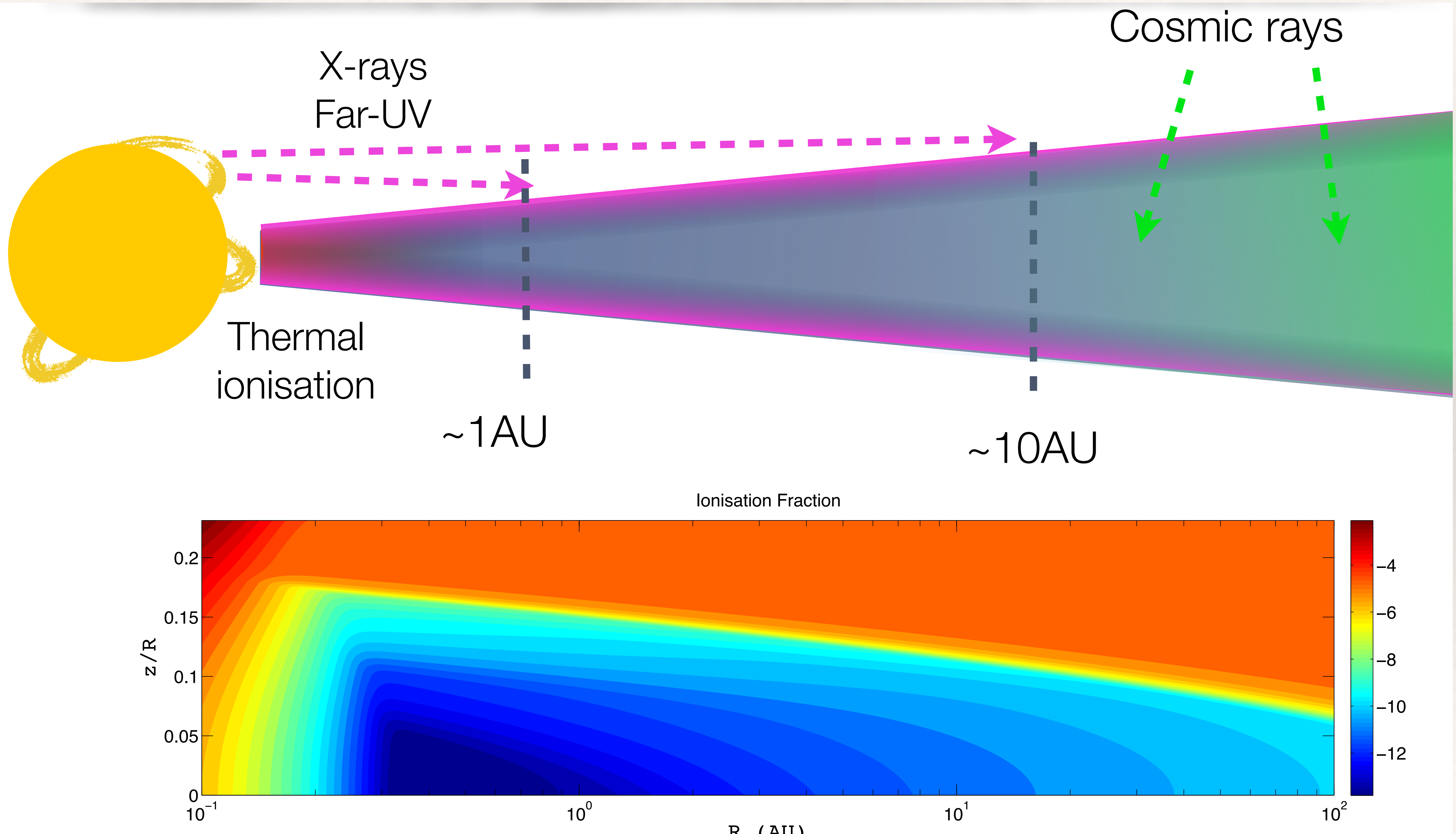
- diversity of scales between radius and thickness, makes thin disk very hard to simulate
- Resolution constraints mean turbulence doesn't 'exist' (i.e. injection scale too close to grid scale)
- alpha ~ 10^-3 - 10^-2
- but can describe global accretion flow; large-scale features, oscillations, jets, etc



[Flock+ 2011]

Beyond ideal MHD

- **protoplanetary disks**



(credit: Jeff Lesur)

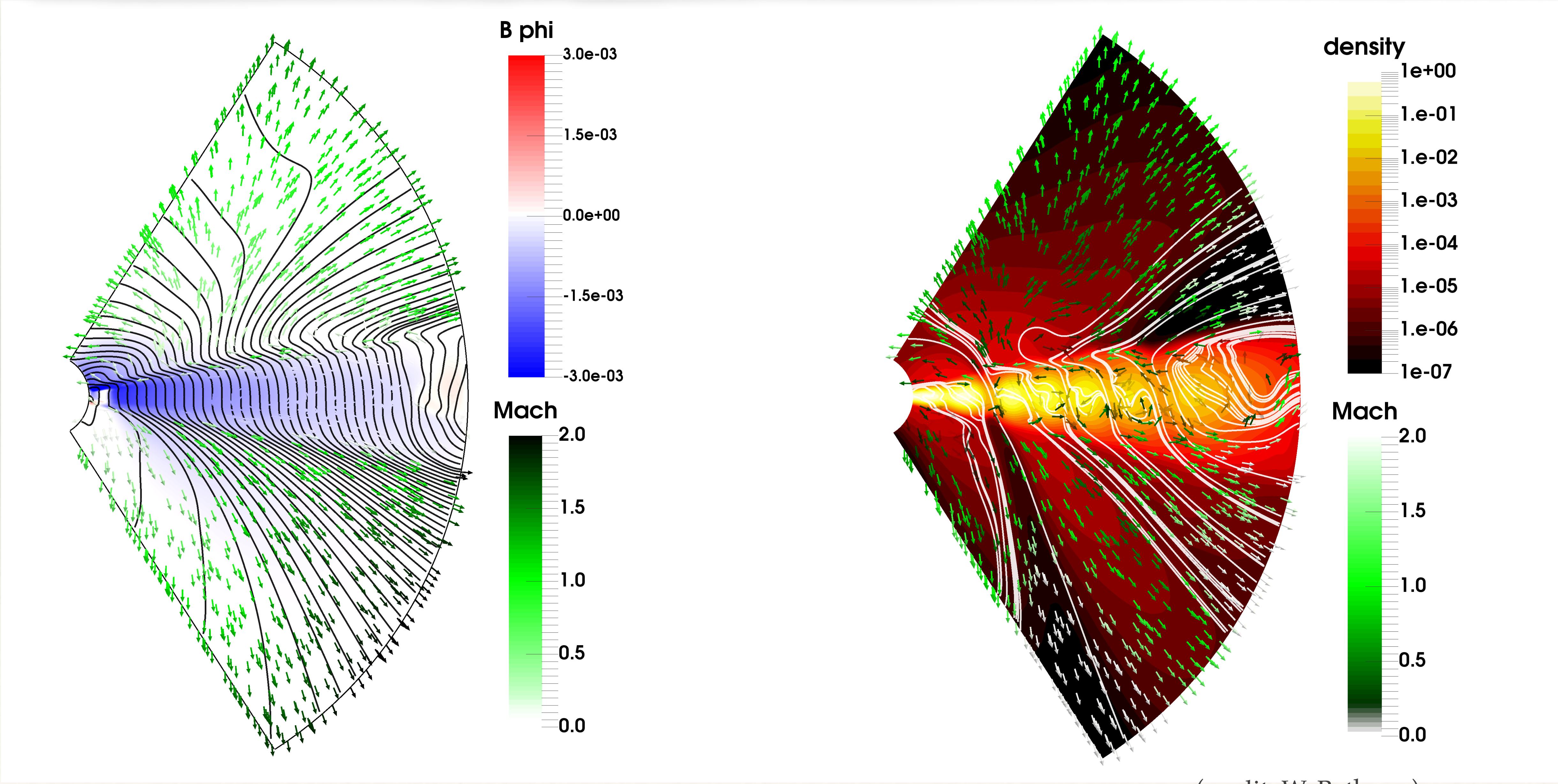
Beyond ideal MHD

- Three non-ideal MHD effects appear:
  - Ohmic resistivity (can be important at mid plane, at smaller radii)
  - Ambipolar diffusion (dominates upper layers)
  - Hall effect (dominates mid plane)

- The MRI is, in general, **KILLED** (except maybe in outer radii)
- because Am < 1, and Rm also too low…

Beyond ideal MHD

- *Instead* you get:
  - self-organisation near the mid plane (rings and zonal flows)
  - wind launching at the better ionised surface layers



(credit: W. Bethune)

Finally

- If you want to know more - do not hesitate to approach me for a chat
- I can also put together reading lists for you, depending on your interests