The Particle-In-Cell (PIC) simulation of plasmas

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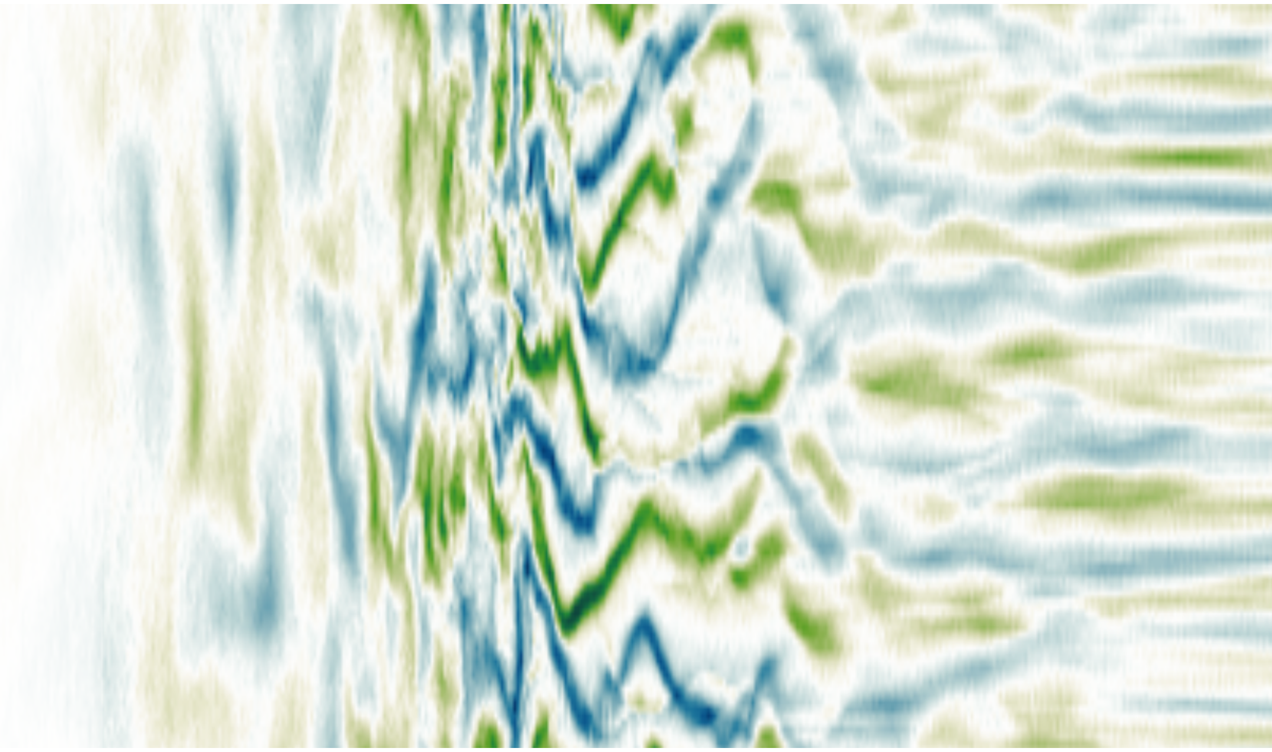

References

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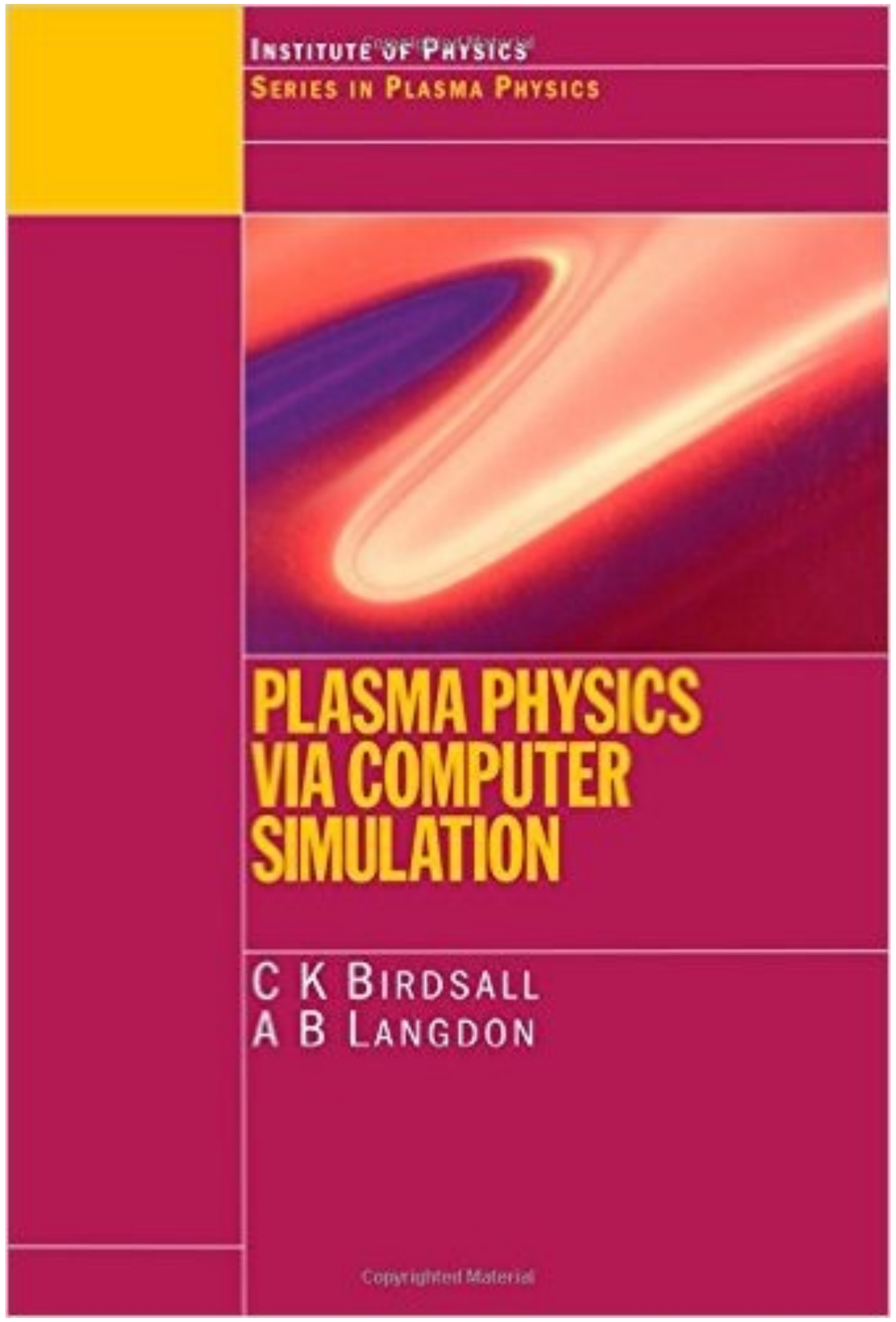

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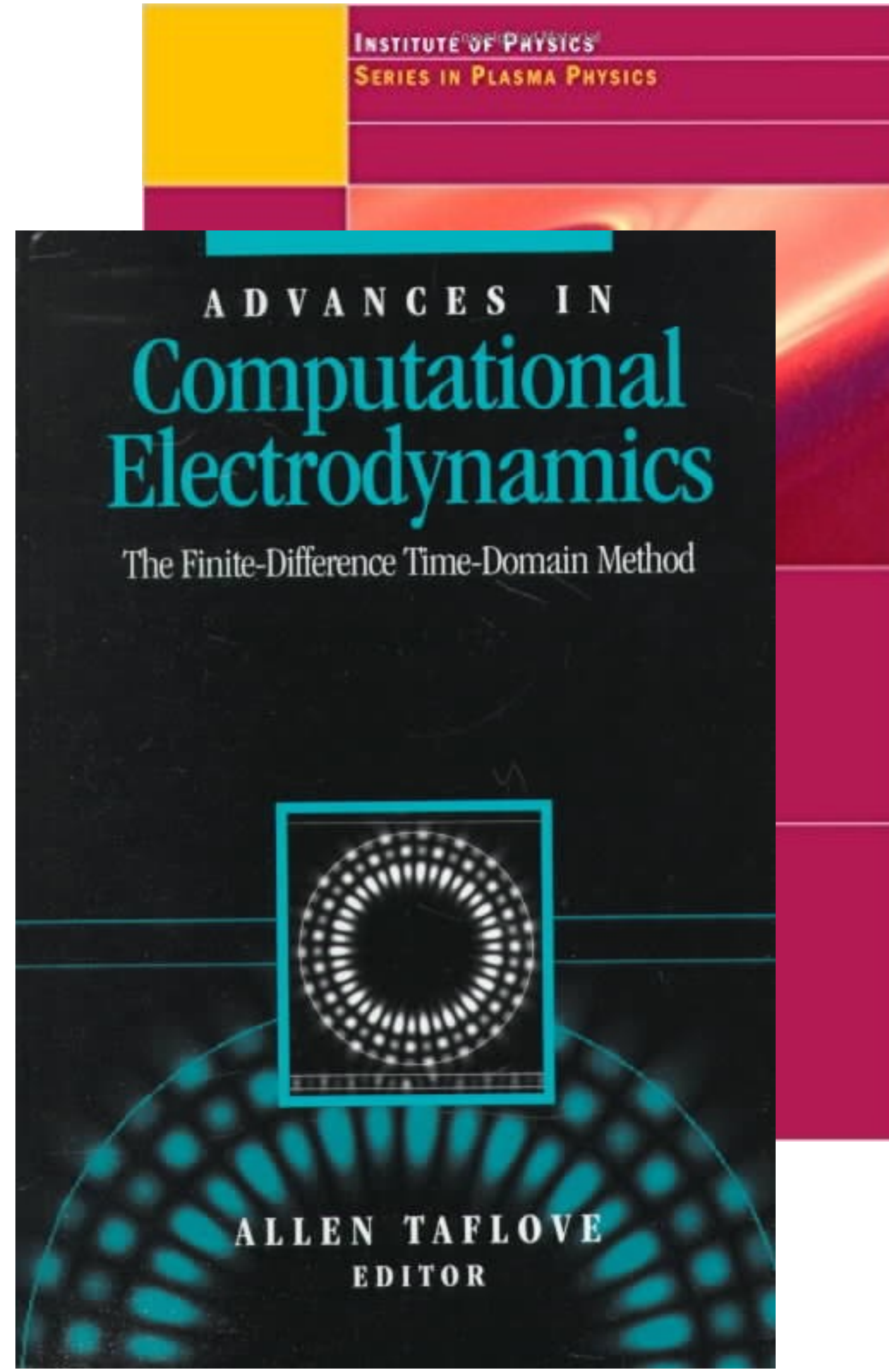

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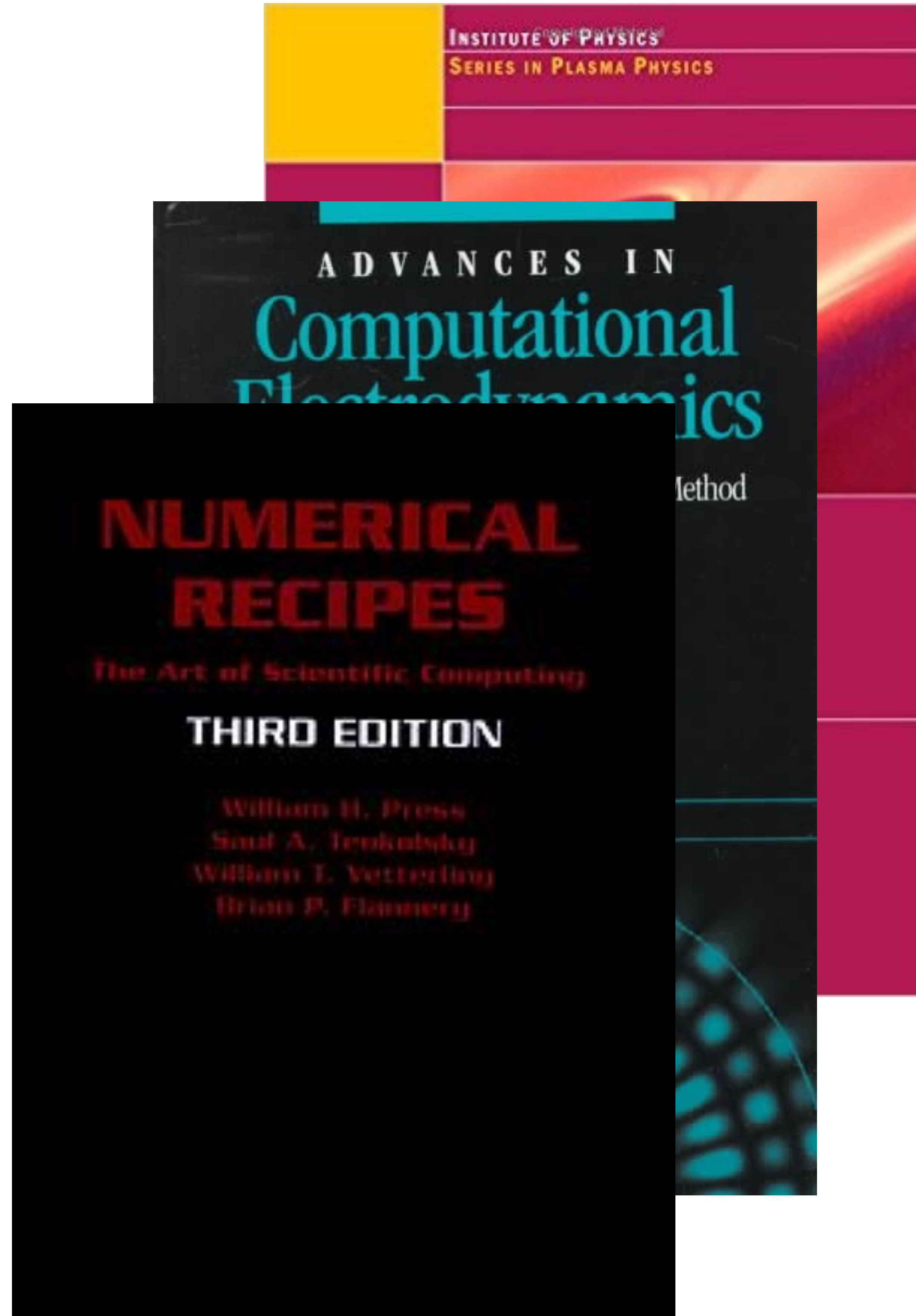
Numerical Recipies
W. H. Press *et al.*



The Particle-In-Cell (PIC) method is a central tool for simulation over a wide range of physics studies

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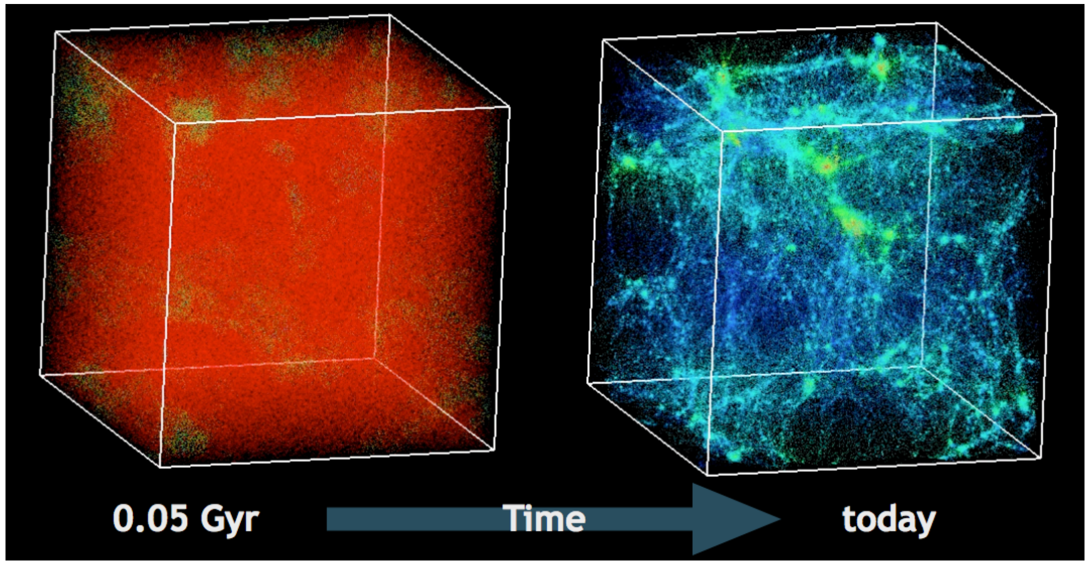
Cosmology



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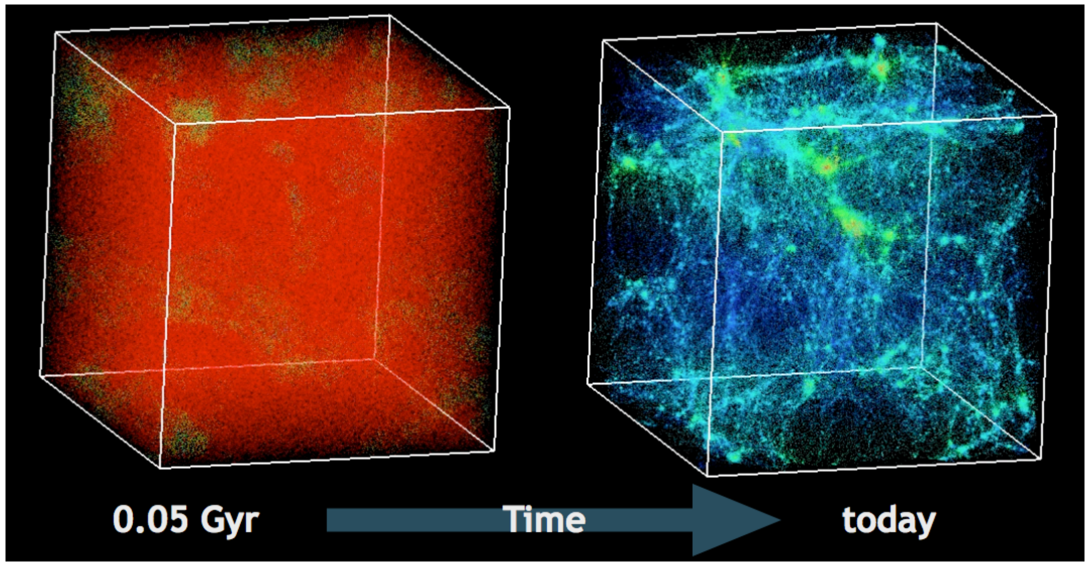

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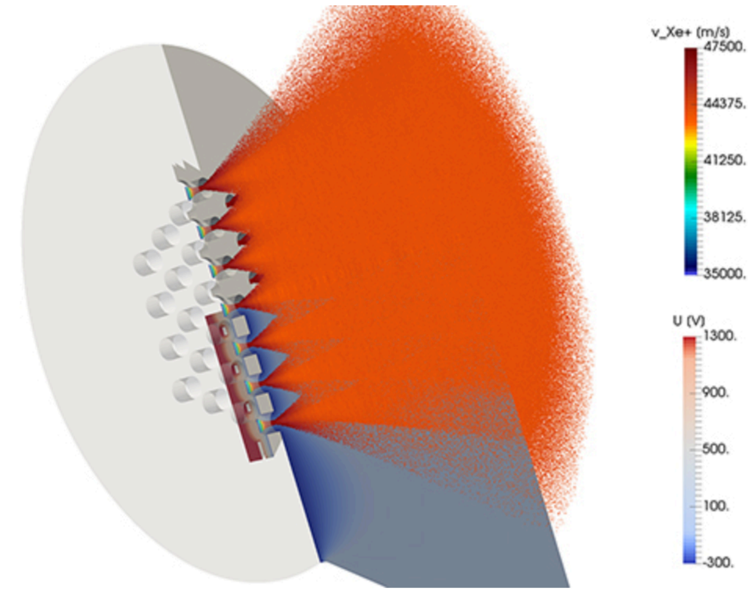
Space propulsion (Plasma thruster)



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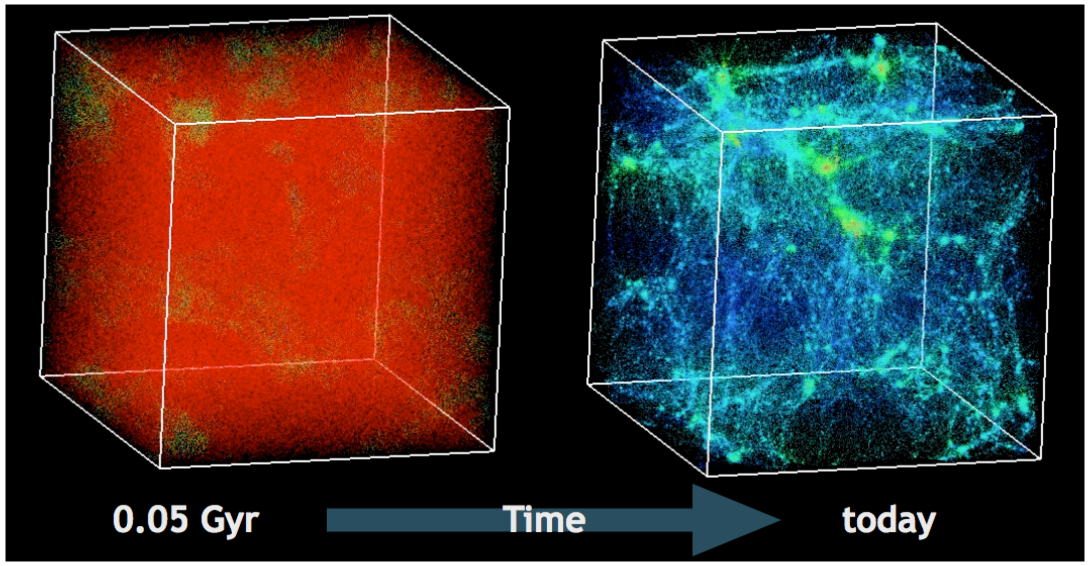

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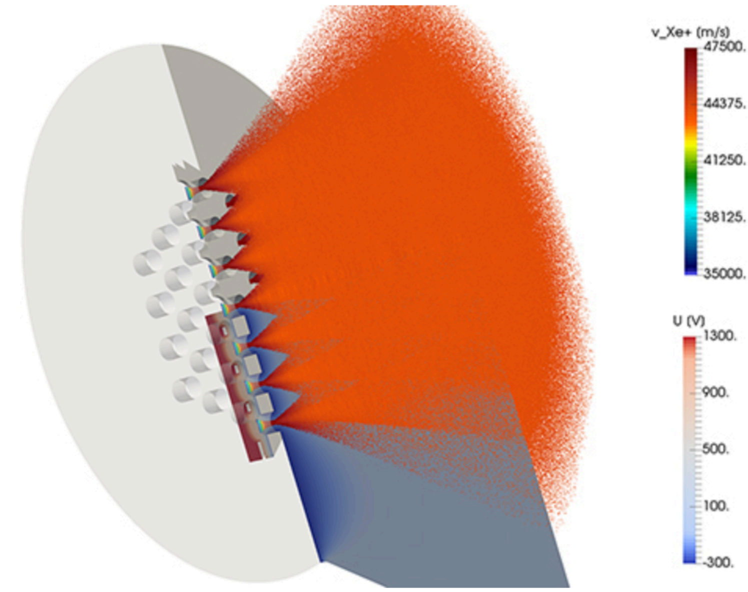

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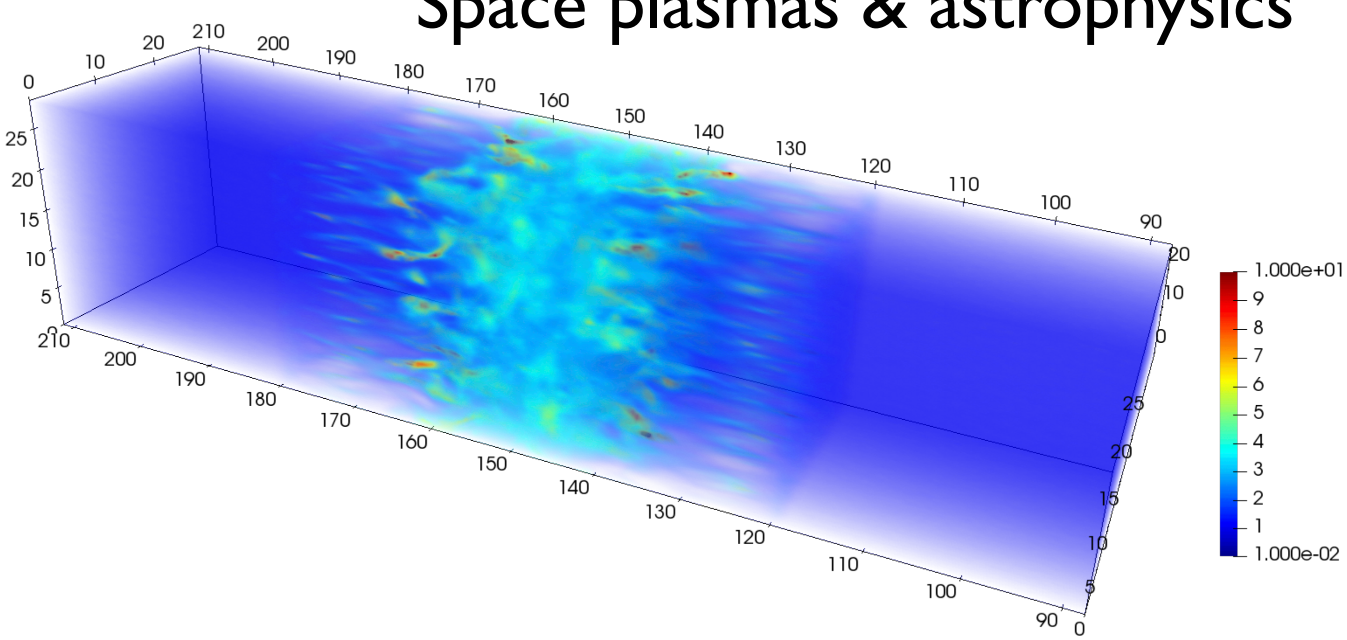
Space plasmas & astrophysics



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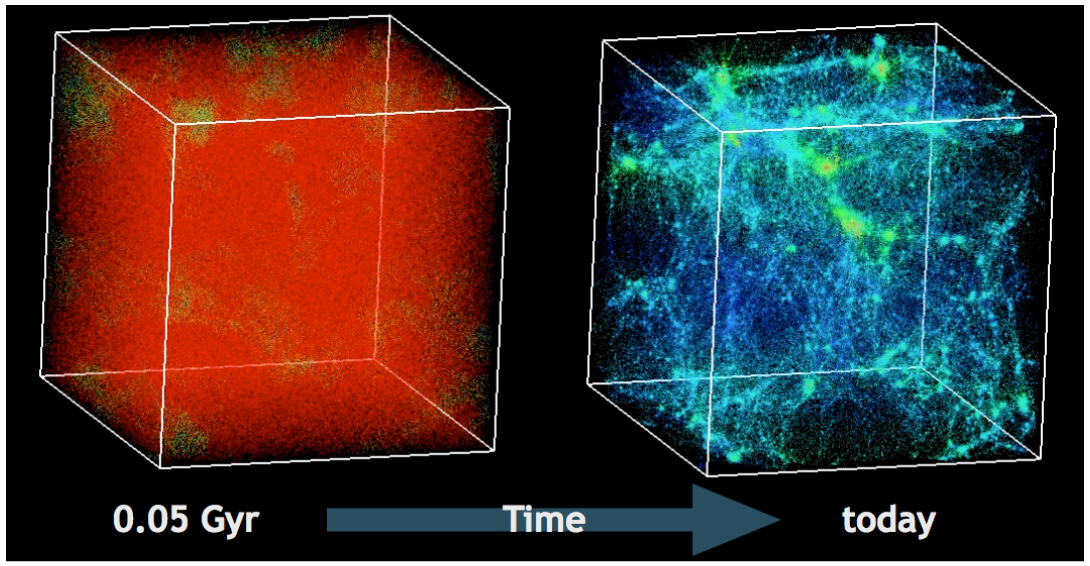

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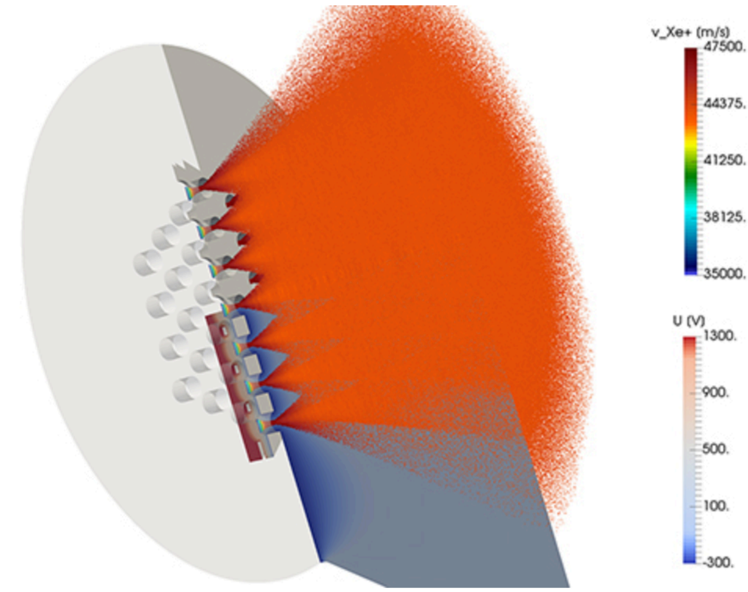

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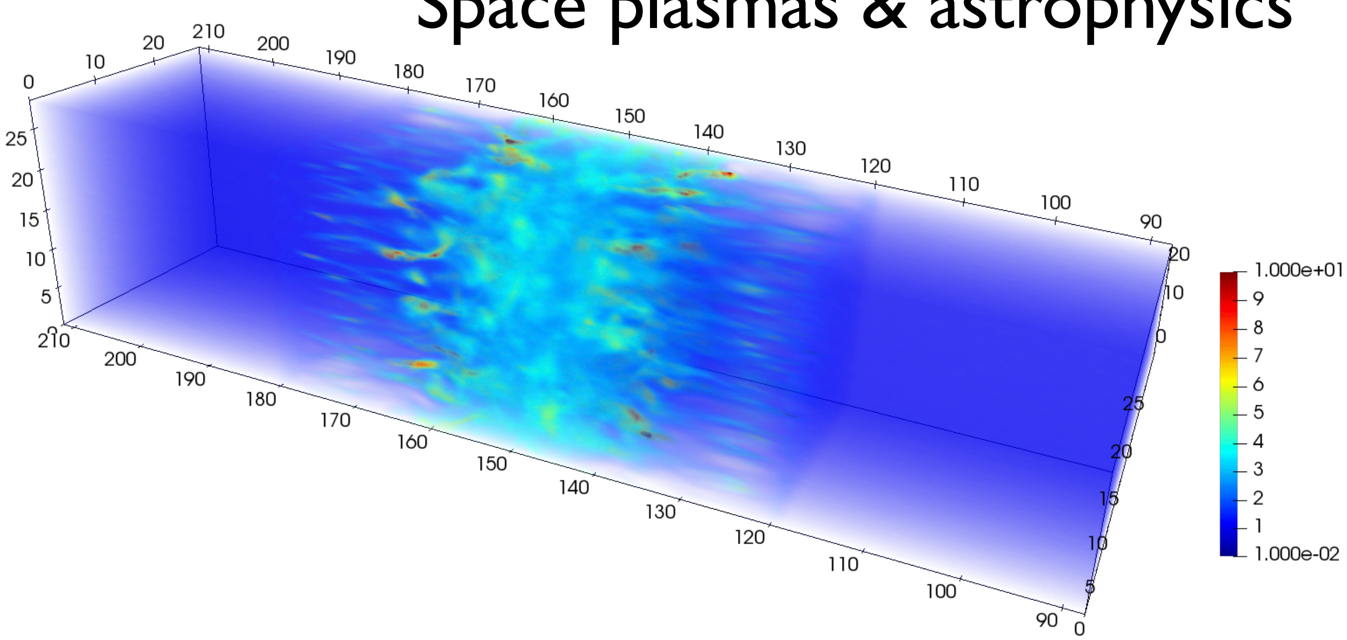

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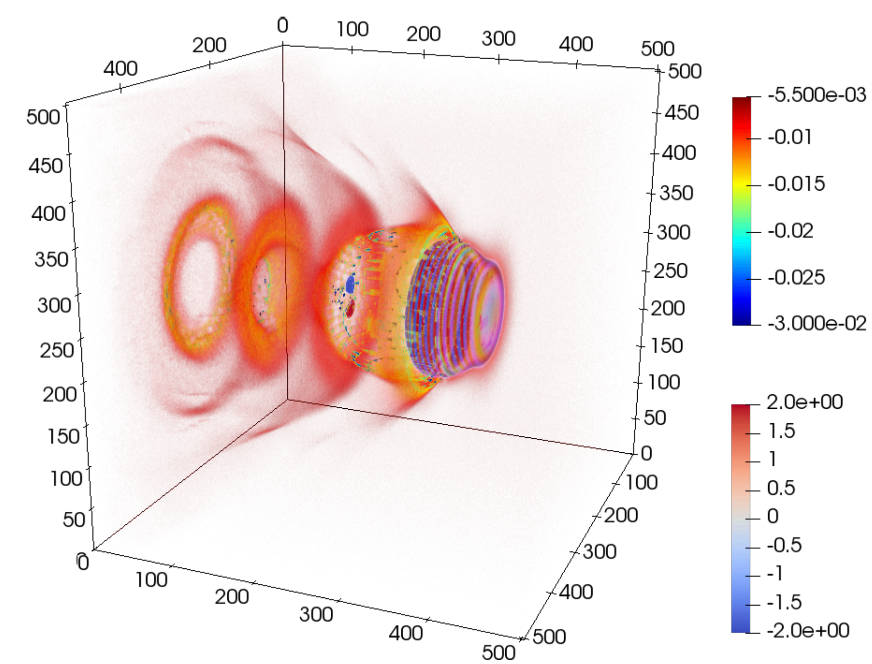
Laser plasma interaction



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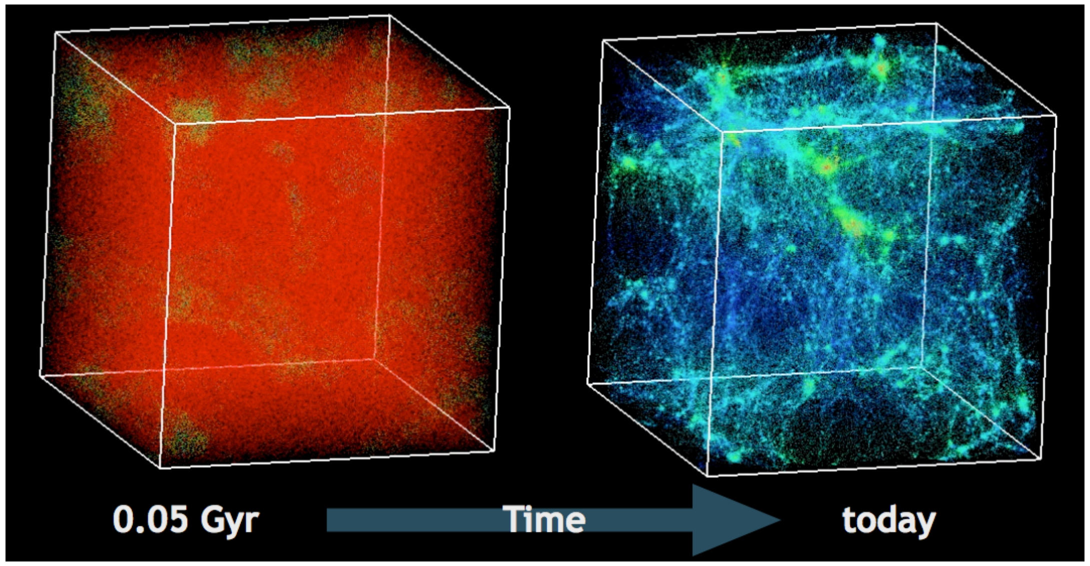

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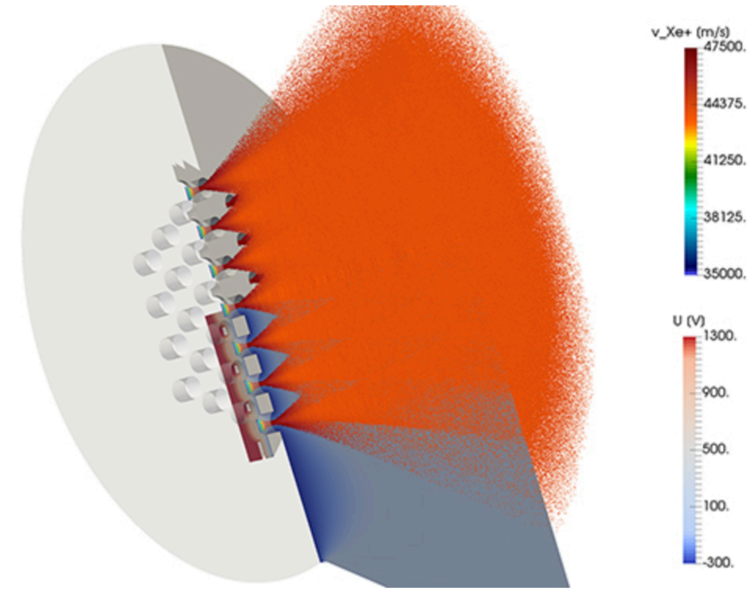

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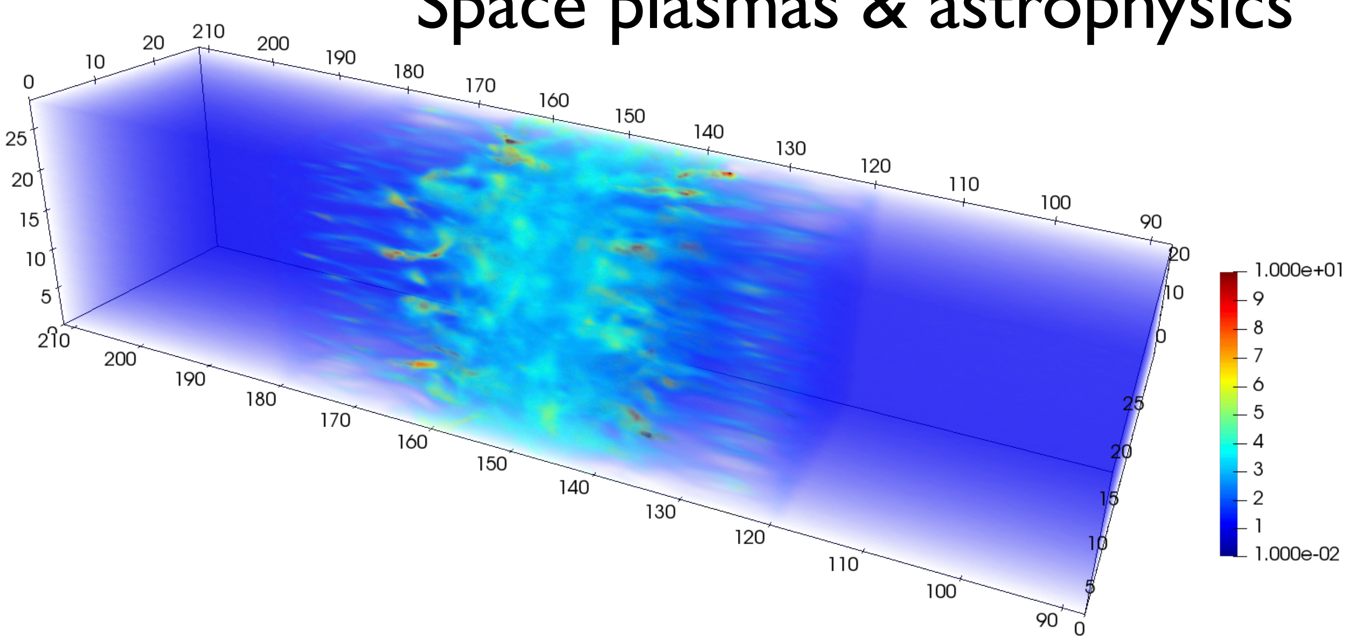

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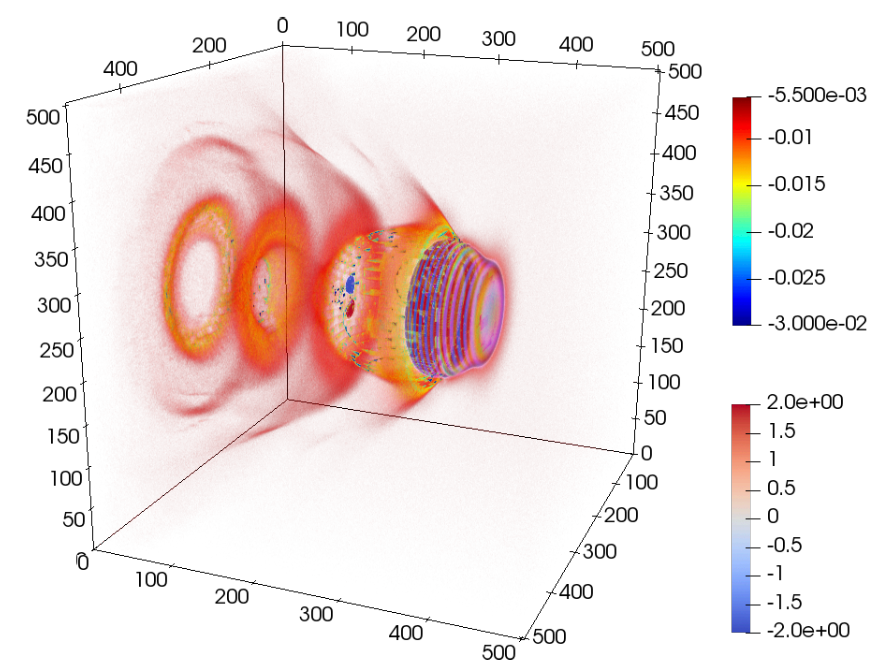

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- Conceptually simple
- Efficiently implemented on (massively) parallel super-computers

Our starting point is the Vlasov-Maxwell description for a *collisionless* plasma

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Plasma

$$\partial_t f_s + \frac{\mathbf{p}}{m_s \gamma} \cdot \nabla f_s + \mathbf{F}_L \cdot \nabla_{\mathbf{p}} f_s = 0$$

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Electromagnetic Field

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \partial_t \mathbf{E} = -\frac{1}{\epsilon_0} \mathbf{J} + c^2 \nabla \times \mathbf{B}$$

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$$\mathbf{F}_L = q_s \left(\mathbf{E} + \frac{\mathbf{p}}{m_s \gamma} \times \mathbf{B} \right)$$

$$\rho(t, \mathbf{x}) = \int d\mathbf{p} f_s(t, \mathbf{x}, \mathbf{p})$$

$$J(t, \mathbf{x}) = q_s \int d\mathbf{p} \frac{\mathbf{p}}{m_s \gamma} f_s(t, \mathbf{x}, \mathbf{p})$$

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1st Remark

Normalization: the Vlasov-Maxwell (relativistic) description provides us with a set of natural units

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Velocity	c
Charge	e
Mass	m_e
Momentum	$m_e\, c$
Energy, Temperature	$m_e c^2$
Time	ω_r^{-1}
Length	c/ω_r
Number density	$n_r = \epsilon_0\, m_e\, \omega_r^2 / e^2$
Current density	$e\, c\, n_r$
Pressure	$m_e\, c^2\, n_r$
Electric field	$m_e\, c\, \omega_r / e$
Magnetic field	$m_e\, \omega_r / e$
Poynting flux	$m_e\, c^3\, n_r / 2$

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The value of ω_r is not defined *a priori*, and acts as a *scaling factor*.

2nd Remark

The Particle-In-Cell method integrates Vlasov Equation along the trajectories of so-called *quasi-particles*

Vlasov Eq. is a **partial differential equation** (PDE) in *Ns*+*Nv* phase-space:

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The **PIC ansatz** consists in decomposing the distribution fct:

$$f_s(t, \mathbf{x}, \mathbf{p}) = \sum_{p=1}^{N} w_p \, S(\mathbf{x} - \mathbf{x}_p(t)) \, \delta(\mathbf{p} - \mathbf{p}_p(t))$$

Shape-function Dirac-distribution

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Injecting this *ansatz* in Vlasov Eq., multiplying by $\mathbf{p}$ and integrating over all momenta $\mathbf{p}$

$$\sum_{p=1}^{N_s} w_p \, \frac{\mathbf{p}_p}{m_s \gamma_p} \, \mathbf{p}_p \cdot \left[\partial_{\mathbf{x}_p} S(\mathbf{x}-\mathbf{x}_p) + \partial_{\mathbf{x}} S(\mathbf{x}-\mathbf{x}_p)\right]$$

$$+ \sum_{p=1}^{N_s} w_p \, S(\mathbf{x}-\mathbf{x}_p) \, \left[\partial_t \mathbf{p}_p - q_s \left(\mathbf{E} + \mathbf{v}_p \times \mathbf{B}\right)\right] = 0$$

Let us now integrate in space:

$$\sum_{p=1}^{N_s} w_p \, \frac{\mathbf{p}_p}{m_s \gamma_p} \, \mathbf{p}_p \cdot \cancel{\int d\mathbf{x} \left[\partial_{\mathbf{x}_p} S(\mathbf{x}-\mathbf{x}_p) + \partial_{\mathbf{x}} S(\mathbf{x}-\mathbf{x}_p)\right]}$$

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Finally leading to solving for all $\mathbf{p}$:

$$\partial_t \mathbf{p}_p = q_s \left(\mathbf{E}_p + \mathbf{v} \times \mathbf{B}_p\right) \quad \text{with} \quad (\mathbf{E},\mathbf{B})_p \equiv \int d\mathbf{x} \, (\mathbf{E},\mathbf{B})(\mathbf{x}) \, S(\mathbf{x}-\mathbf{x}_p)$$

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Take the divergence of Maxwell-Ampère's Eq. :

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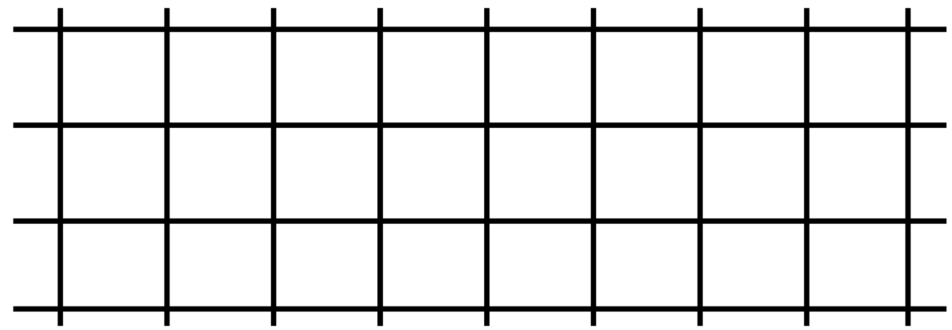
If at time t=0, Poisson & Gauss Eqs. are satisfied,
and if current deposition is made in a way that conserve charge,

then solving only Maxwell-Ampère & Maxwell-Faraday ensures that both Eqs. remain satisfied at later time.

Initialization of a PIC simulation



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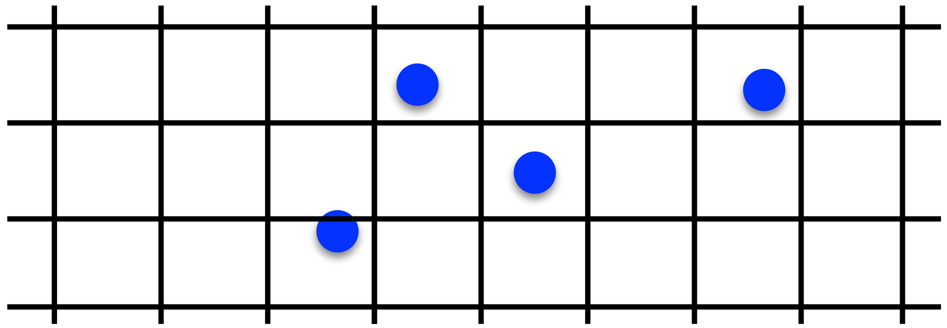

1) **for each species of your plasma, create your quasi-particles**
   e.g. defining the species density, velocity and temperature profiles

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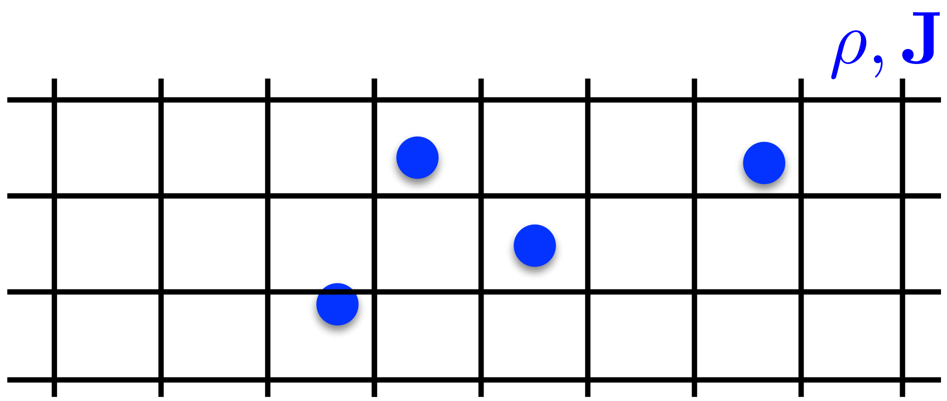

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2) loop over all particles and project charge and current density onto the grid

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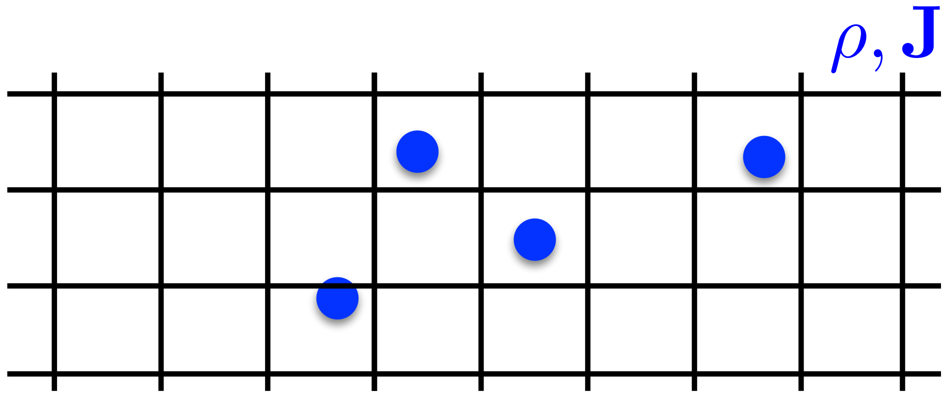

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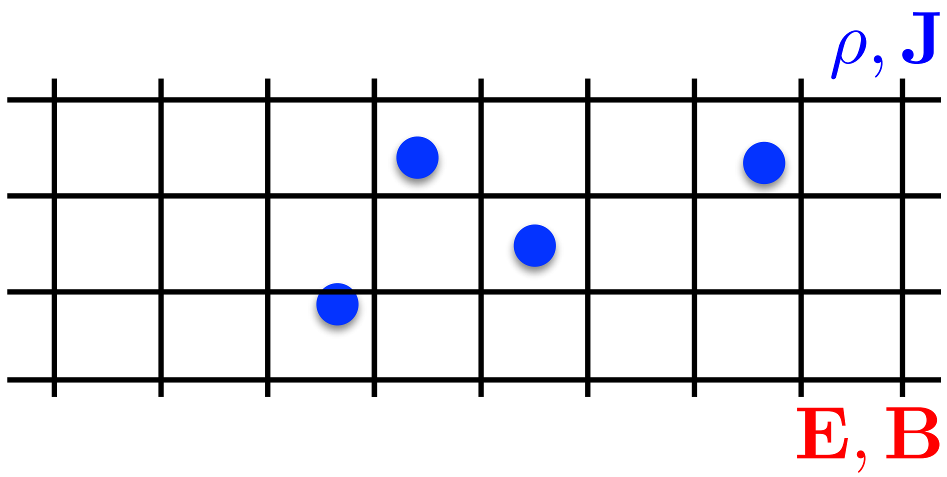

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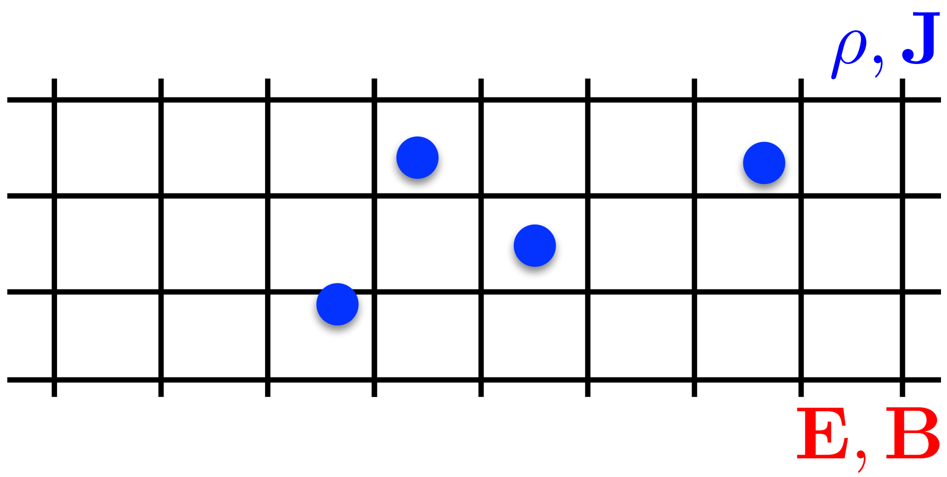
4) add any (user defined) external fields provided they are divergence-free

The Particle-In-Cell loop



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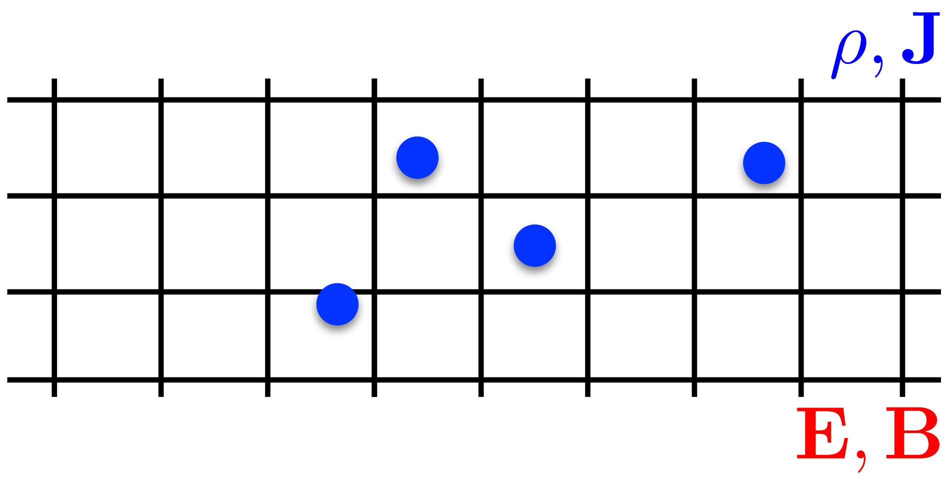

Gather fields at particle position

$$[\mathbf{E}, \mathbf{B}] \rightarrow [\mathbf{E}_p, \mathbf{B}_p]$$

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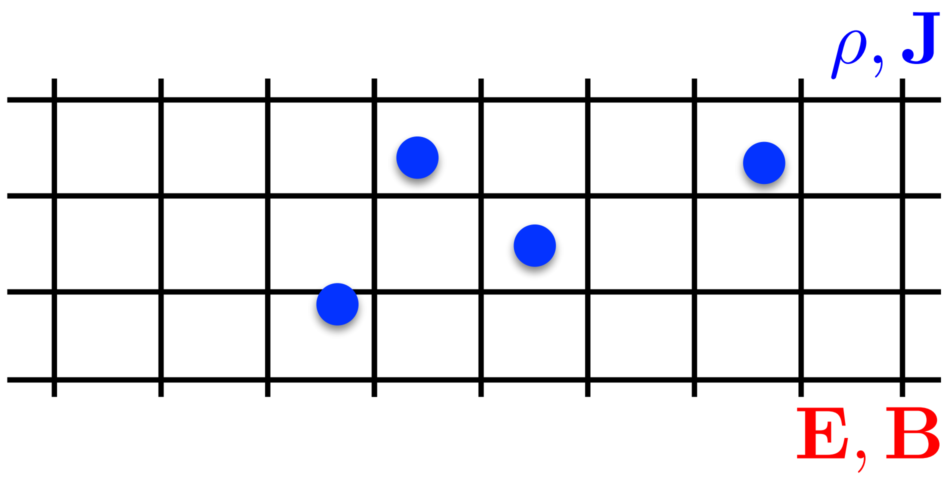

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$$\forall p \quad \begin{aligned} d_t \mathbf{u}_p &= \frac{q_s}{m_s} \mathbf{F}_L \\ d_t \mathbf{x}_p &= \mathbf{u}_p / \gamma_p \end{aligned}$$

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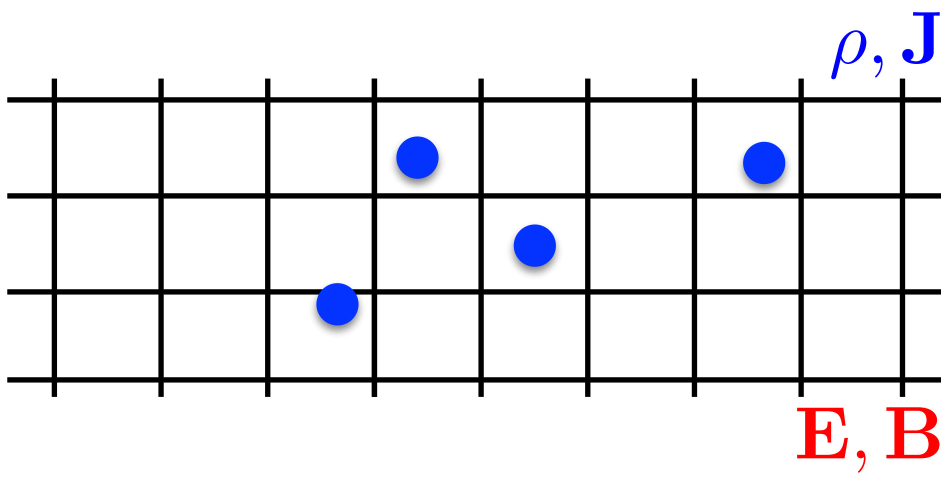

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$$[\mathbf{x}_p, \mathbf{p}_p] \rightarrow [\rho, \mathbf{J}]$$

* using a charge conserving scheme

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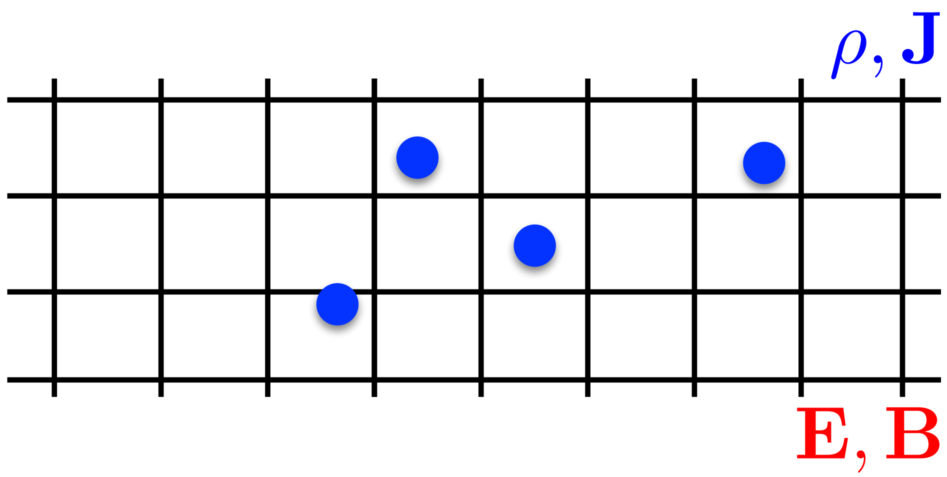

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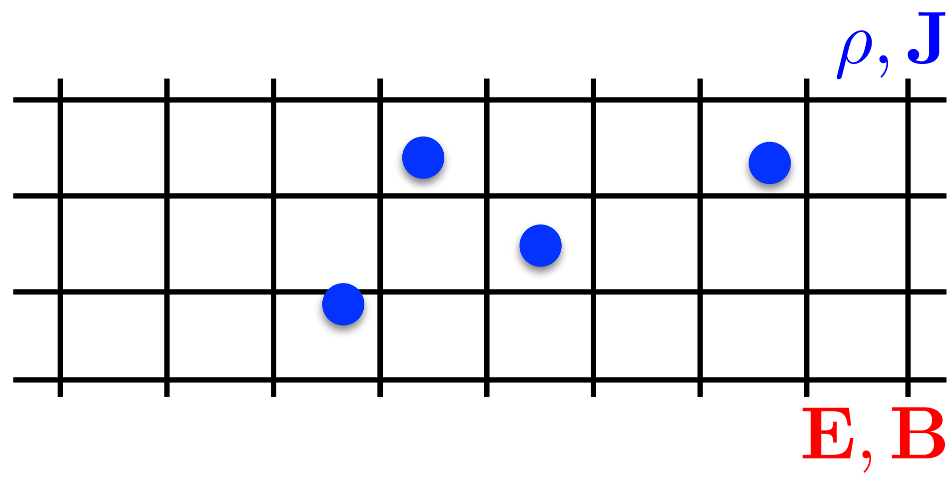

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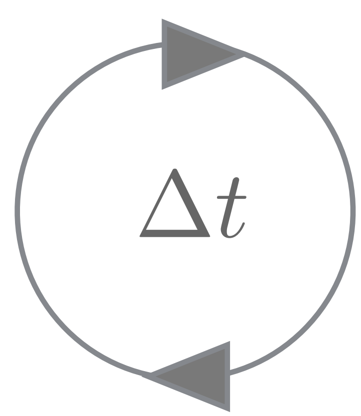

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- Numerical approach: how to build up your PIC code
- Parallelization: getting ready for the super-computers
- Additional modules: beyond the *collisionless* plasma
- Some physics highlights: what you can do with a PIC code

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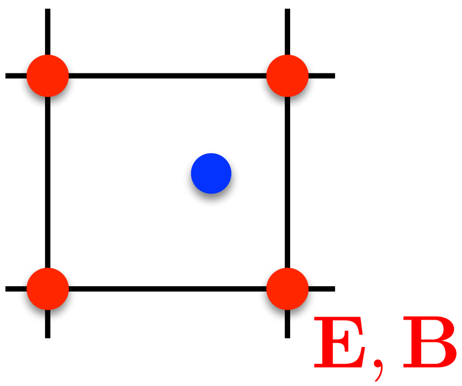

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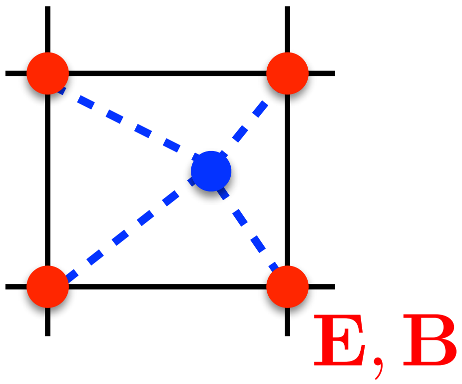

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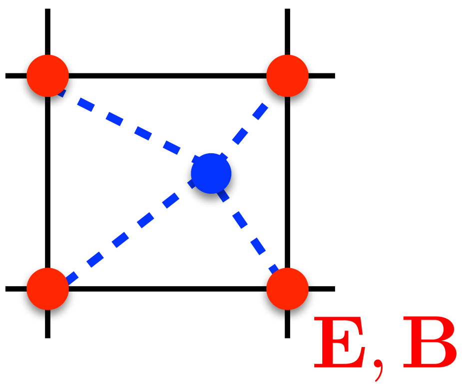

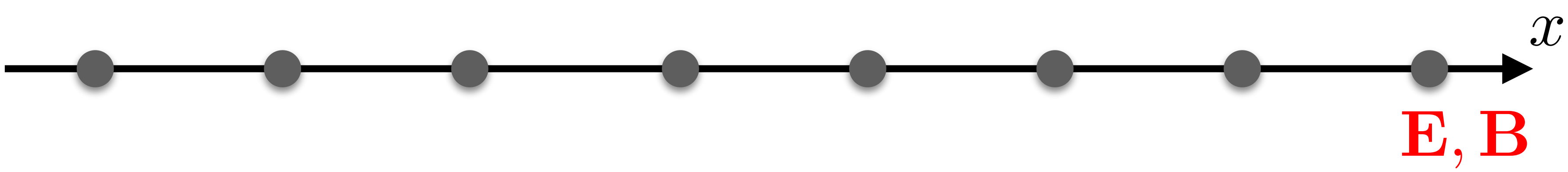

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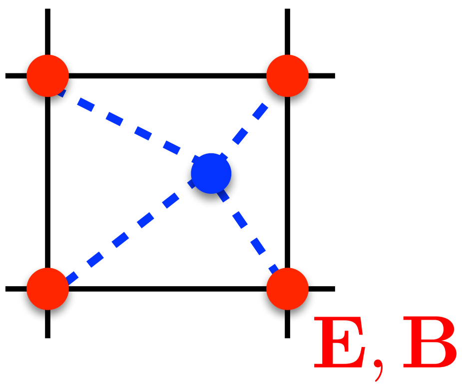

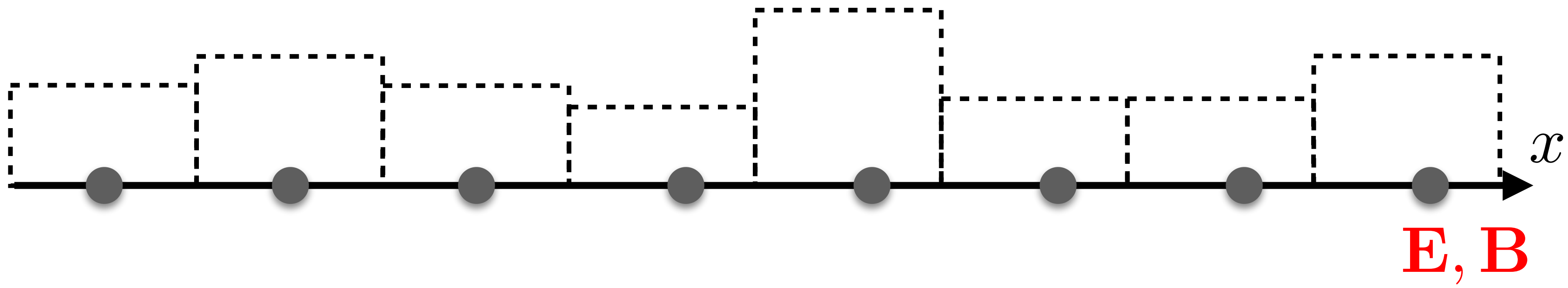

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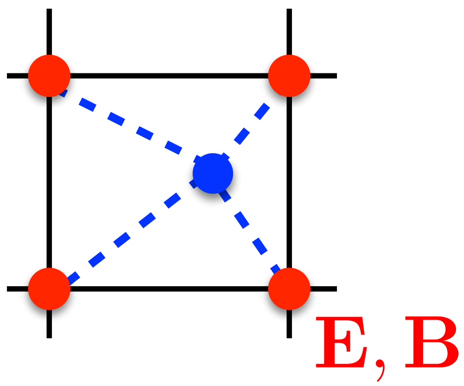

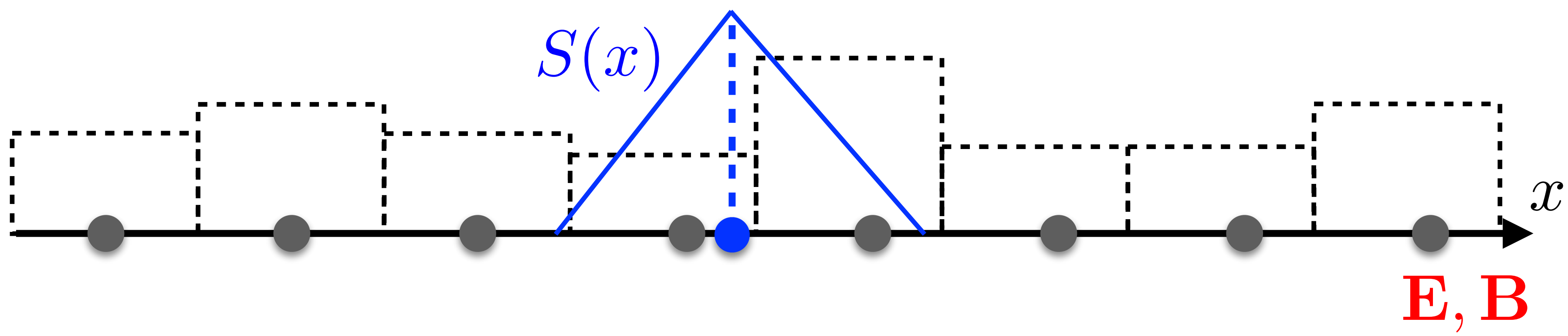

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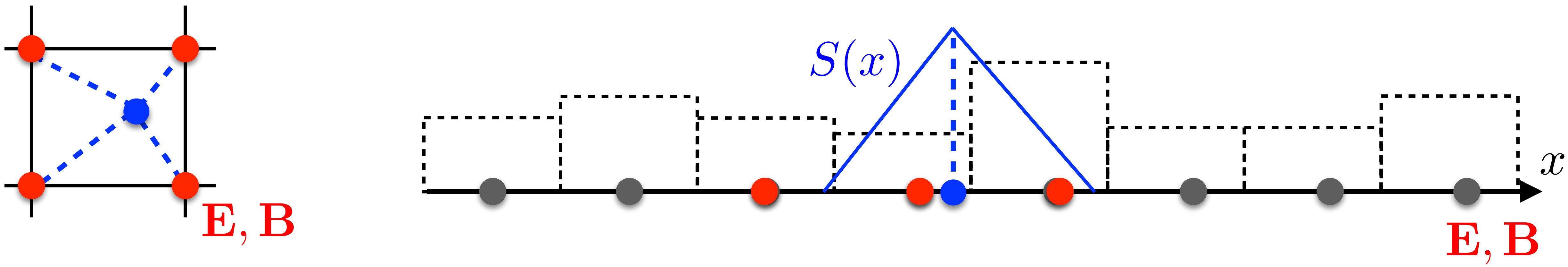

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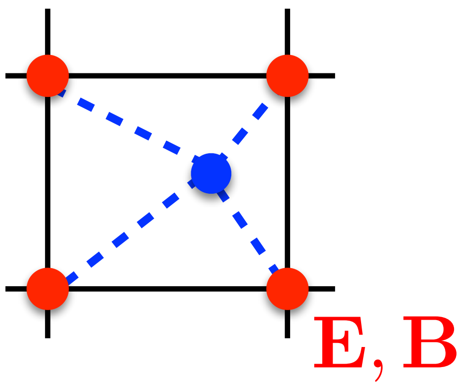

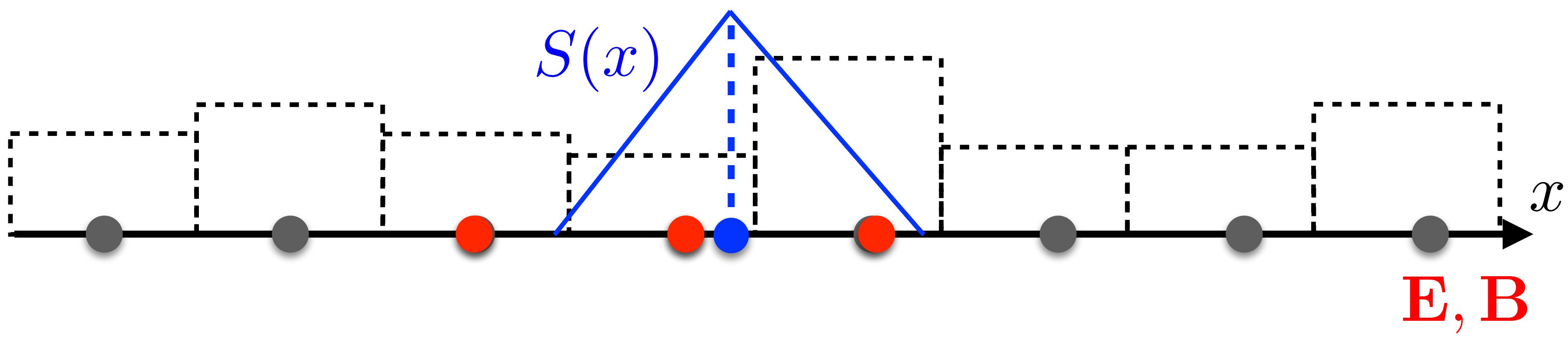

$$(\mathbf{E},\mathbf{B})_p \equiv \int d\mathbf{x}\,(\mathbf{E},\mathbf{B})(\mathbf{x})\,S(\mathbf{x}-\mathbf{x}_p)$$

$$\hat{s}^{(0)}(x) = \Delta x\,\delta(x),$$

$$\hat{s}^{(1)}(x) = \begin{cases} 1 & \text{if } |x| \le \frac{1}{2}\Delta x, \\ 0 & \text{otherwise}, \end{cases}$$

$$\hat{s}^{(2)}(x) = \begin{cases} \left(1 - \left|\frac{x}{\Delta x}\right|\right) & \text{if } |x| \le \Delta x, \\ 0 & \text{otherwise}, \end{cases}$$

$$\hat{s}^{(3)}(x) = \begin{cases} \frac{3}{4}\left[1 - \frac{4}{3}\left(\frac{x}{\Delta x}\right)^2\right] & \text{if } |x| \le \frac{1}{2}\Delta x, \\ \frac{9}{8}\left(1 - \frac{2}{3}\left|\frac{x}{\Delta x}\right|\right)^2 & \text{if } \frac{1}{2}\Delta x < |x| \le \frac{3}{2}\Delta x, \\ 0 & \text{otherwise}, \end{cases}$$

$$\hat{s}^{(4)}(x) = \begin{cases} \frac{2}{3}\left[1 - \frac{3}{2}\left(\frac{x}{\Delta x}\right)^2 + \frac{3}{4}\left|\frac{x}{\Delta x}\right|^3\right] & \text{if } |x| \le \Delta x, \\ \frac{4}{3}\left(1 - \frac{1}{2}\left|\frac{x}{\Delta x}\right|\right)^3 & \text{if } \Delta x < |x| \le 2\,\Delta x, \\ 0 & \text{otherwise}. \end{cases}$$

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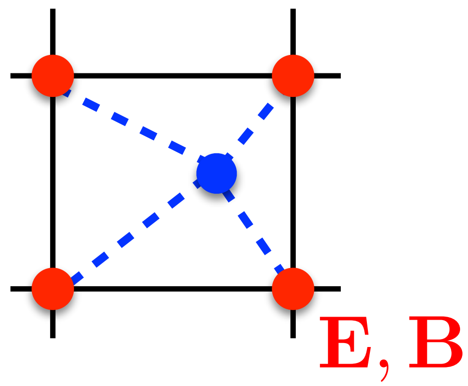

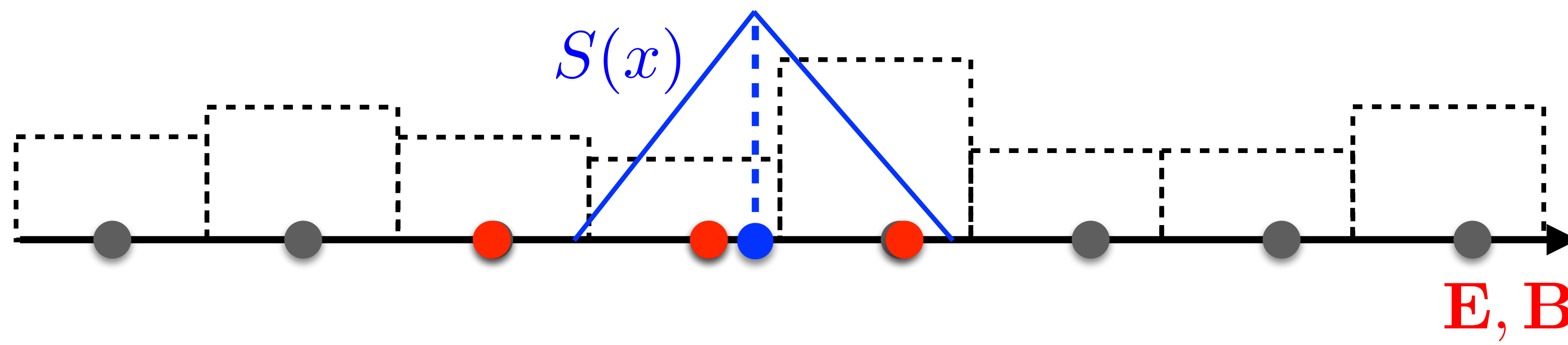

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Step 2

The Boris leap-frog *pusher* is a very popular method to advance particles

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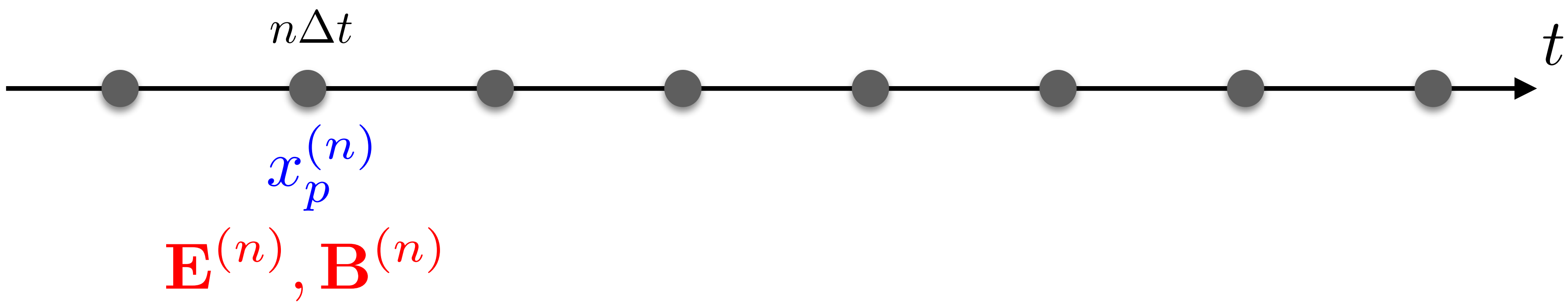

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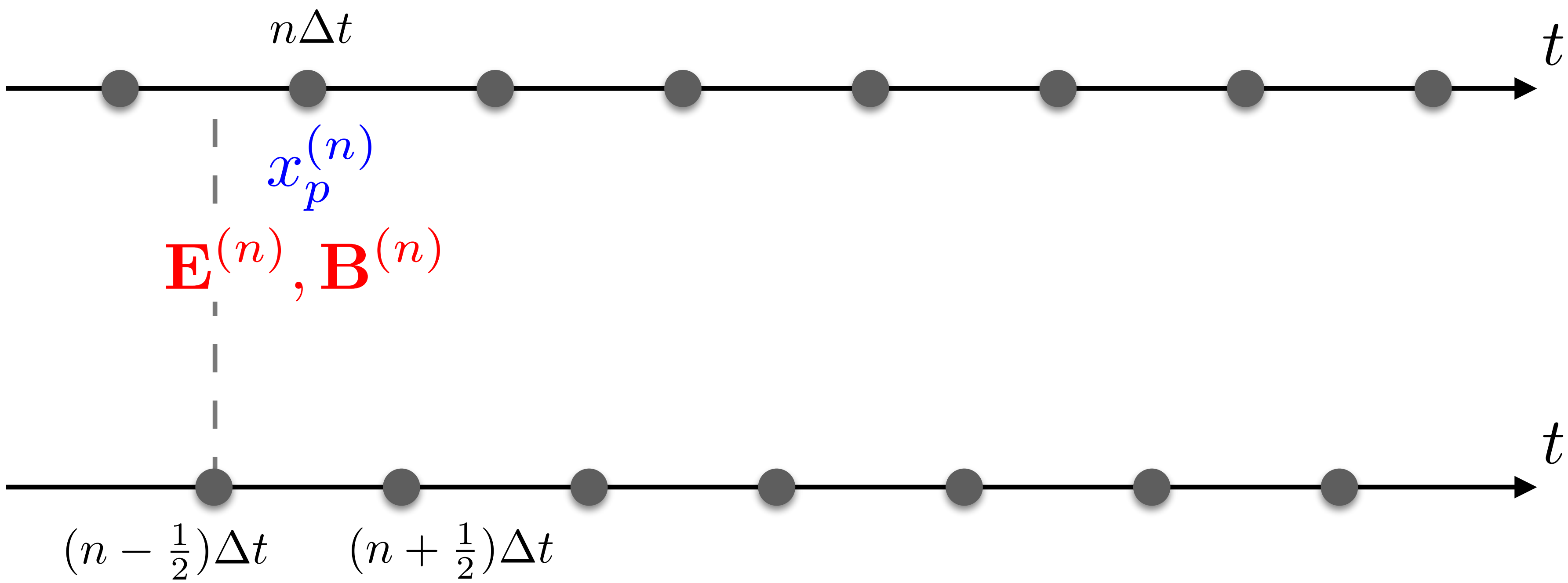

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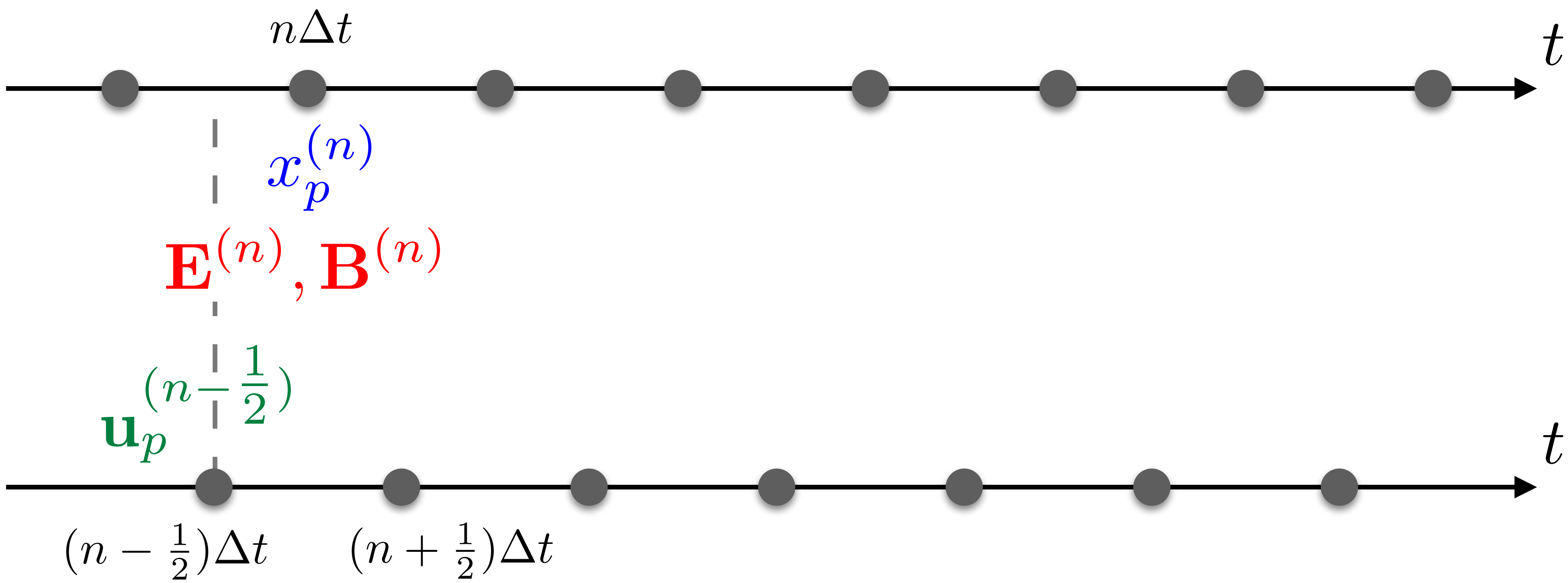

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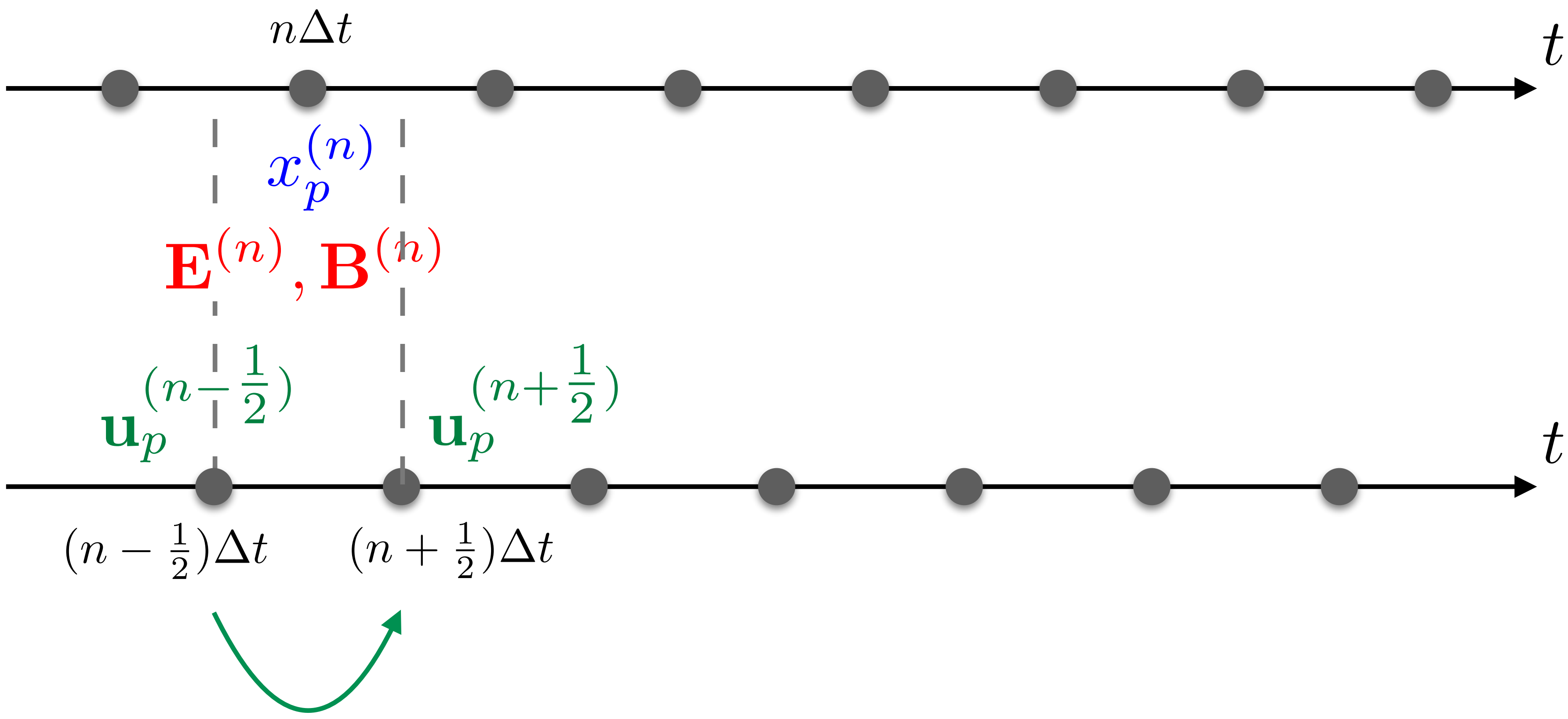

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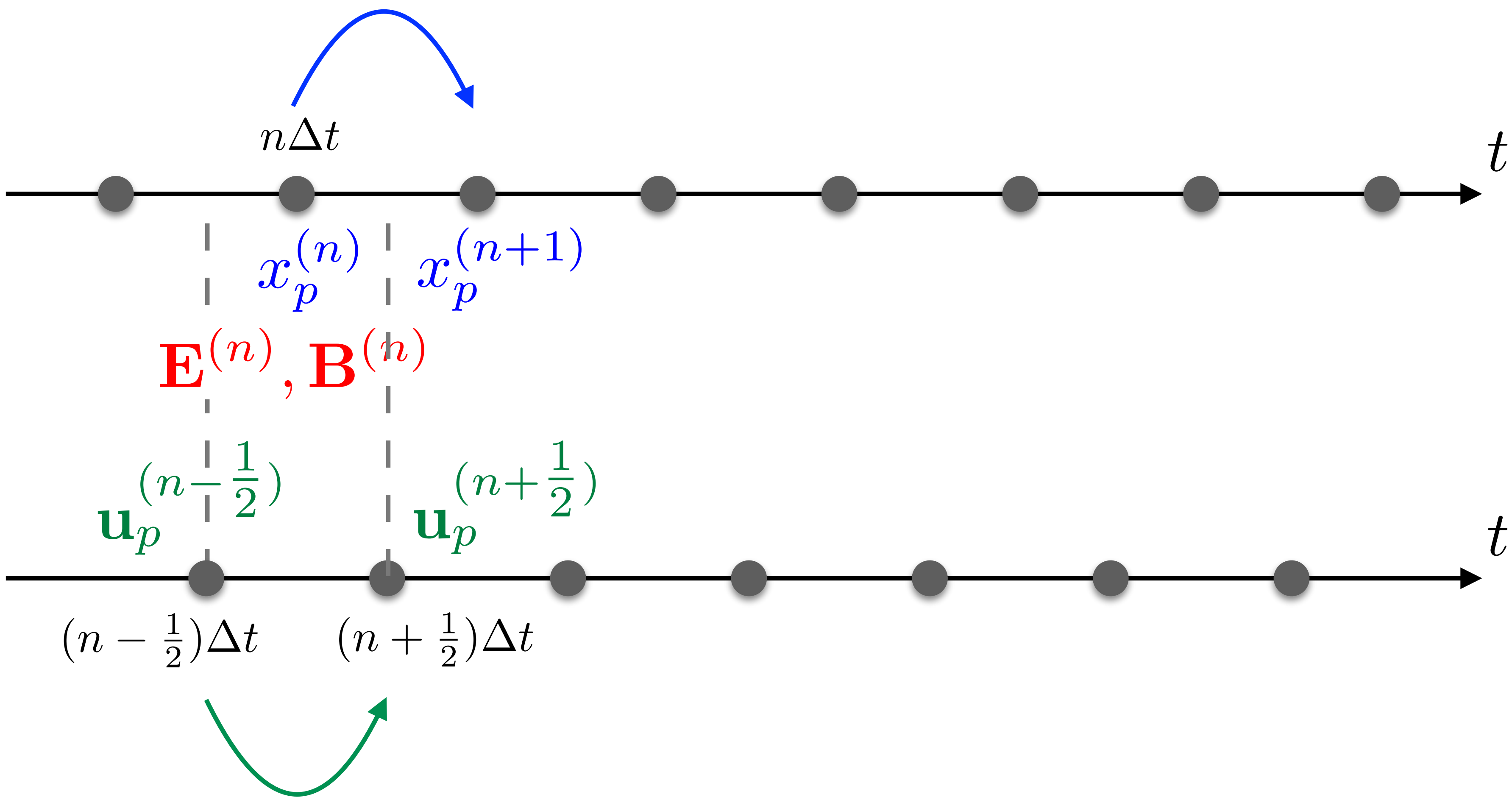

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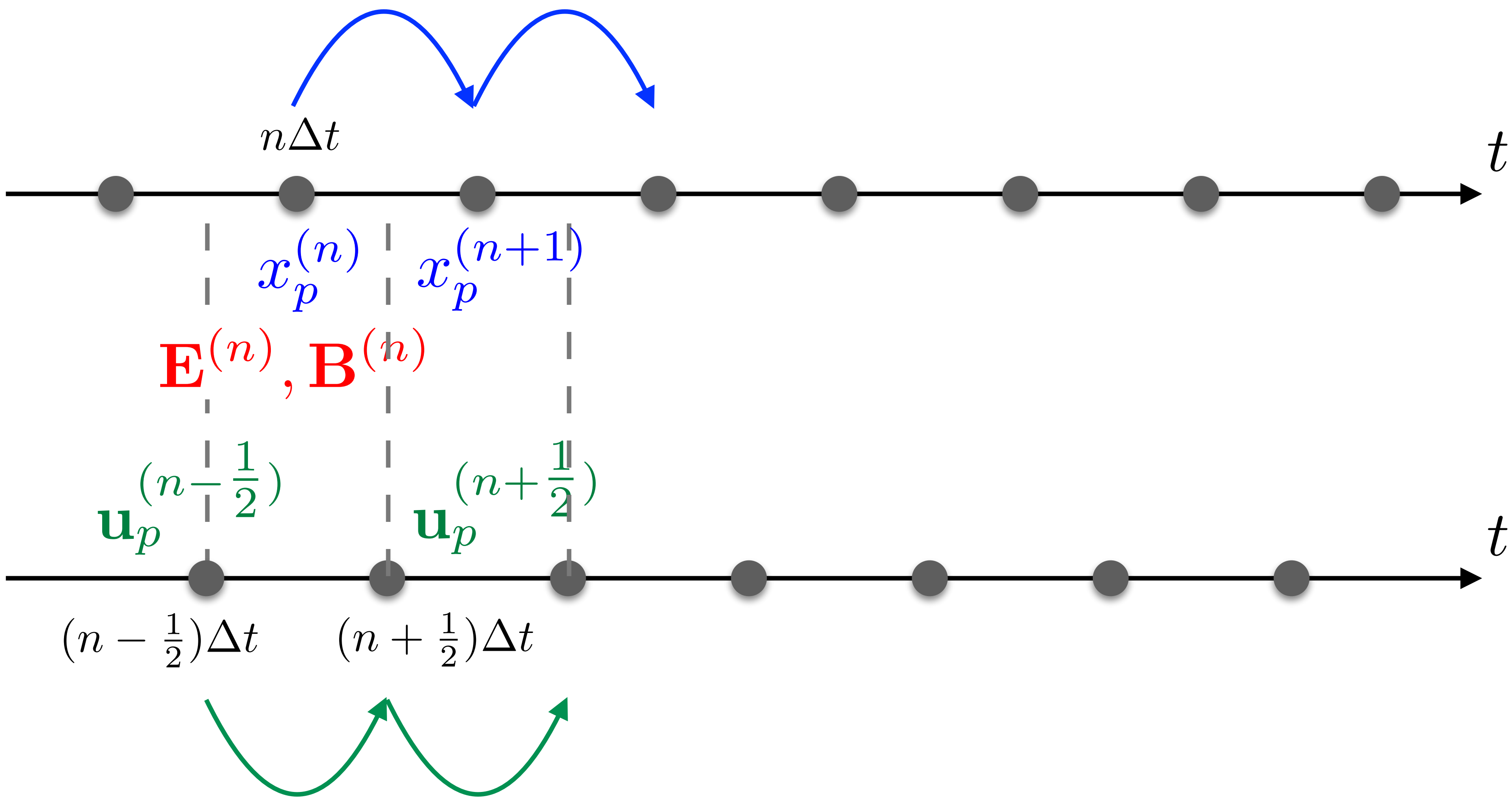

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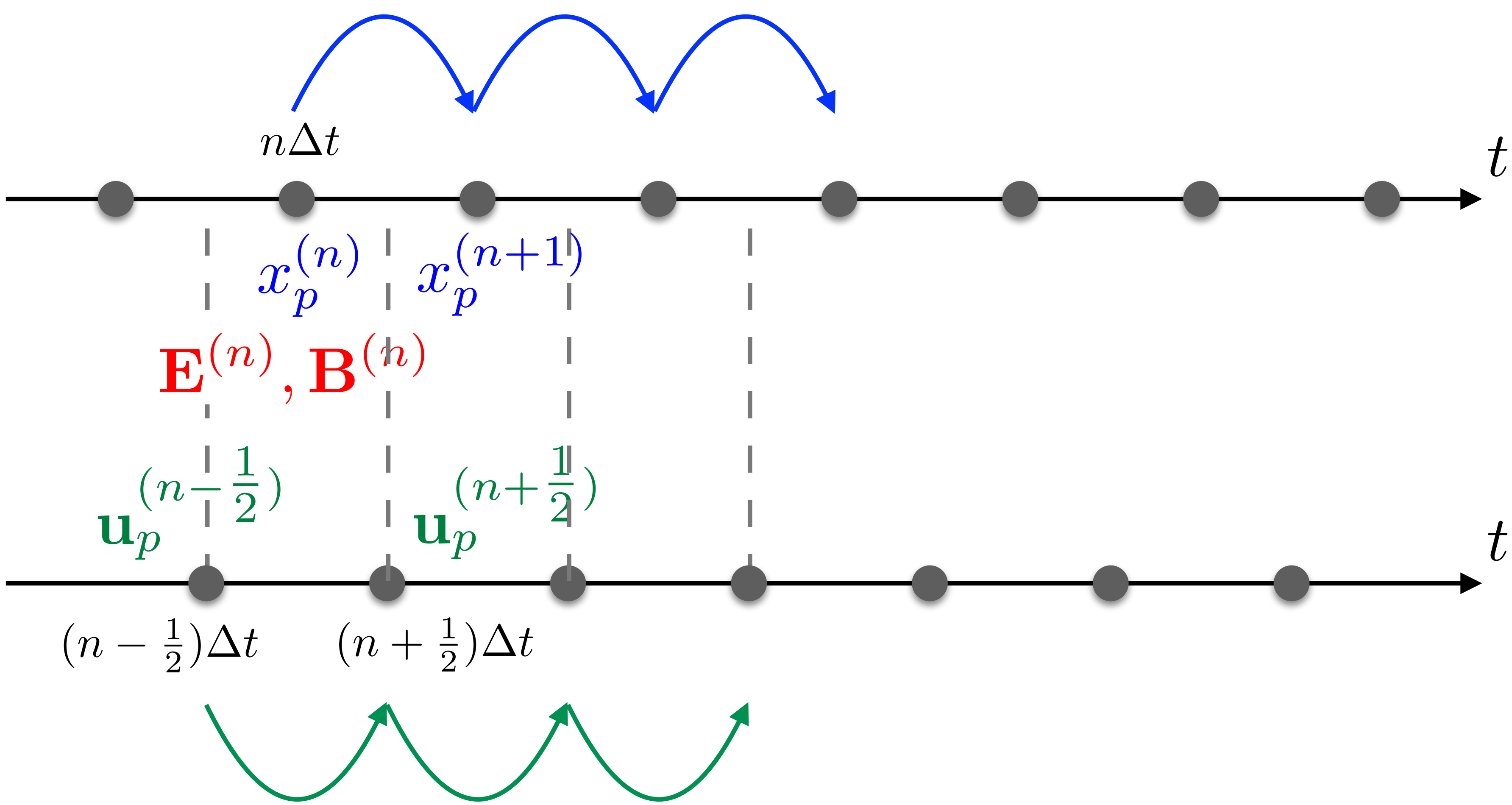

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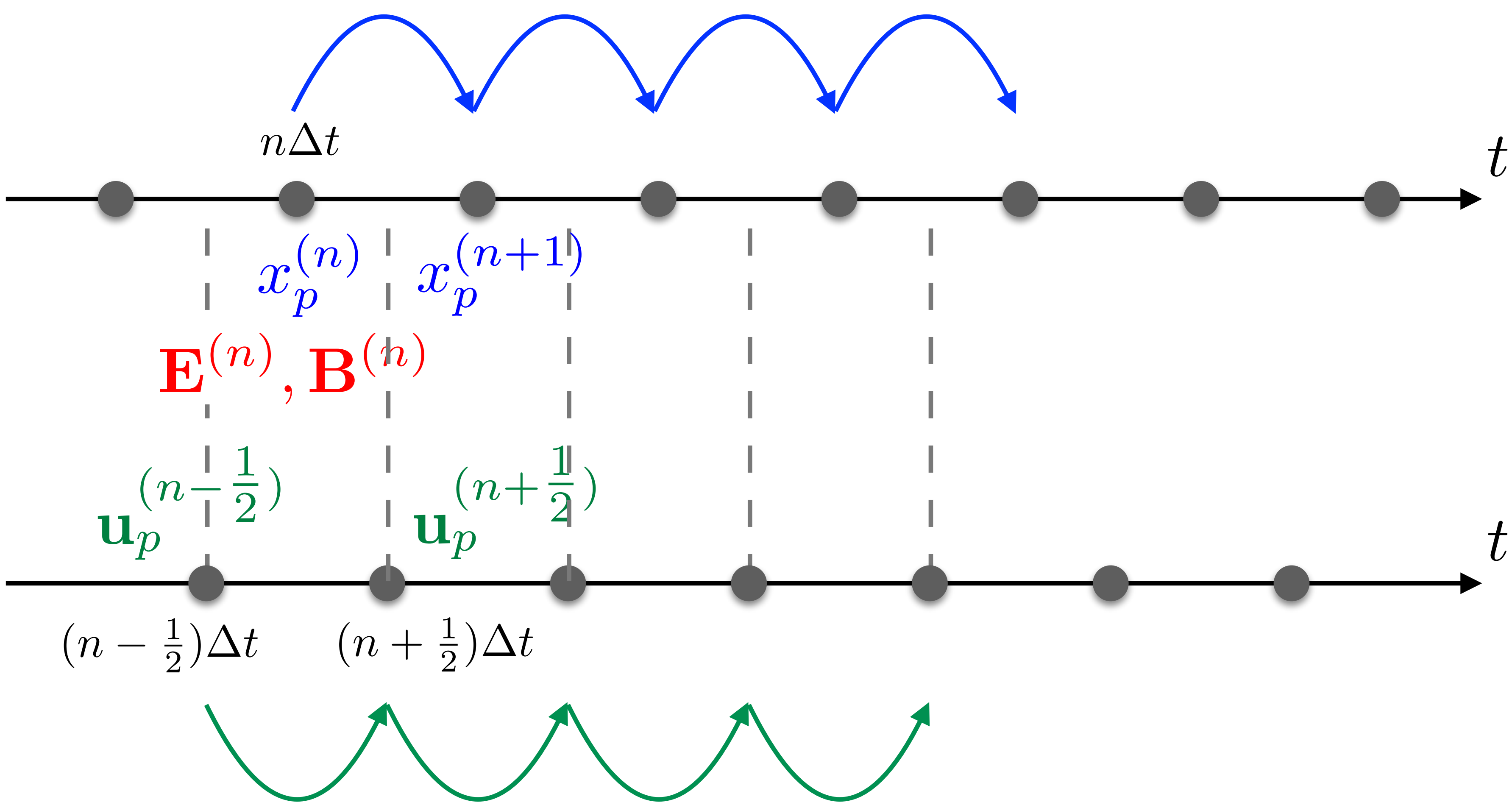

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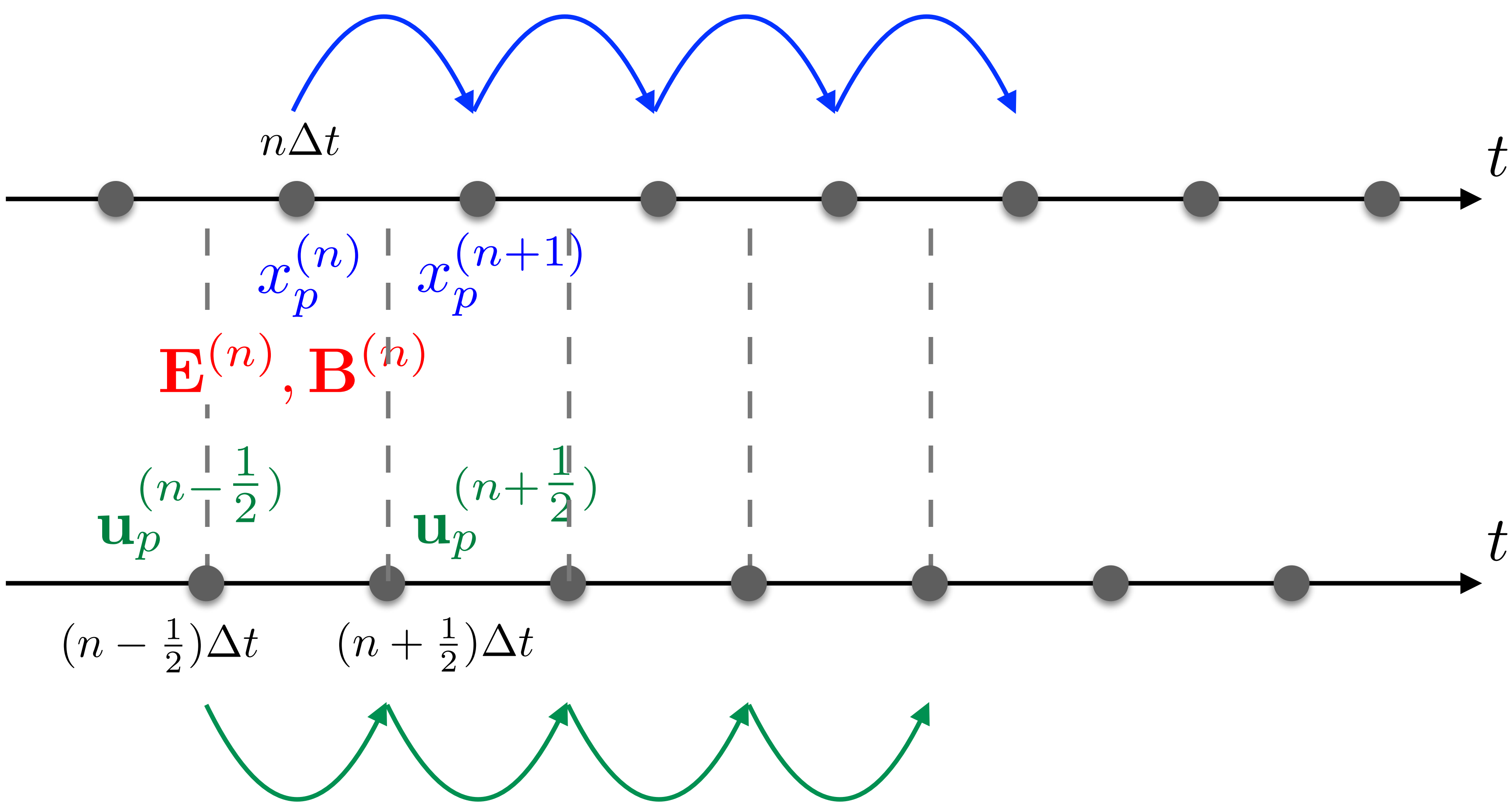

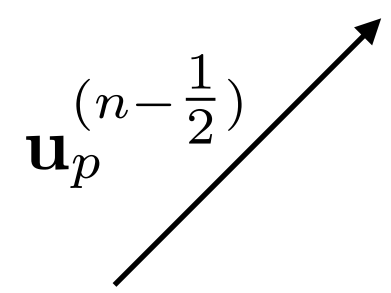

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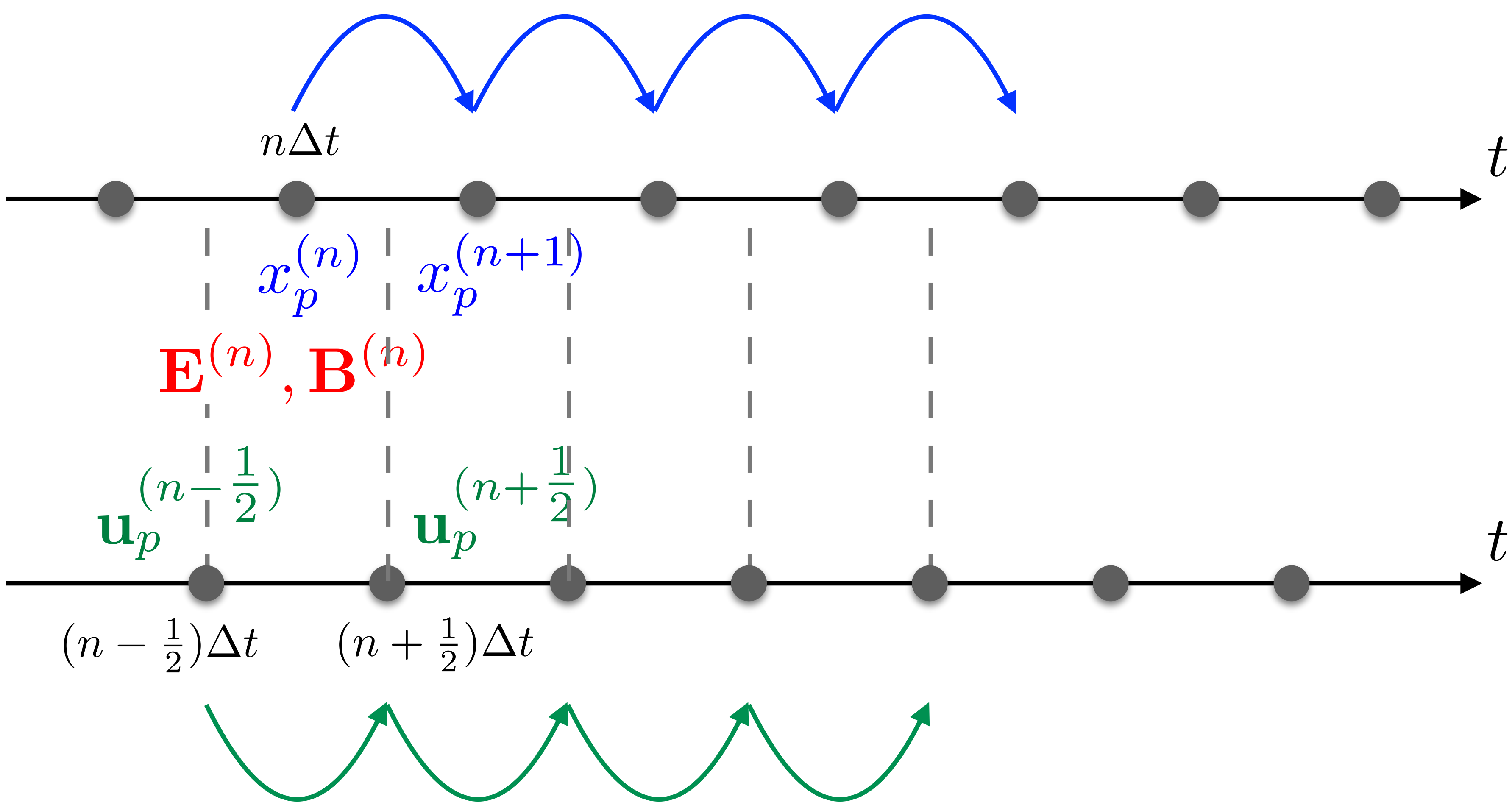
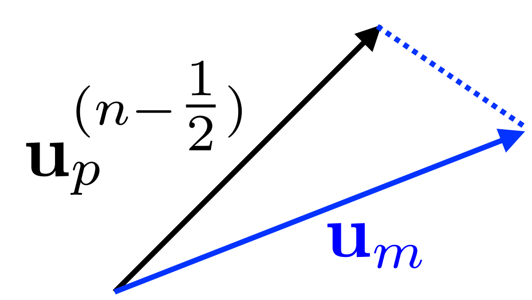


$$\mathbf{u}_m = \mathbf{u}_p^{(n-\frac{1}{2})} + \frac{q_s}{m_s}\frac{\Delta t}{2}\mathbf{E}_p$$



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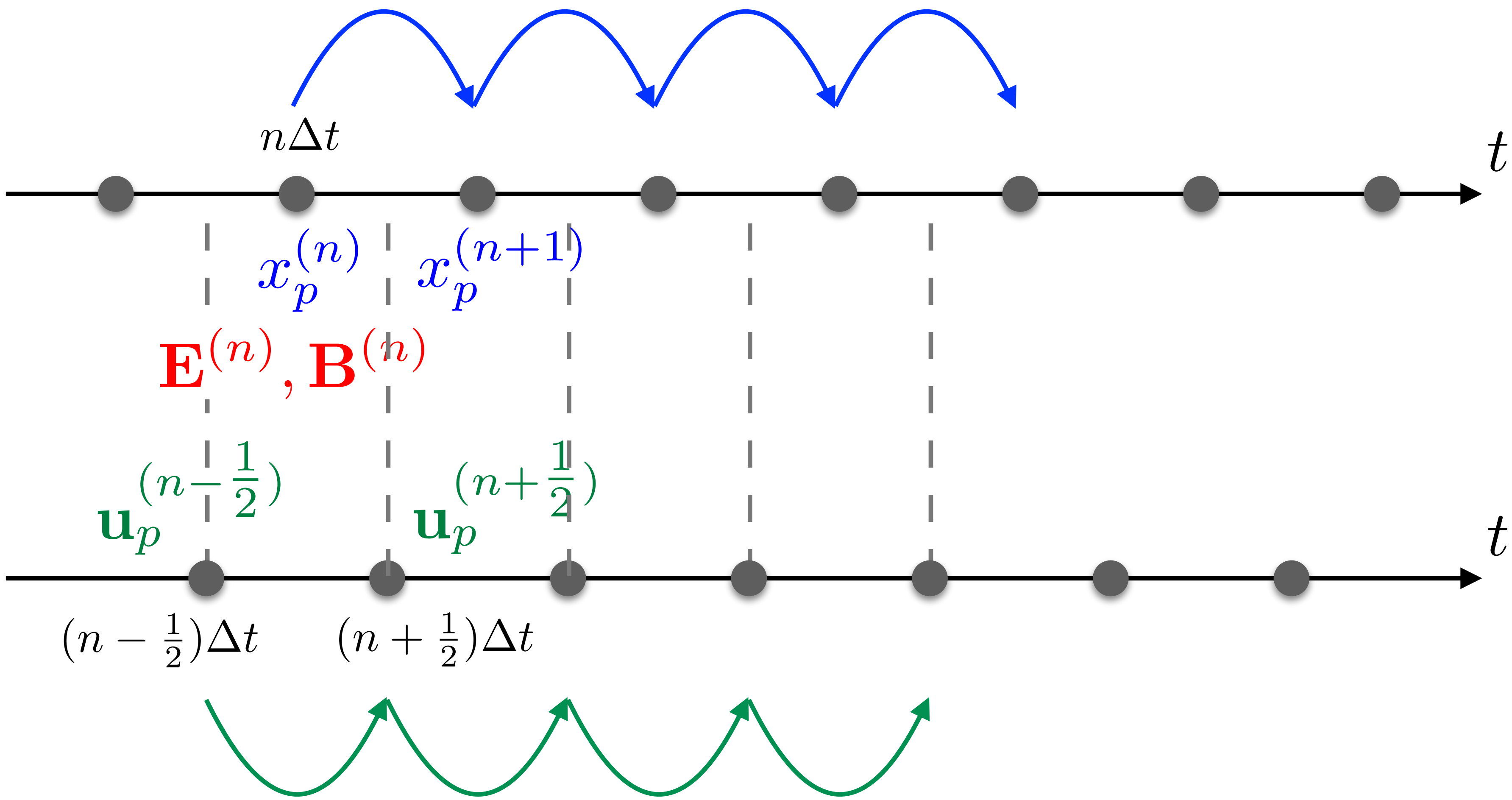
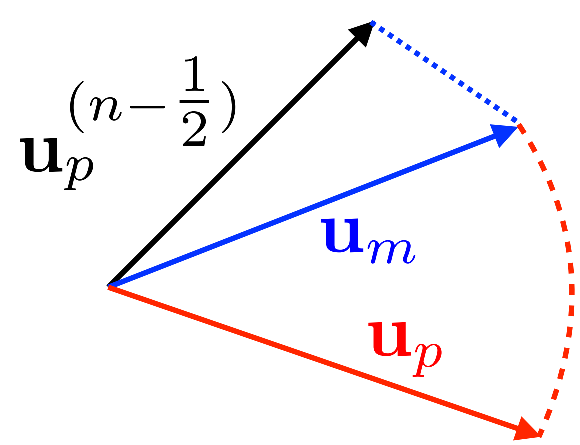


$$\mathbf{u}_m = \mathbf{u}_p^{(n-\frac{1}{2})} + \frac{q_s}{m_s}\frac{\Delta t}{2}\mathbf{E}_p$$

$$\mathbf{u}_p = \mathbf{u}_p^{(n-\frac{1}{2})} + \frac{q_s}{m_s}\Delta t\,\mathcal{M}(\mathbf{B}_p)\,\mathbf{u}_m$$



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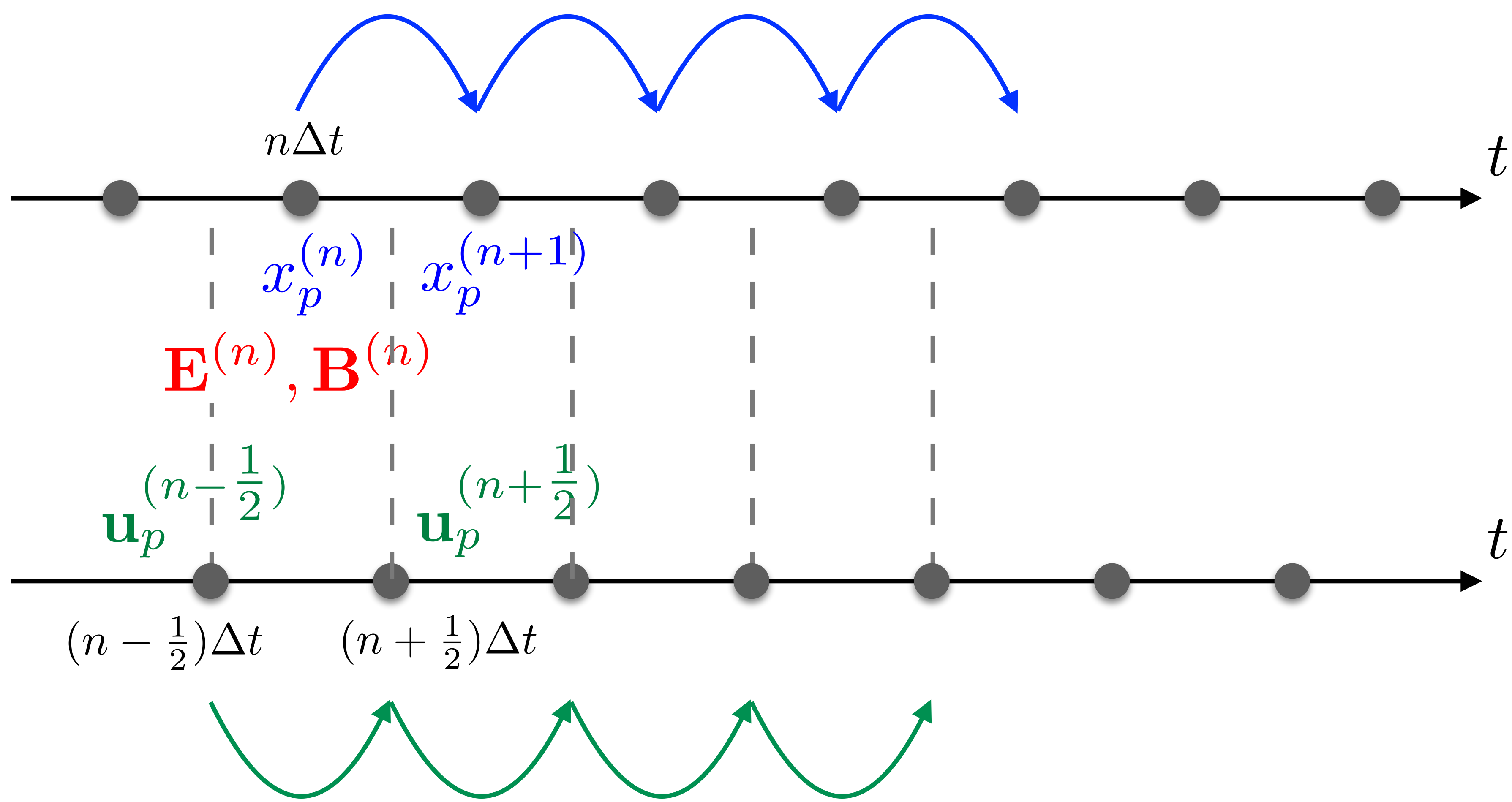

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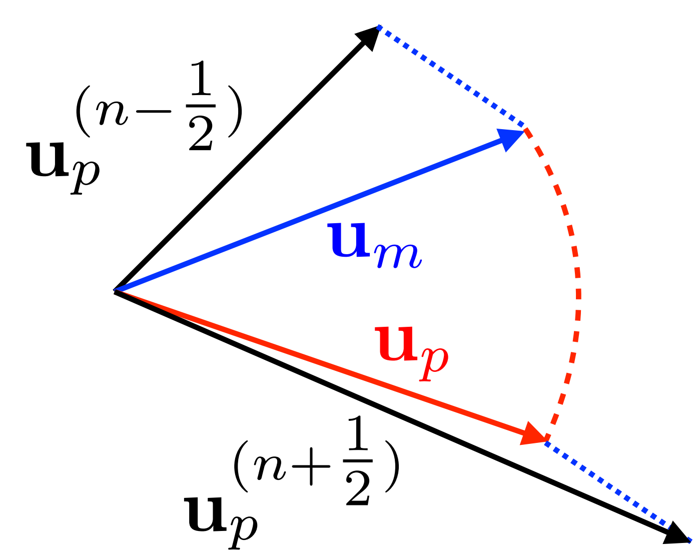
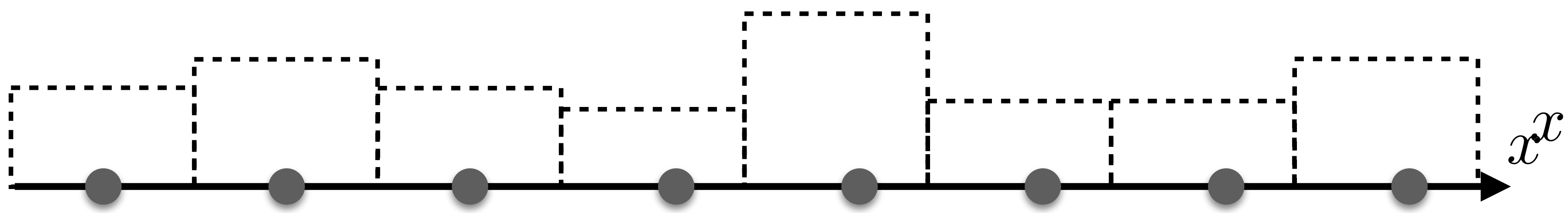
$$\mathbf{u}_p^{(n+\frac{1}{2})} = \mathbf{u}_p + \frac{q_s}{m_s}\frac{\Delta t}{2}\mathbf{E}_p$$



Step 3

Charge-conserving current deposition scheme are available among which Esirkepov's is 'most' popular



Esirkepov, Comp. Phys. Comm. **135**, 144 (2001)

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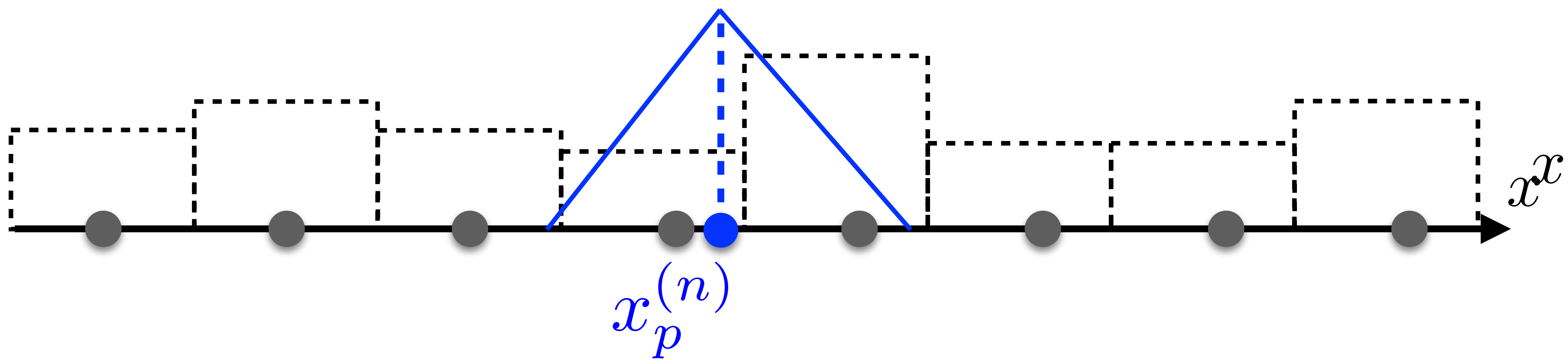

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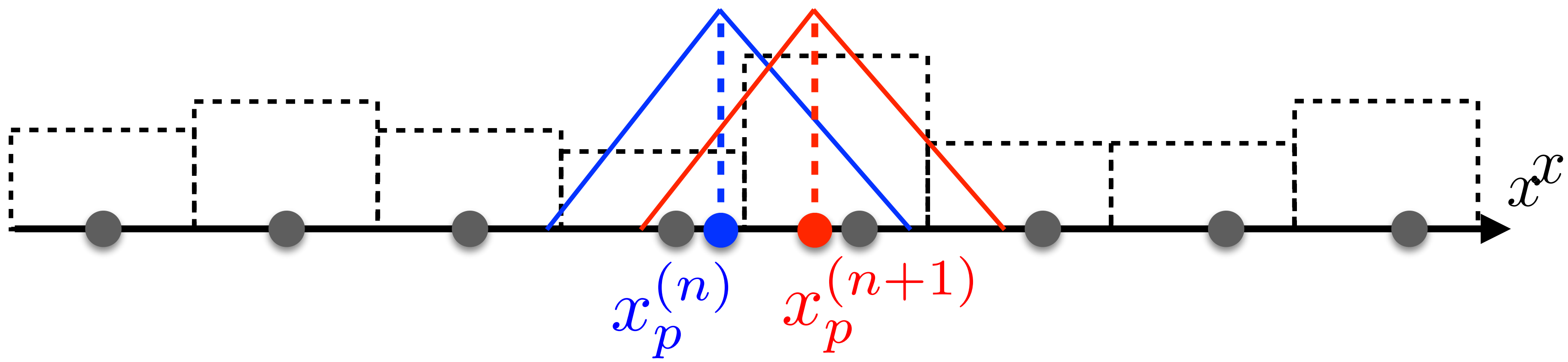

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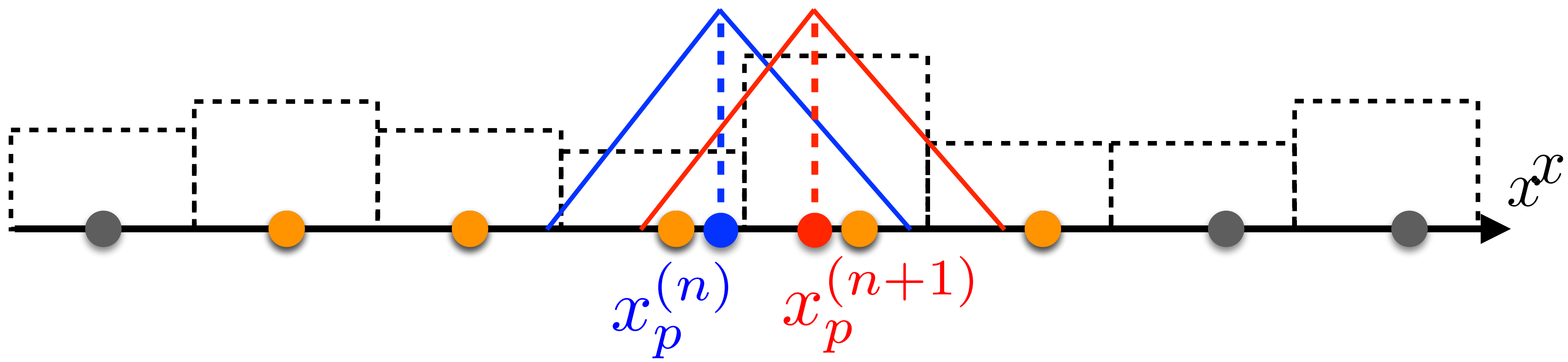

In 1D, current deposition is easily done directly from charge conservation:

$$\partial_x J_x = -\partial_t \rho$$

while other component are 'directly' projected onto the grid (see interpolation)

Esirkepov, Comp. Phys. Comm. **135**, 144 (2001)

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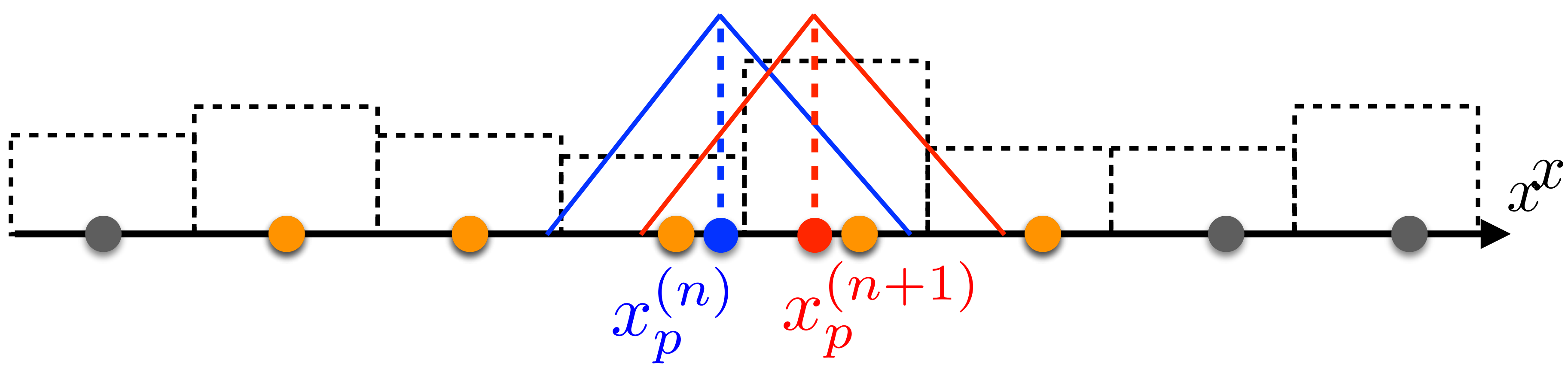

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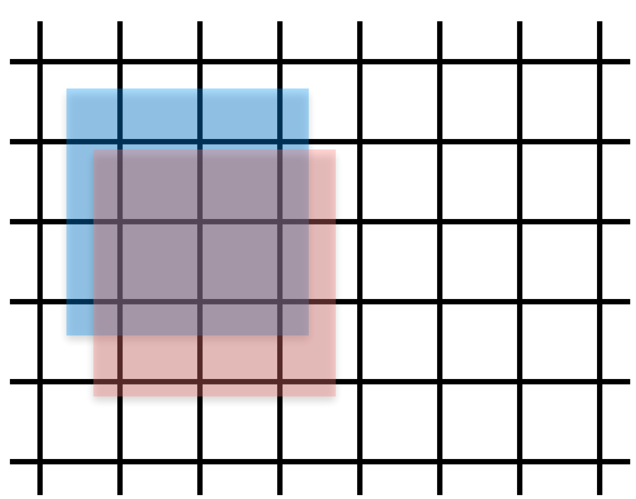
while other component are 'directly' projected onto the grid (see interpolation)

In 2D & 3D, Esirkepov's method allows to conserve charge (within machine presicion)



$$(J_{x,p})_{i+\frac{1}{2},j}^{(n+\frac{1}{2})} = (J_{x,p})_{i-\frac{1}{2},j}^{(n+\frac{1}{2})} + q_s\, w_p\, \frac{\Delta x}{\Delta t}\, (W_x)_{i+\frac{1}{2},j}^{(n+\frac{1}{2})}$$

$$(J_{y,p})_{i,j+\frac{1}{2}}^{(n+\frac{1}{2})} = (J_{y,p})_{i,j-\frac{1}{2}}^{(n+\frac{1}{2})} + q_s\, w_p\, \frac{\Delta y}{\Delta t}\, (W_y)_{j,i+\frac{1}{2}}^{(n+\frac{1}{2})}$$

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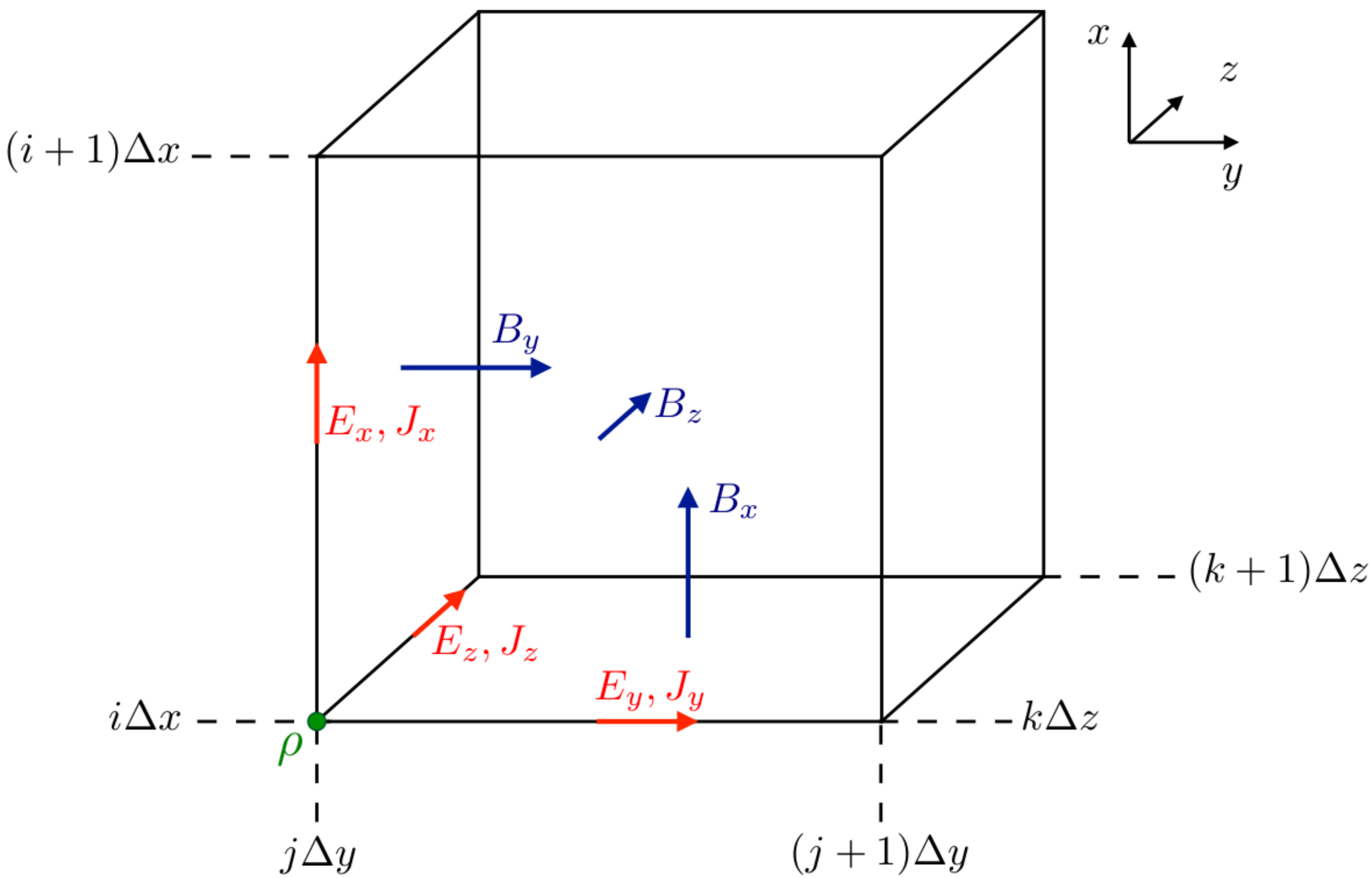
Step 4

The Finite-Difference Time-Domain (FDTD) method is a popular method for solving Maxwell's Equations



A. Taflove, *Computation electrodynamics: The finite-difference time-domain method*, 3rd Ed. (2005)

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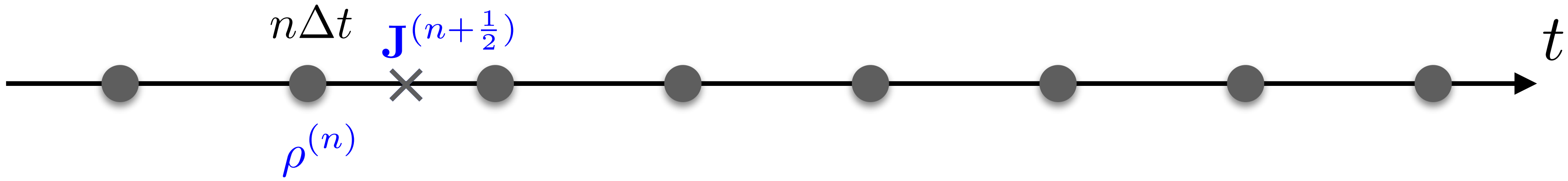

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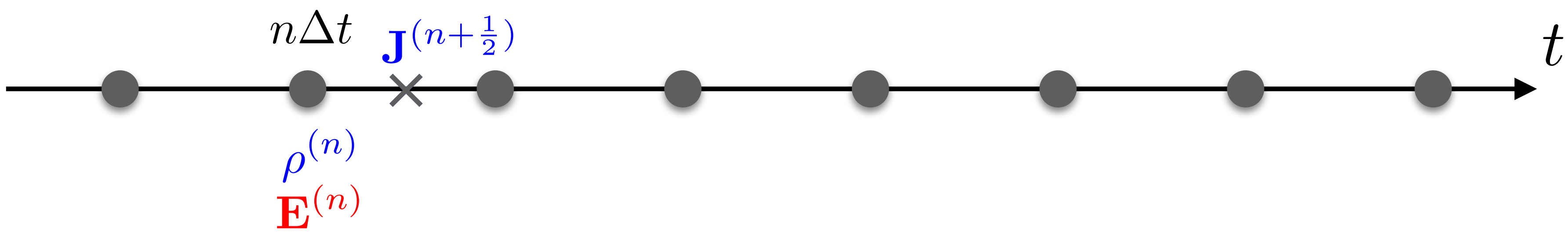

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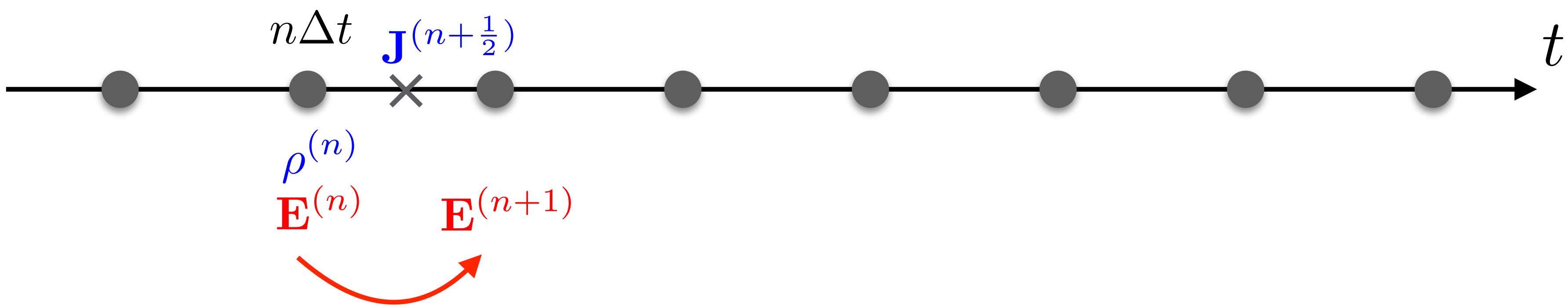

Solving Ampère's equation: $\partial_t E_y = -J_y - \partial_x B_z$

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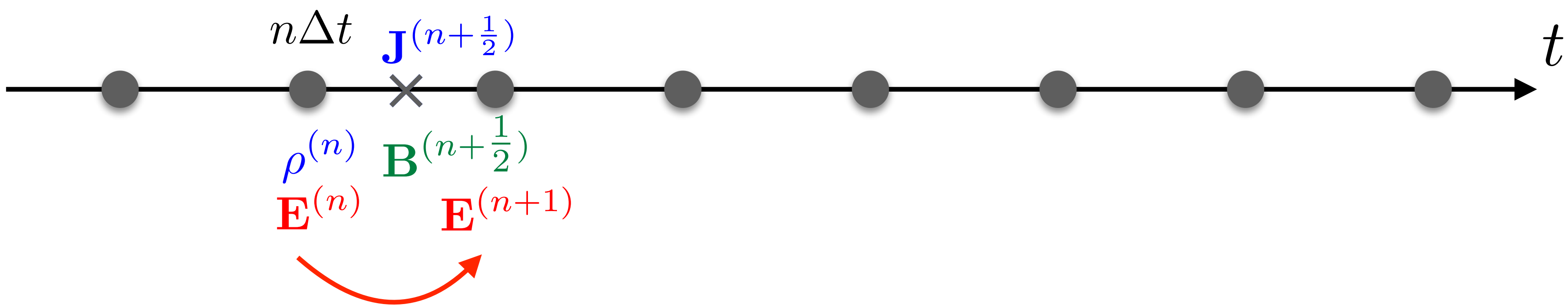

Solving Ampère's equation: $\partial_t E_y = -J_y - \partial_x B_z$

time-centering
$$\frac{(E_y)^{(n+1)} - (E_y)^{(n)}}{\Delta t} = -J_y^{(n+\frac{1}{2})} - (\partial_x B_z)^{(n+\frac{1}{2})}$$

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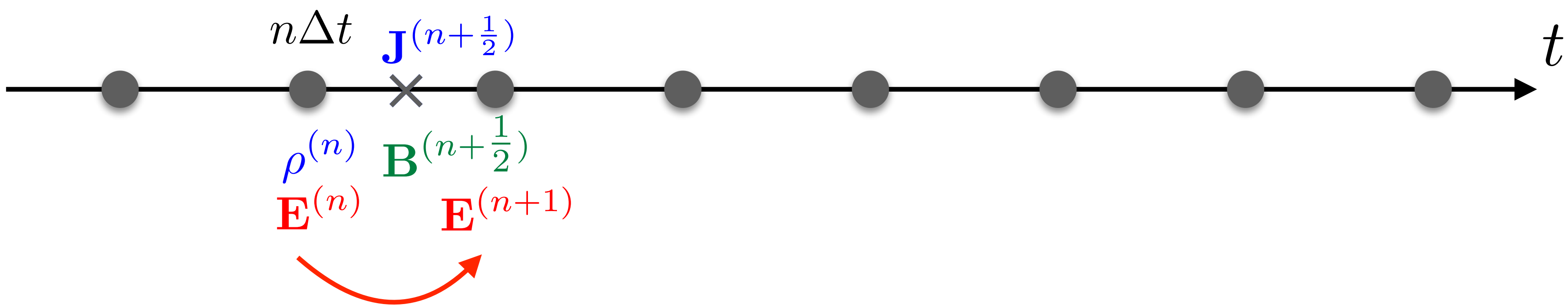

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time-centering
$$\frac{(E_y)^{(n+1)} - (E_y)^{(n)}}{\Delta t} = -J_y^{(n+\frac{1}{2})} - (\partial_x B_z)^{(n+\frac{1}{2})}$$

space-centering
$$\frac{(E_y)_i^{(n+1)} - (E_y)_i^{(n)}}{\Delta t} = -(J_y)_i^{(n+\frac{1}{2})} - \frac{(B_z)_{i+\frac{1}{2}}^{(n+\frac{1}{2})} - (B_z)_{i+\frac{1}{2}}^{(n-\frac{1}{2})}}{\Delta x}$$

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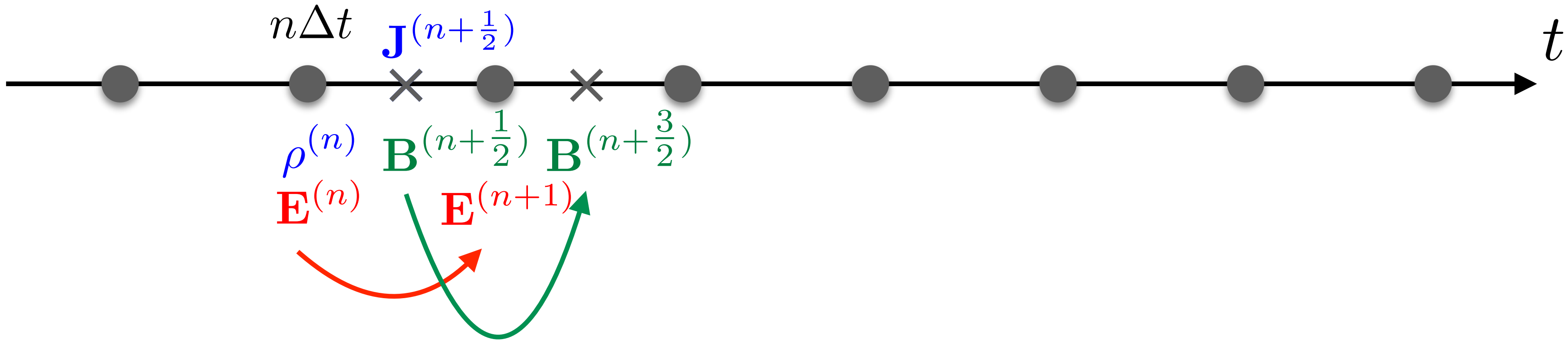

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space-centering $$\frac{(E_y)_i^{(n+1)} - (E_y)_i^{(n)}}{\Delta t} = -(J_y)_i^{(n+\frac{1}{2})} - \frac{(B_z)_{i+\frac{1}{2}}^{(n+\frac{1}{2})} - (B_z)_{i+\frac{1}{2}}^{(n-\frac{1}{2})}}{\Delta x}$$

Solving Faraday's equation: $\partial_t B_z = \partial_x E_y$

A. Taflove, *Computation electrodynamics: The finite-difference time-domain method*, 3rd Ed. (2005)

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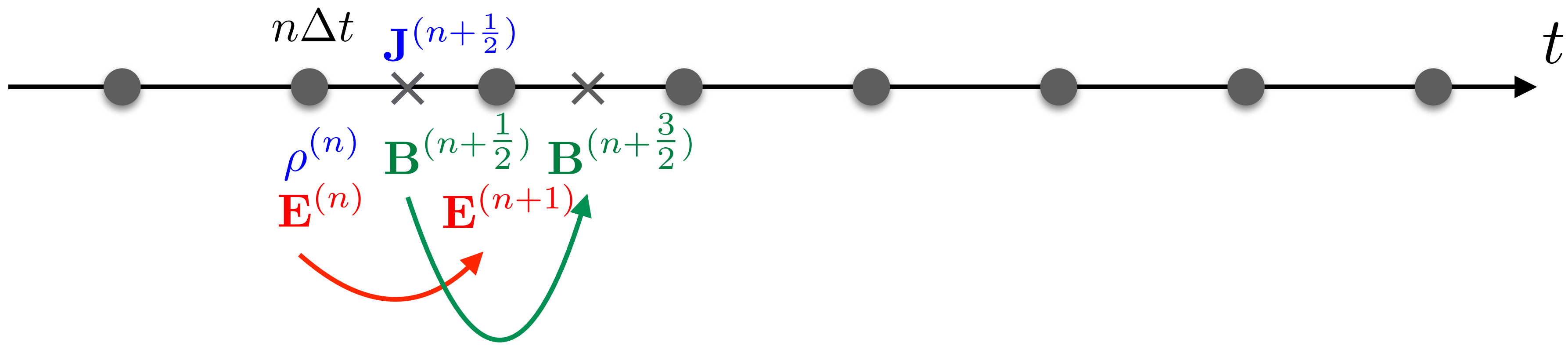

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space-centering
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Solving Faraday's equation: $\partial_t B_z = \partial_x E_y$

space/time-centering
$$\frac{(B_z)_{i+\frac{1}{2}}^{(n+\frac{3}{2})} - (B_z)_{i+\frac{1}{2}}^{(n+\frac{1}{2})}}{\Delta t} = \frac{(E_y)_{i+1}^{(n+1)} - (E_y)_i^{(n+1)}}{\Delta x}$$

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Step 4

Numerical analysis of the FDTD solvers gives you access to the numerical dispersion relation & CFL condition

Nuter et al., Eur. Phys. J. D (2014); All papers by B. Godfrey, from the 70's up to now !!!

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Numerical analysis of the FDTD solvers gives you access to the numerical dispersion relation & CFL condition

The *numerical* electromagnetic wave equation in a vacuum

$$\partial_t^N \mathbf{E} = +\nabla^N \times \mathbf{B}$$

$$\partial_t^N \mathbf{B} = -\nabla^N \times \mathbf{E}$$

Nuter et al., Eur. Phys. J. D (2014); All papers by B. Godfrey, from the 70's up to now !!!

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Numerical analysis of the FDTD solvers gives you access to the numerical dispersion relation & CFL condition

The *numerical* electromagnetic wave equation in a vacuum

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Looking for *numerical* solution in the form:

$$(E_y)^{(n)}_{i,j+\frac{1}{2},k} = E_{y0}\,\exp\{\imath\left[ik_x\Delta x + (j+\tfrac{1}{2})k_y\Delta y \right.$$
$$\left. + kk_z\Delta z - n\omega\Delta t\right]\}$$

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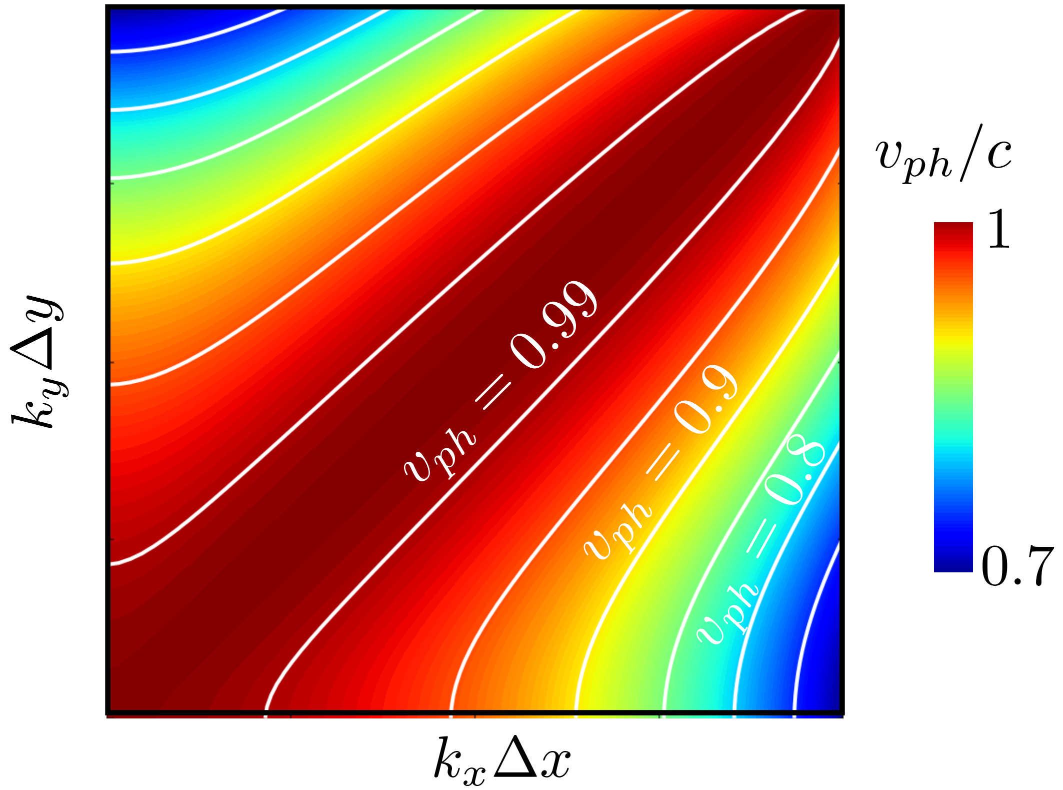
The FDTD solver is subject to *numerical dispersion* as the numerical light wave velocity is found to depend on its wavenumber and orientation.



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A quick summary

The PIC approach in a nutshell

Initialization	time step $n = 0$, time $t = 0$

Particle loading $\quad \forall p$, define $(\mathbf{x}_p)^{n=0}$, $(\mathbf{u}_p)^{n=-\frac{1}{2}}$

Charge projection on grid $\quad \left[\forall p, (\mathbf{x}_p)^{n=0}\right] \rightarrow \rho^{(n=0)}(\mathbf{x})$

Compute initial fields

- solve Poisson on grid: $\left[\rho^{(n=0)}(\mathbf{x})\right] \rightarrow \mathbf{E}^{(n=0)}_{\text{stat}}(\mathbf{x})$
- add external fields: $\mathbf{E}^{(n=0)}(\mathbf{x}) = \mathbf{E}^{(n=0)}_{\text{stat}}(\mathbf{x}) + \mathbf{E}^{(n=0)}_{\text{ext}}(\mathbf{x})$

$$\mathbf{B}^{(n=\frac{1}{2})}(\mathbf{x}) = \mathbf{B}^{(n=\frac{1}{2})}_{\text{ext}}(\mathbf{x})$$

PIC loop: from time step n to $n+1$, time $t = (n+1)\,\Delta t$

Restart charge & current densities
Save magnetic fields value (used to center magnetic fields)

Interpolate fields at particle positions $\quad \forall p, \left[\mathbf{x}_p, \mathbf{E}^{(n)}(\mathbf{x}), \mathbf{B}^{(n)}(\mathbf{x})\right] \rightarrow \mathbf{E}^{(n)}_p, \mathbf{B}^{(n)}_p$

Push particles

- compute new velocity $\forall p$, $\mathbf{p}_p^{(n-\frac{1}{2})} \left[\mathbf{E}^{(n)}_p, \mathbf{B}^{(n)}_p\right] \mathbf{p}_p^{(n+\frac{1}{2})}$
- compute new position $\forall p$, $\mathbf{x}_p^{(n)} \left[\mathbf{p}_p^{(n+\frac{1}{2})}\right] \mathbf{x}_p^{(n+1)}$

Project current onto the grid using a charge-conserving scheme

$$\left[\forall p \;\; \mathbf{x}_p^{(n)}, \mathbf{x}_p^{(n+1)}, \mathbf{p}_p^{(n+\frac{1}{2})}\right] \rightarrow \mathbf{J}^{(n+\frac{1}{2})}(\mathbf{x})$$

Solve Maxwell's equations

- solve Maxwell-Faraday: $\mathbf{E}^{(n)}(\mathbf{x}) \left[\mathbf{J}^{(n+\frac{1}{2})(\mathbf{x})}\right] \mathbf{E}^{(n+1)}(\mathbf{x})$
- solve Maxwell-Ampère: $\mathbf{B}^{(n+\frac{1}{2})}(\mathbf{x}) \left[\mathbf{E}^{(n+1)}(\mathbf{x})\right] \mathbf{B}^{(n+\frac{3}{2})}(\mathbf{x})$
- center magnetic fields: $\mathbf{B}^{(n+1)}(\mathbf{x}) = \frac{1}{2}\left(\mathbf{B}^{(n+\frac{1}{2})}(\mathbf{x}) + \mathbf{B}^{(n+\frac{3}{2})}(\mathbf{x})\right)$

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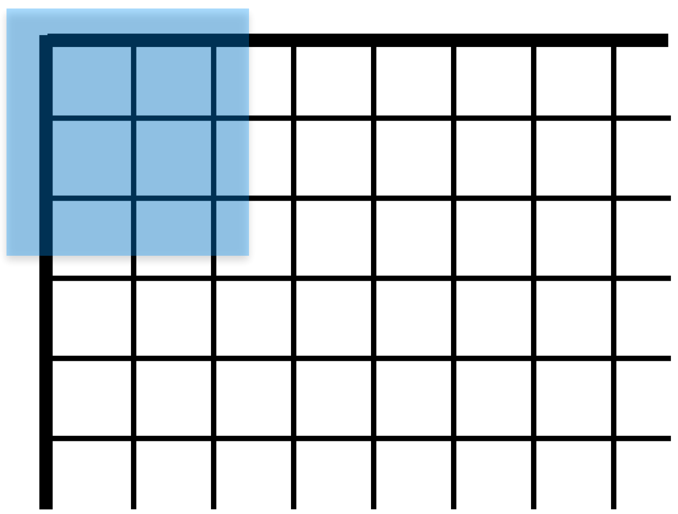

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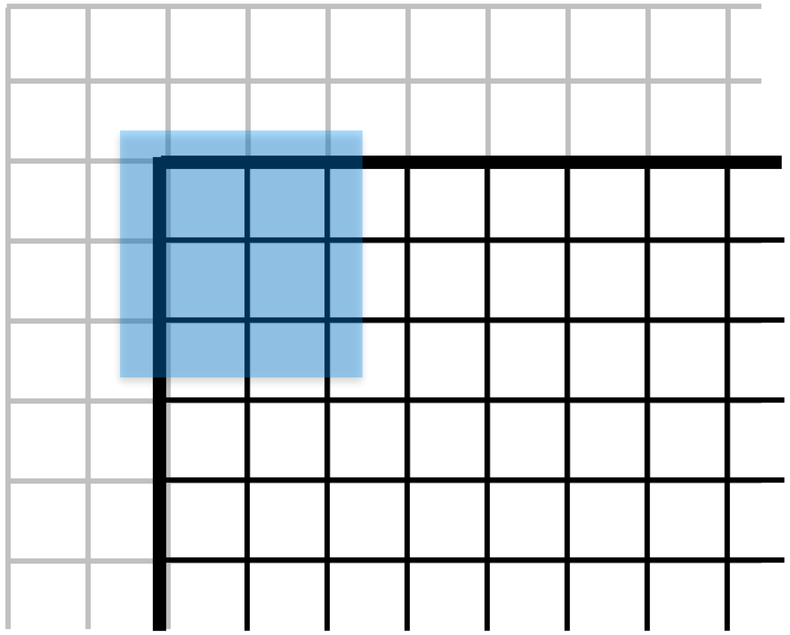

Outlines

- Numerical approach: how to build your PIC code
- High-performance computing:
  getting ready for the super-computers
- Additional modules: beyond the *collisionless* plasma
- Some physics highlights: what you can do with a PIC code

Parallelization is mandatory for large-scale PIC simulation

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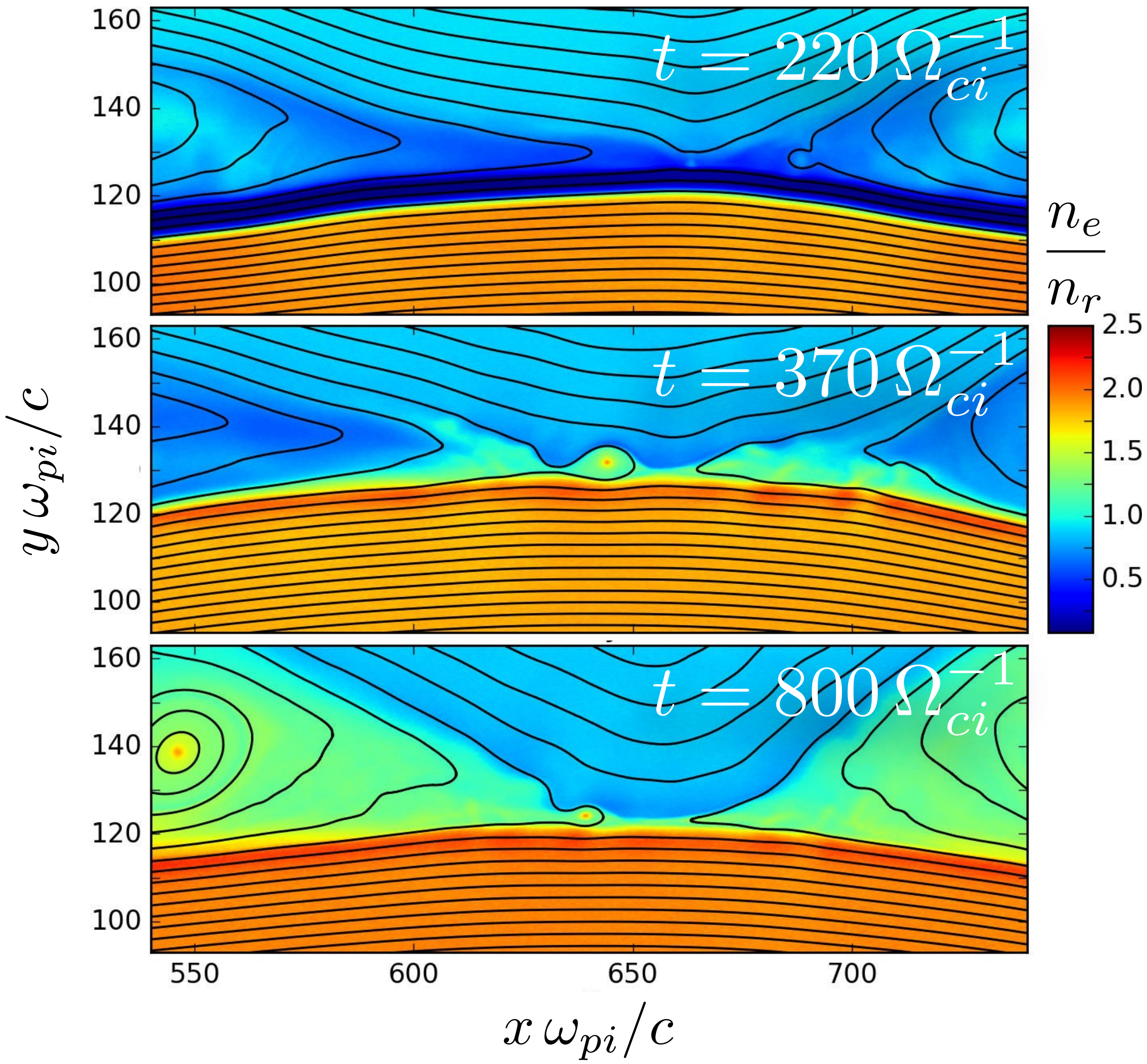
Large scale PIC simulation of
magnetic reconnection at the earth magnetopause



Simulation box: $1280\,\frac{c}{\omega_{pi}} \times 256\,\frac{c}{\omega_{pi}}$

25600×10240 PIC cells

run up to $t = 800\,\Omega_{ci}^{-1}$

$N_t \sim 9.5 \times 10^5$ timesteps

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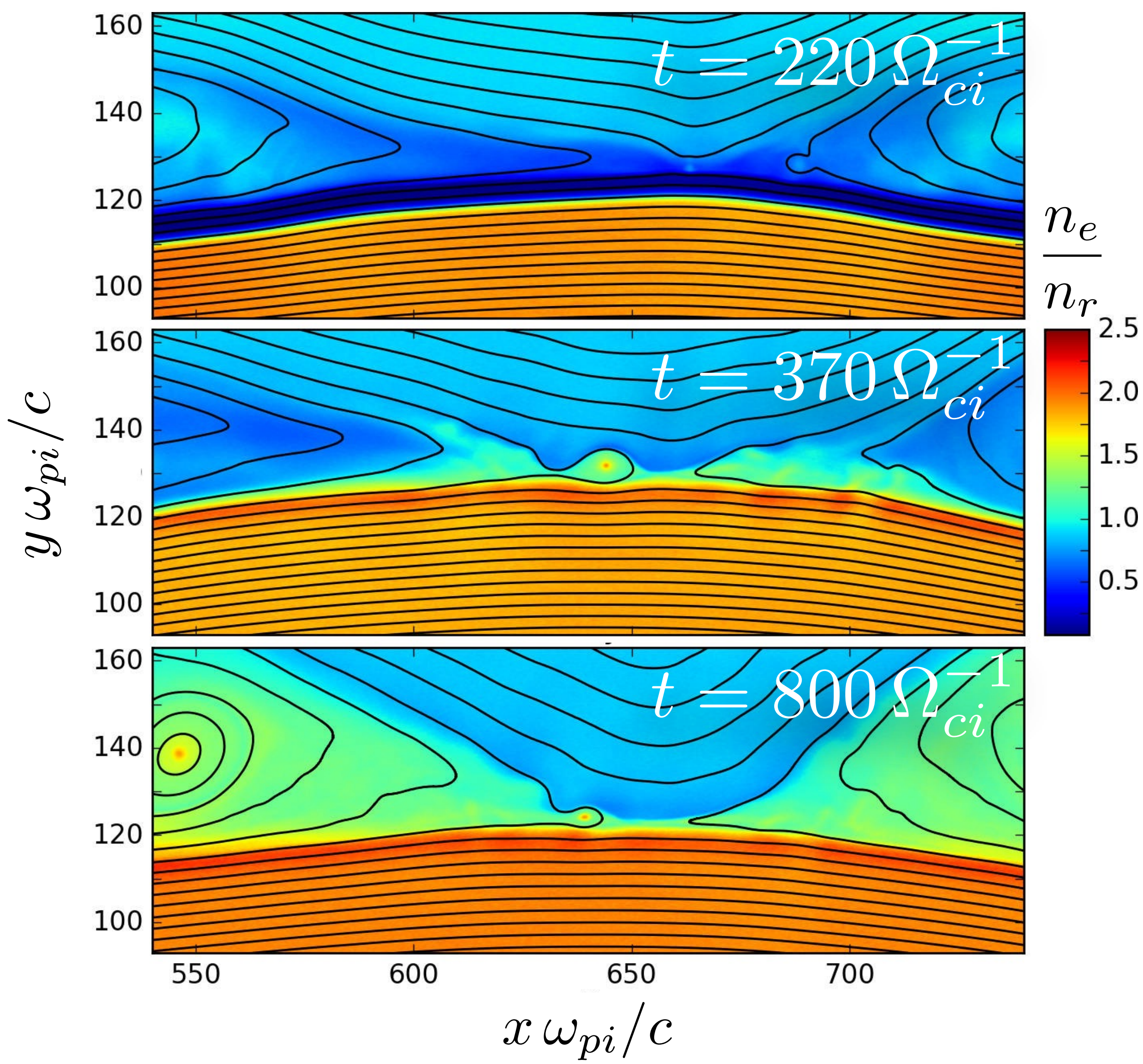

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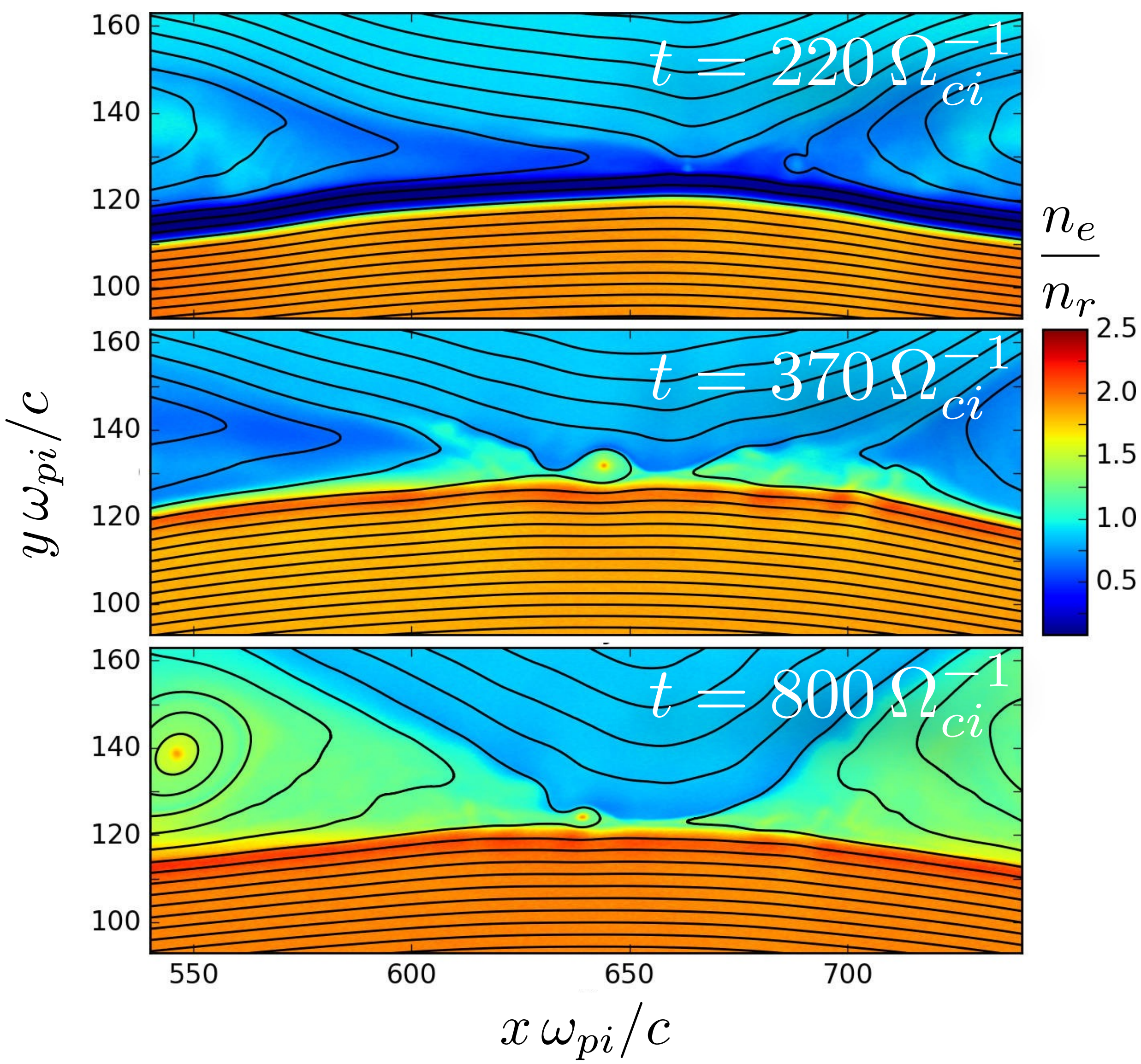

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Solution:
share the work on 16384 CPUs !!!

High-performance computing new paradigms & challenges

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Tianhe-2 34 PF:

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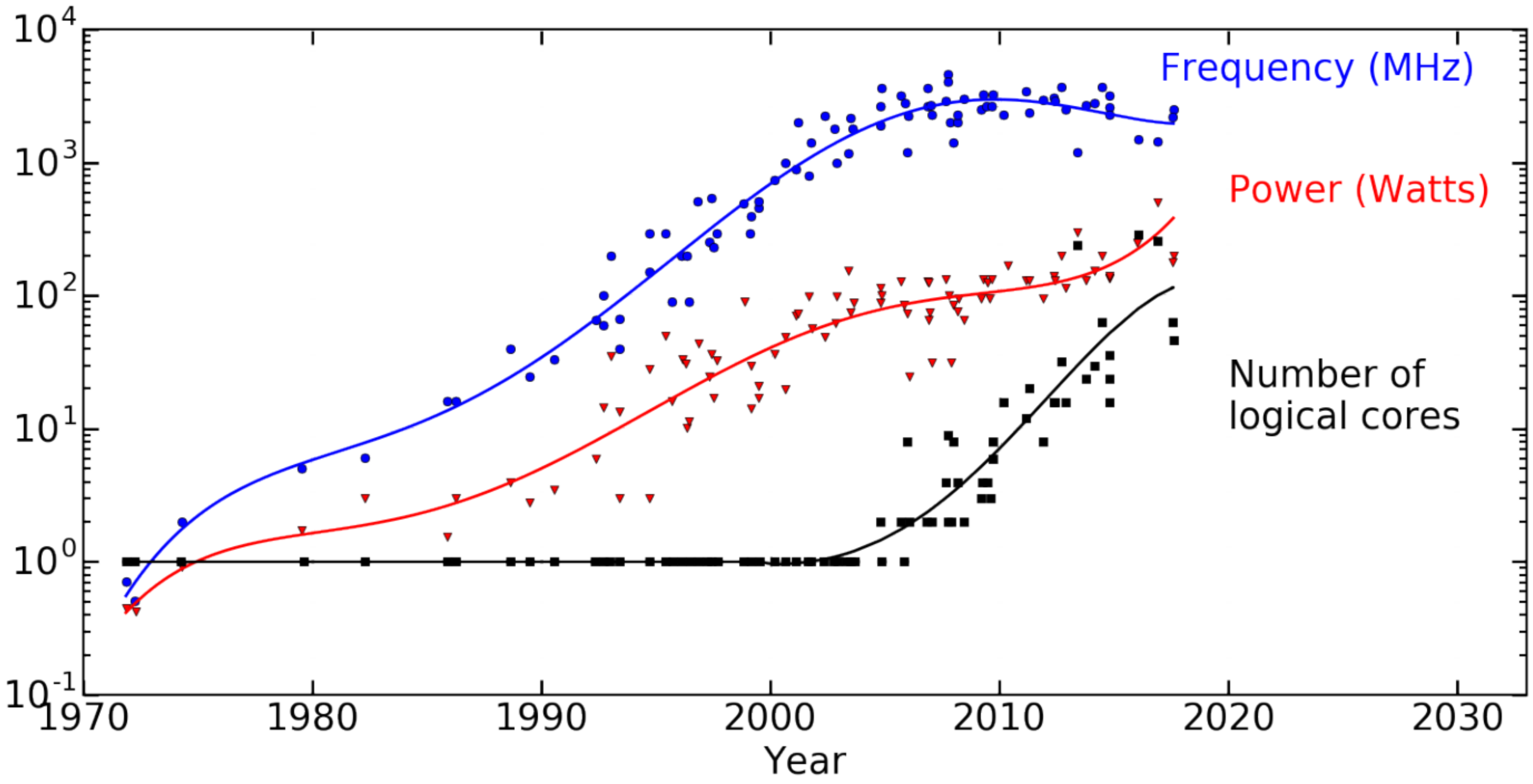

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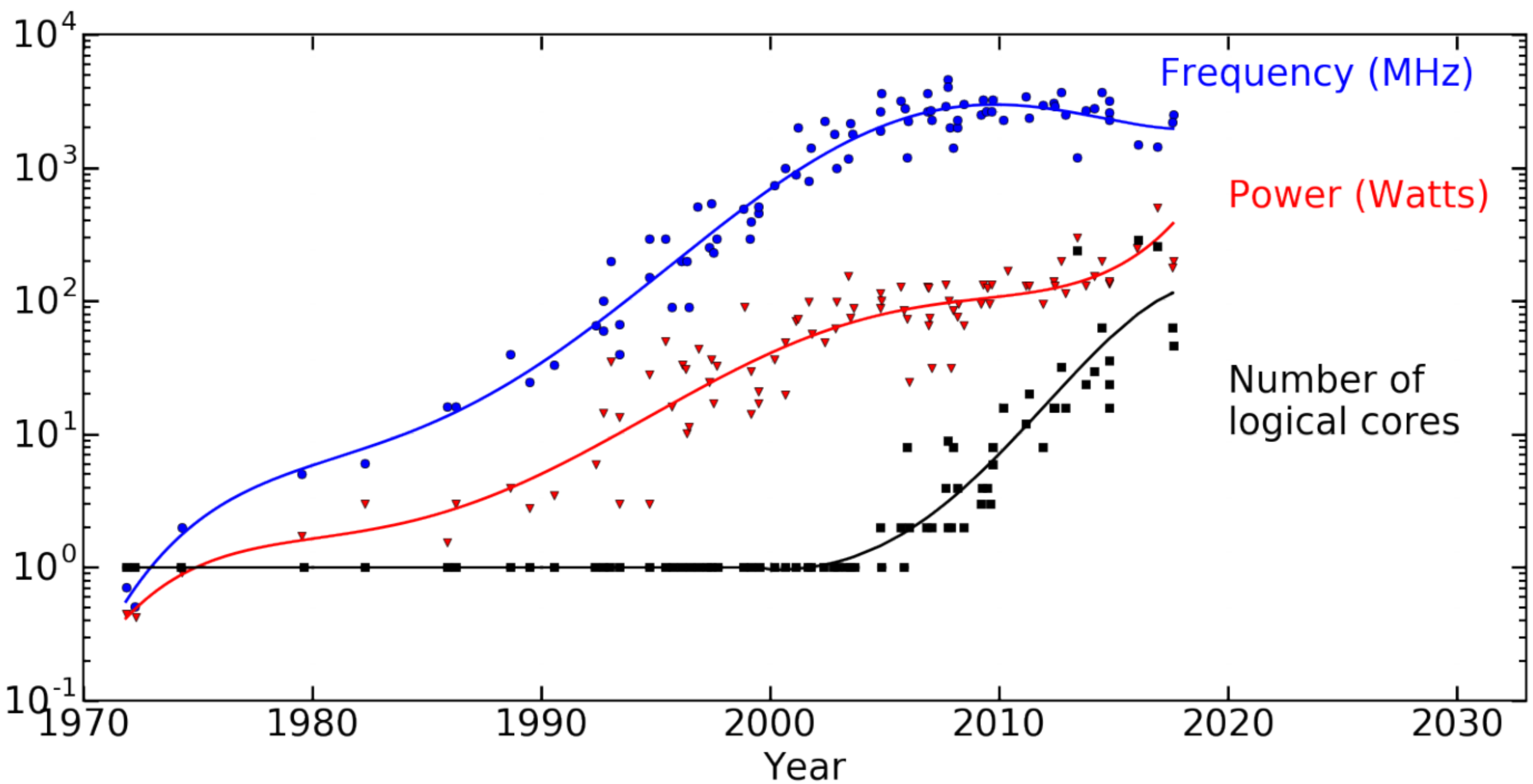

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- massive
- hybrid MPI-OpenMP
- dynamic (load balance)

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- shared vs. distributed
- cache use

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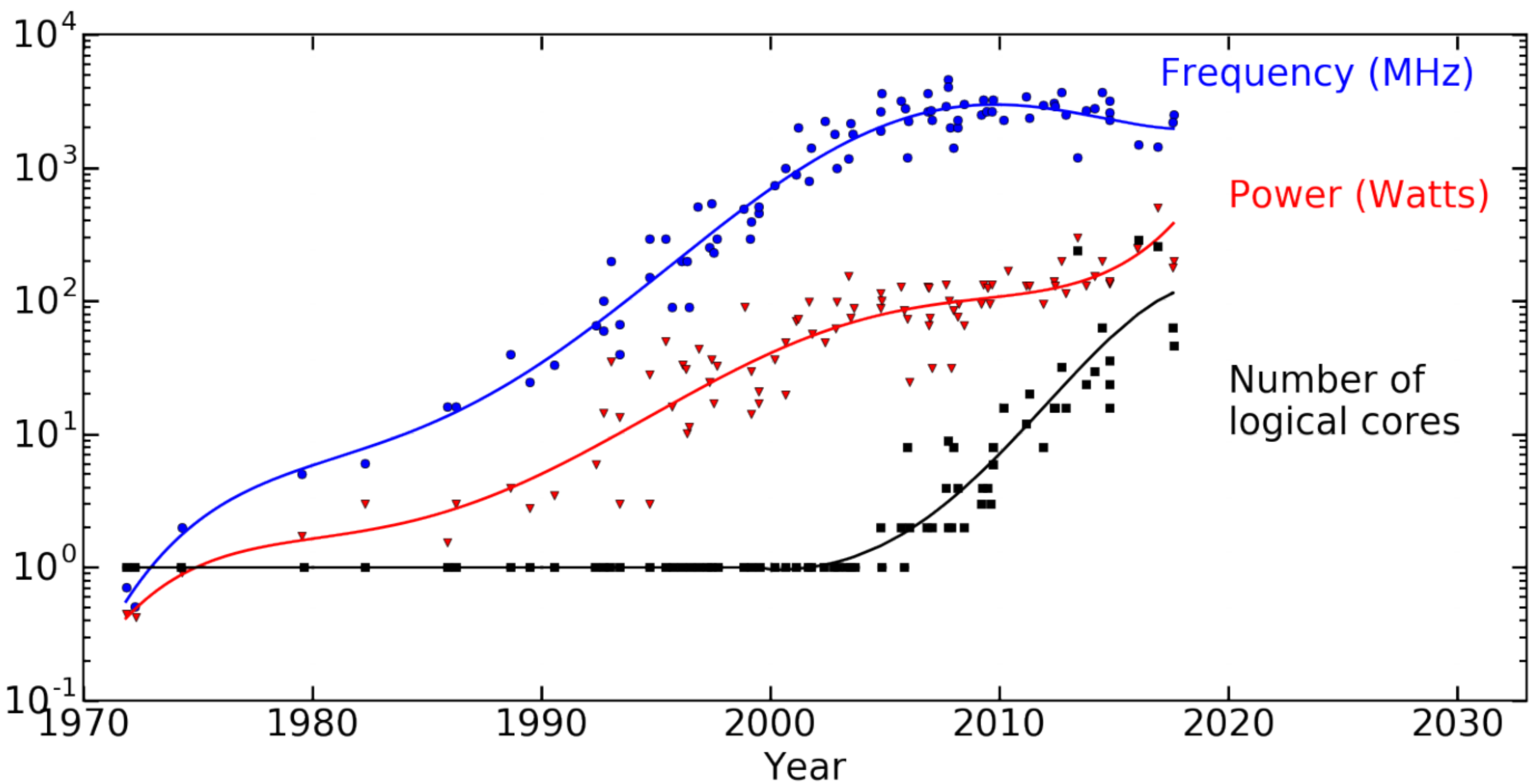

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*Derouillat *et al.*, Comp. Phys. Comm. **222**, 351 (2018)
**Beck *et al.*, arXiv:1810.03949

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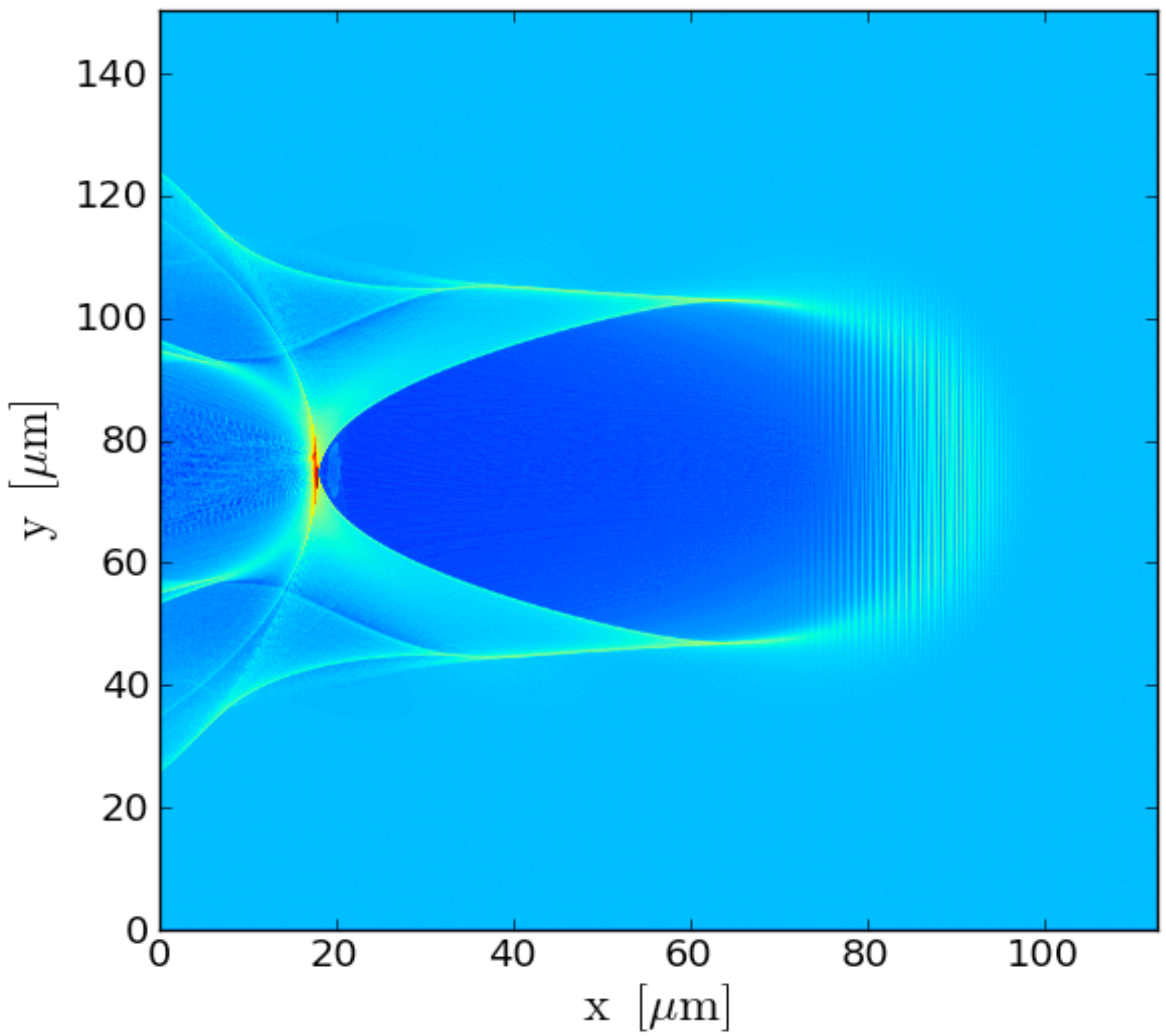

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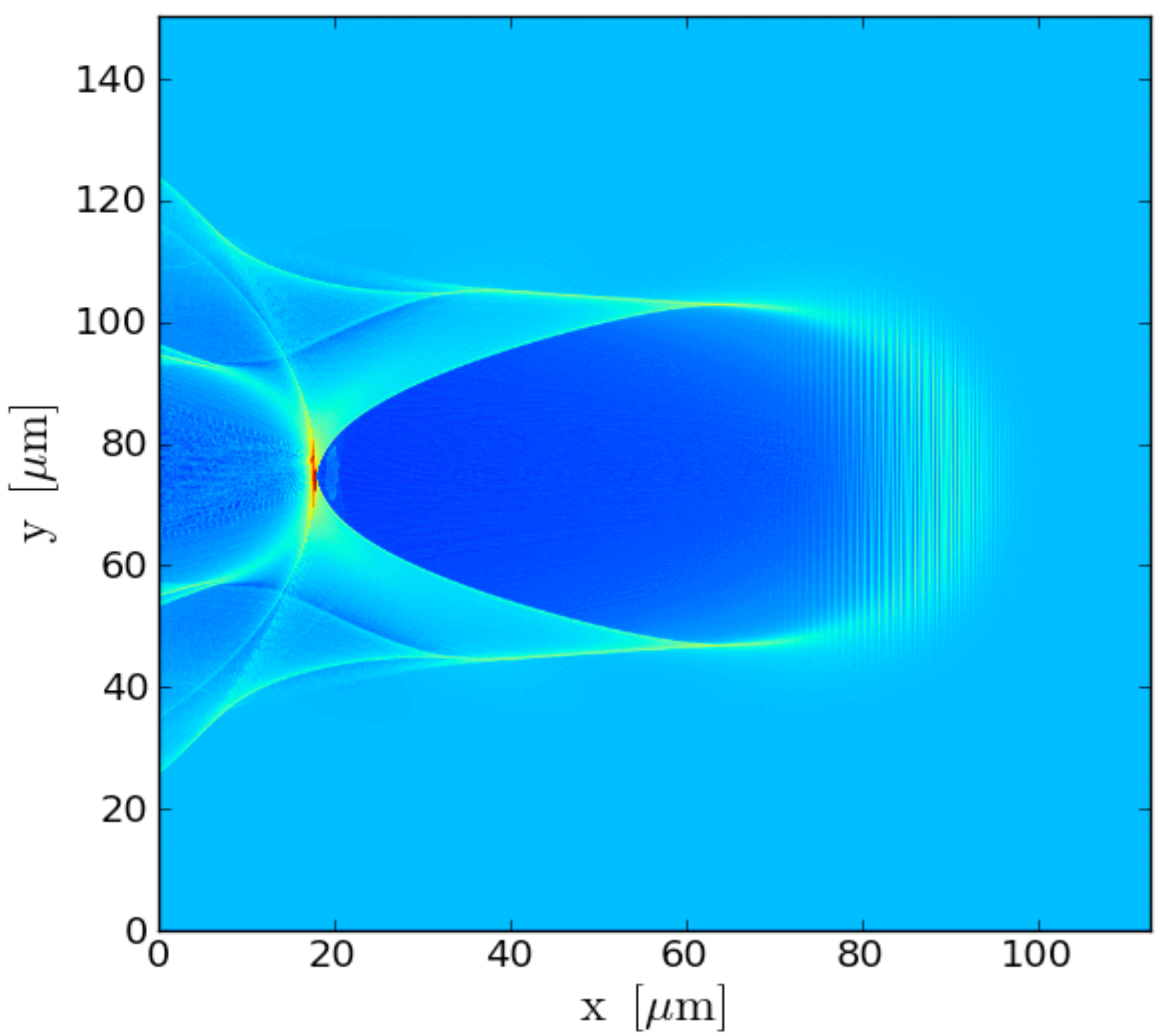

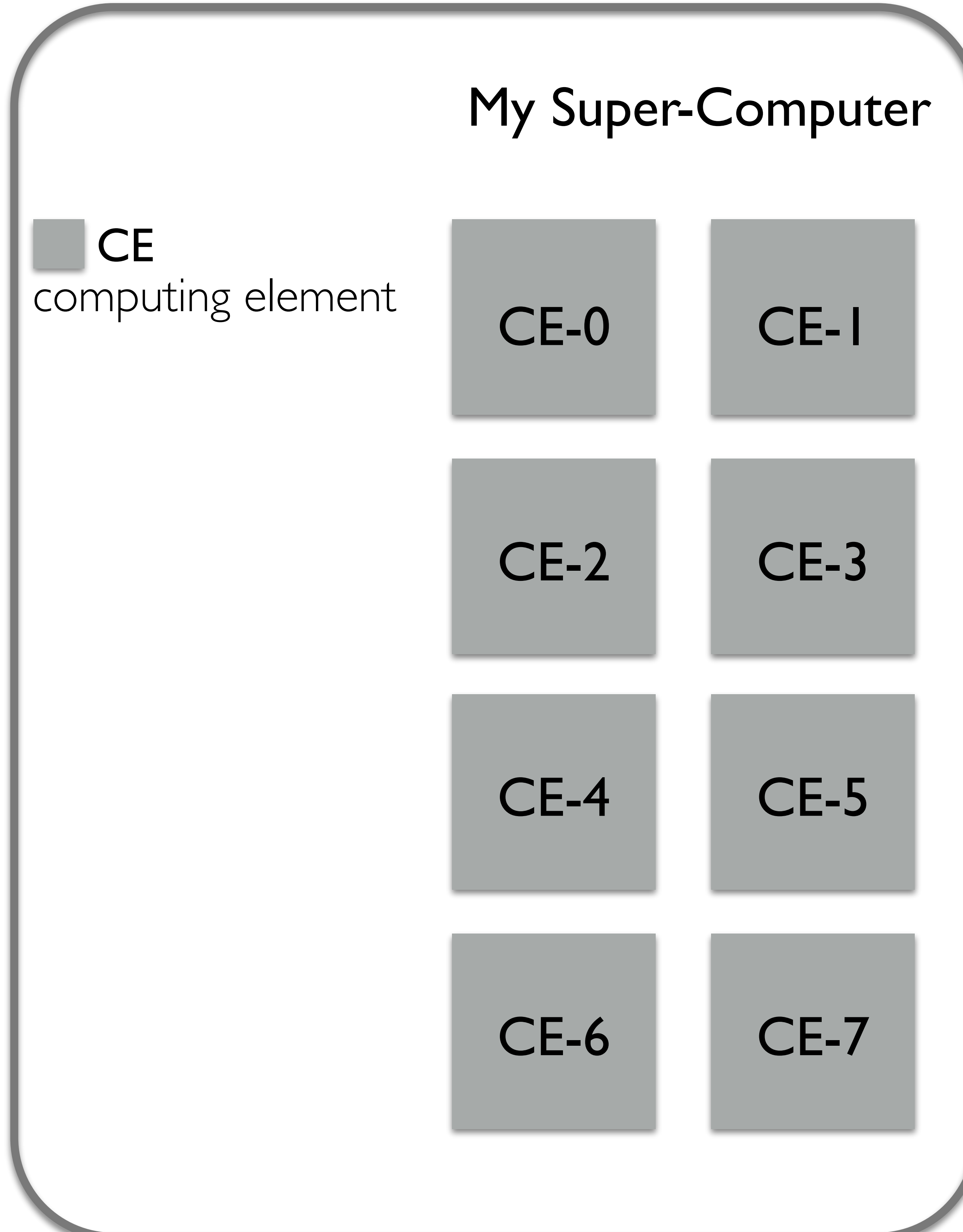

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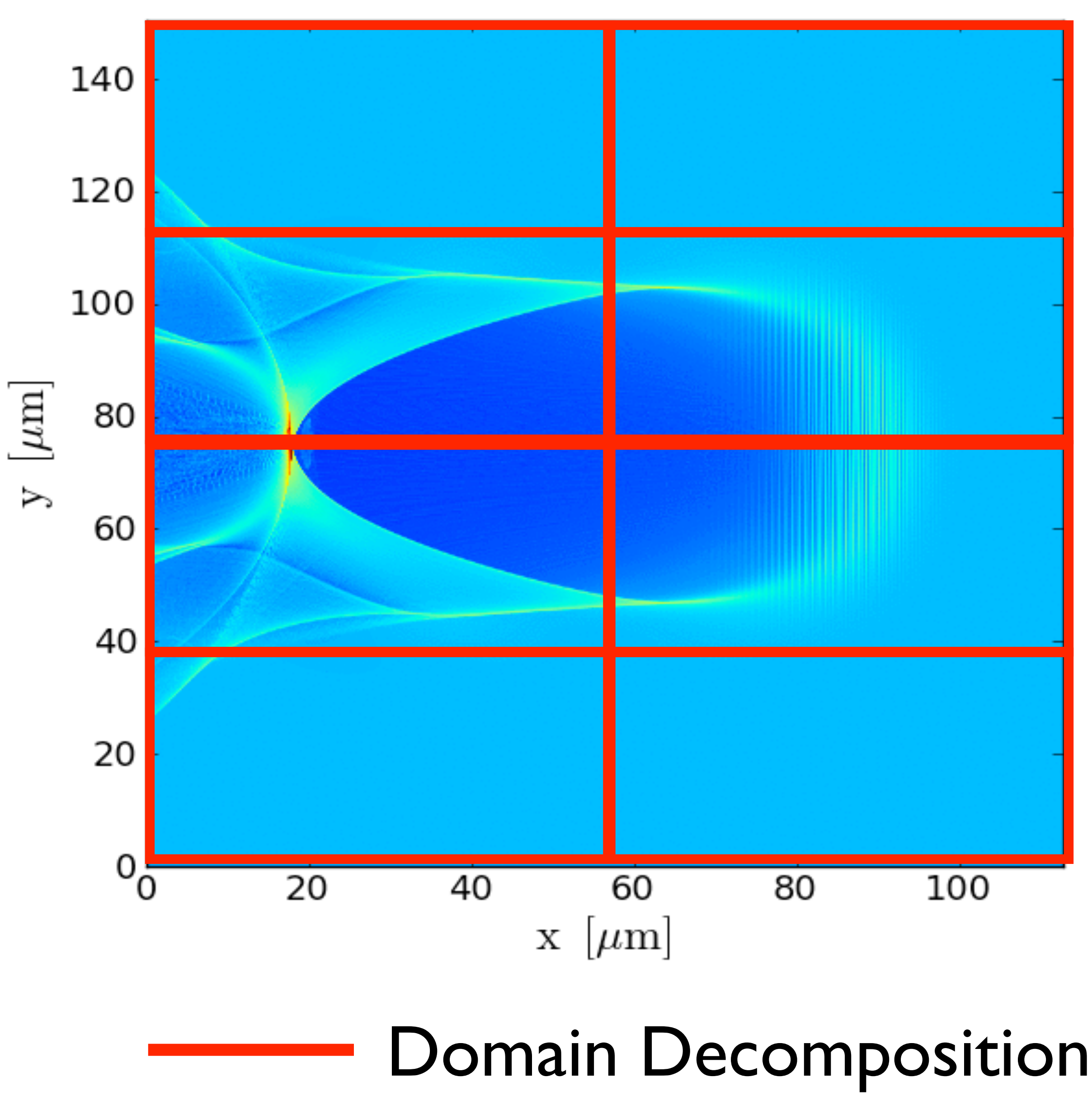

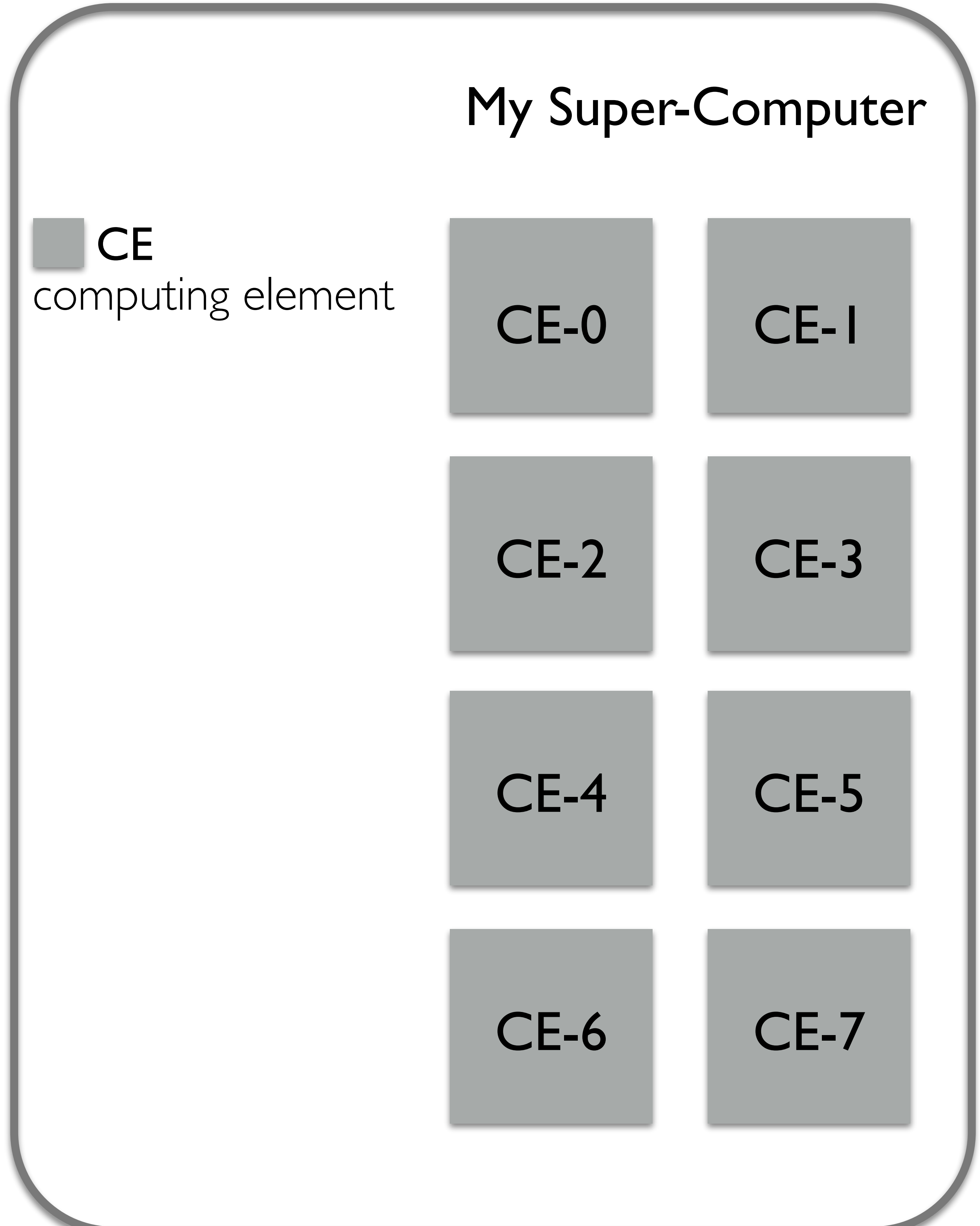

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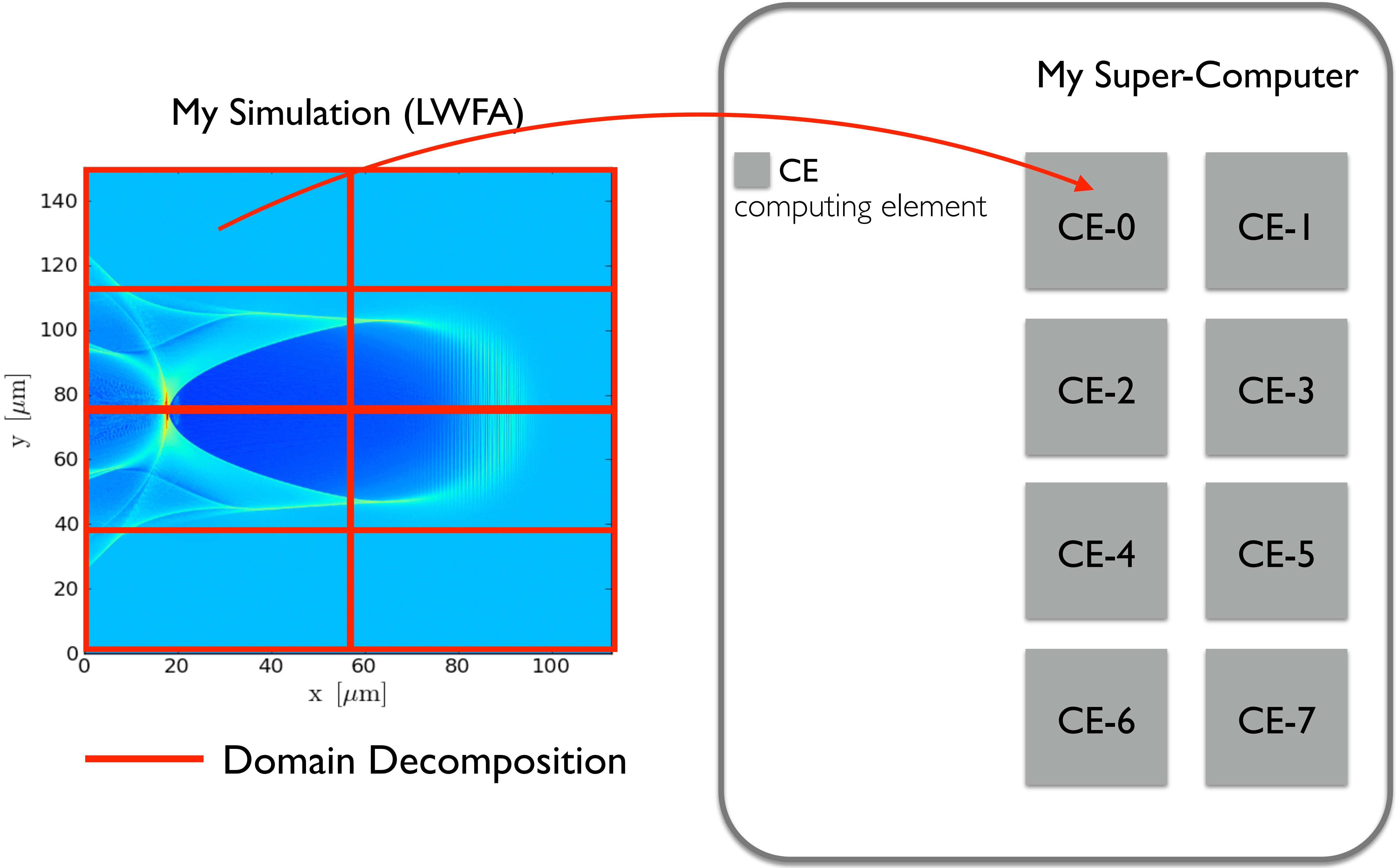

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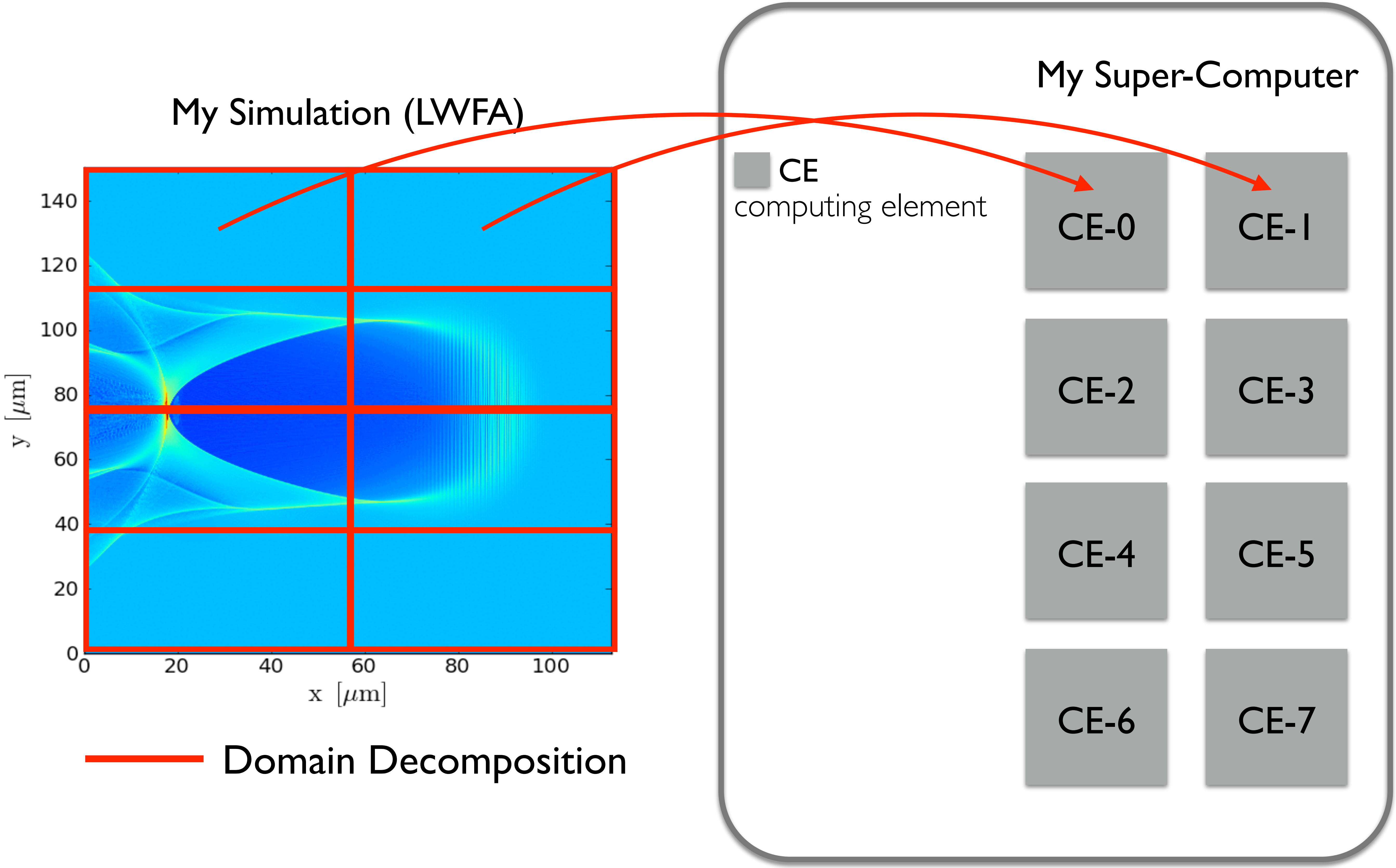

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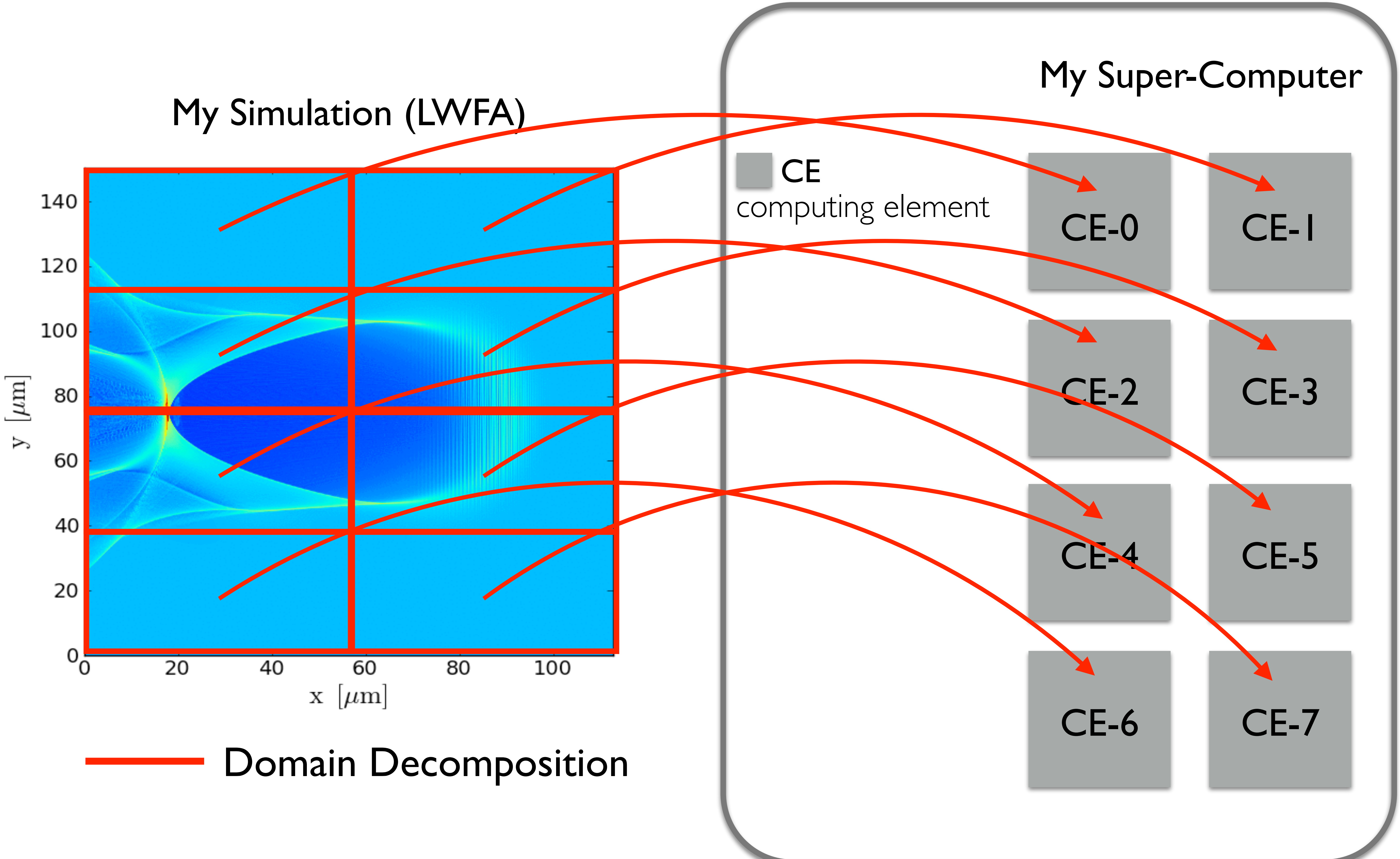

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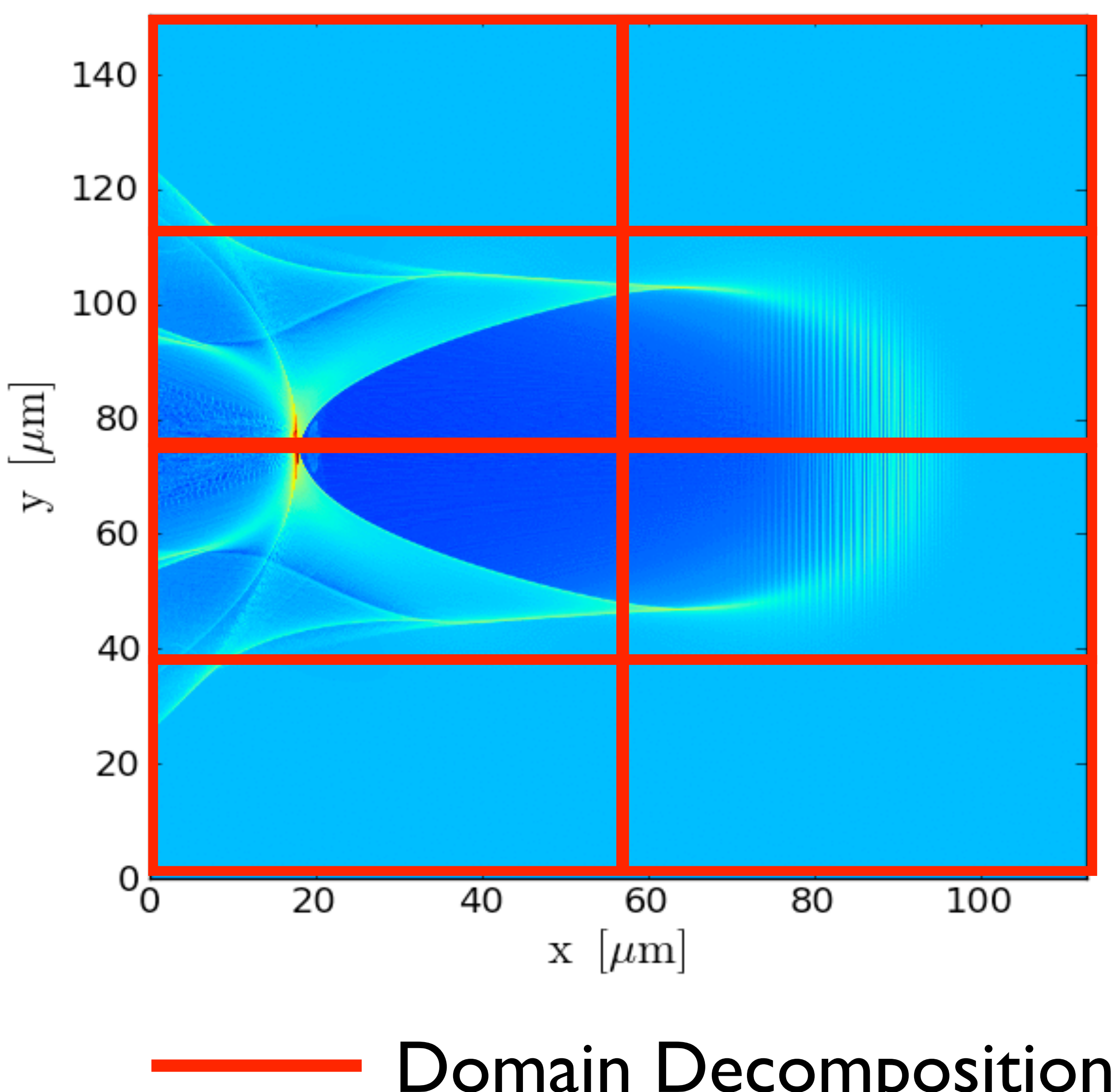

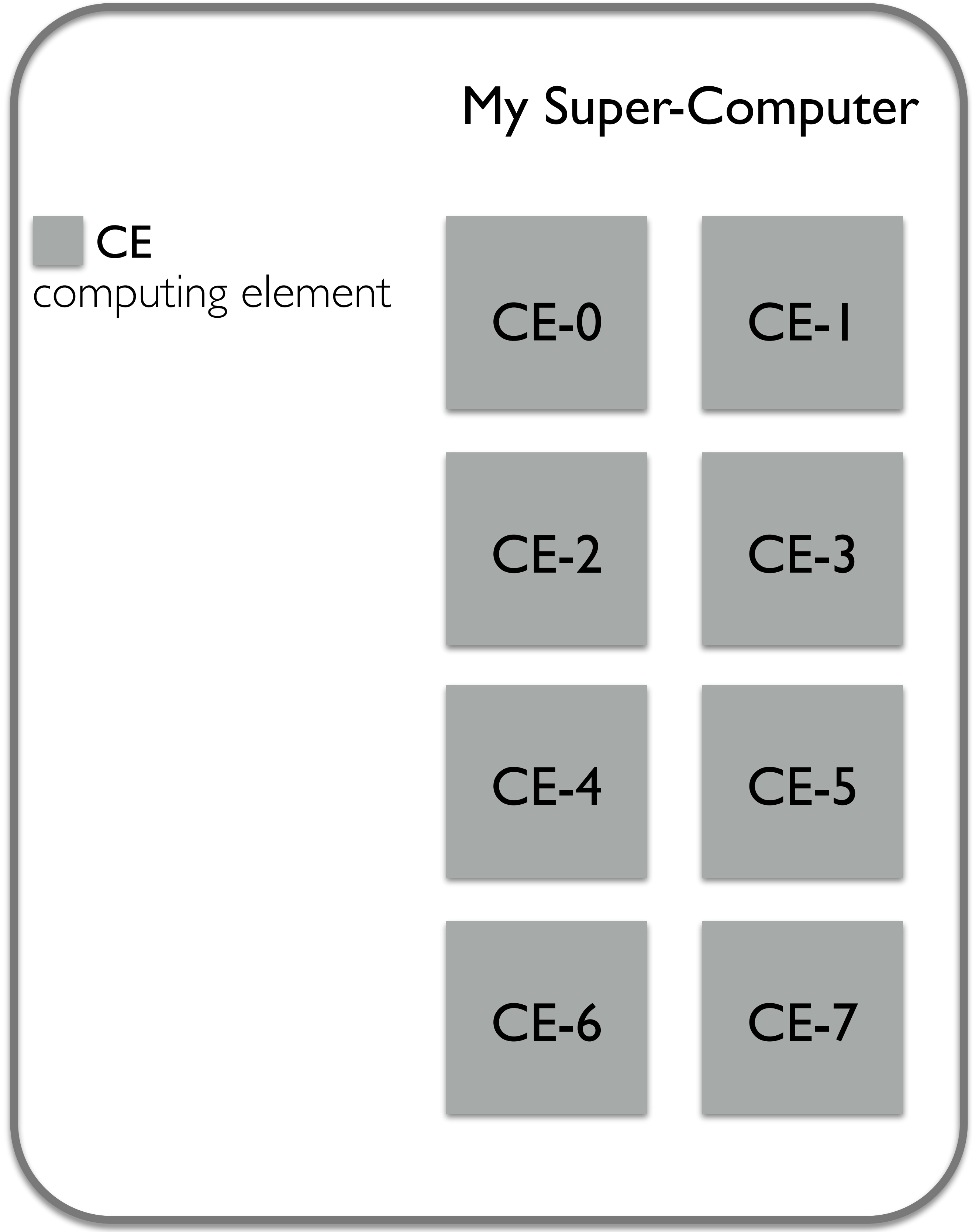

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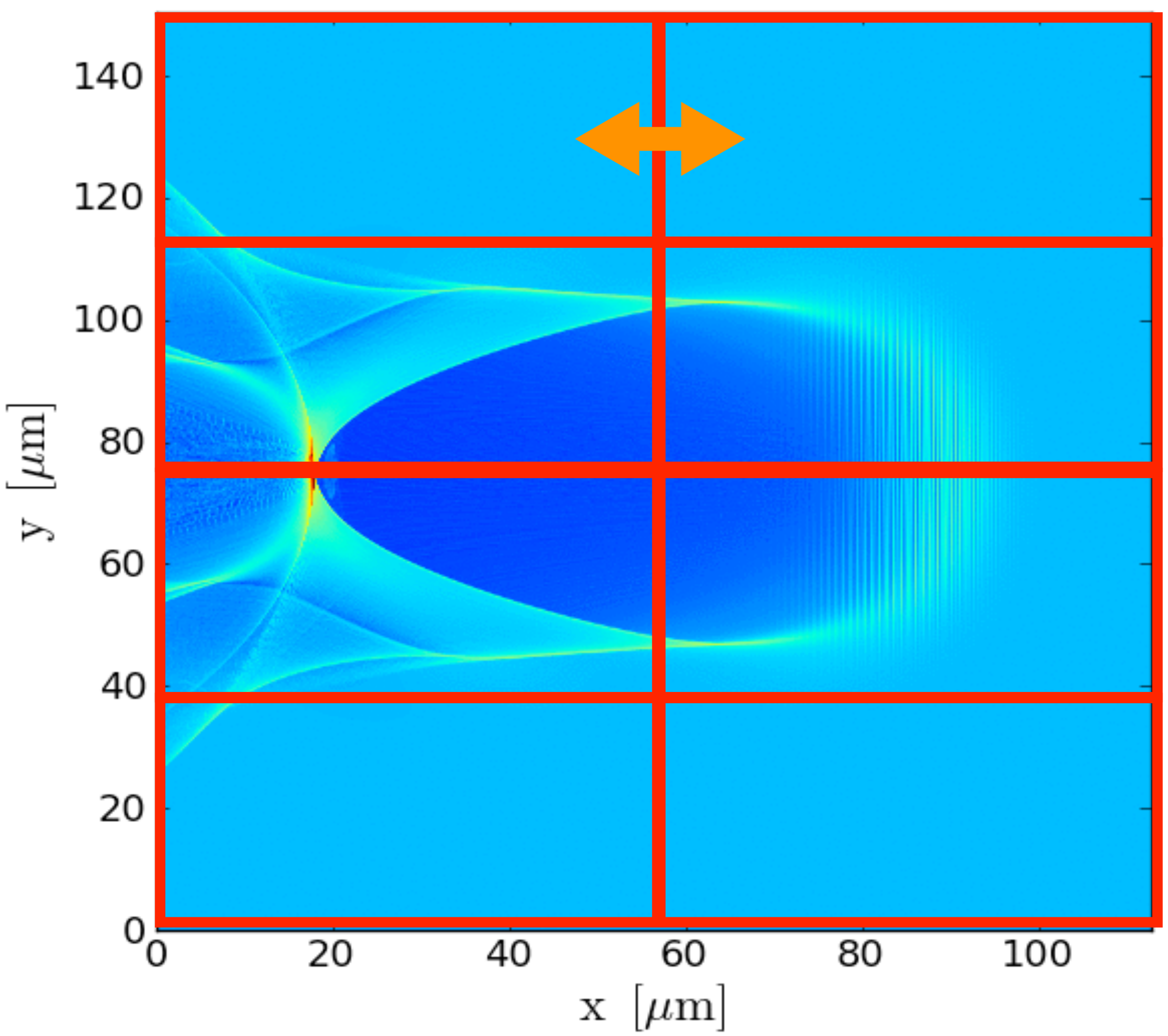
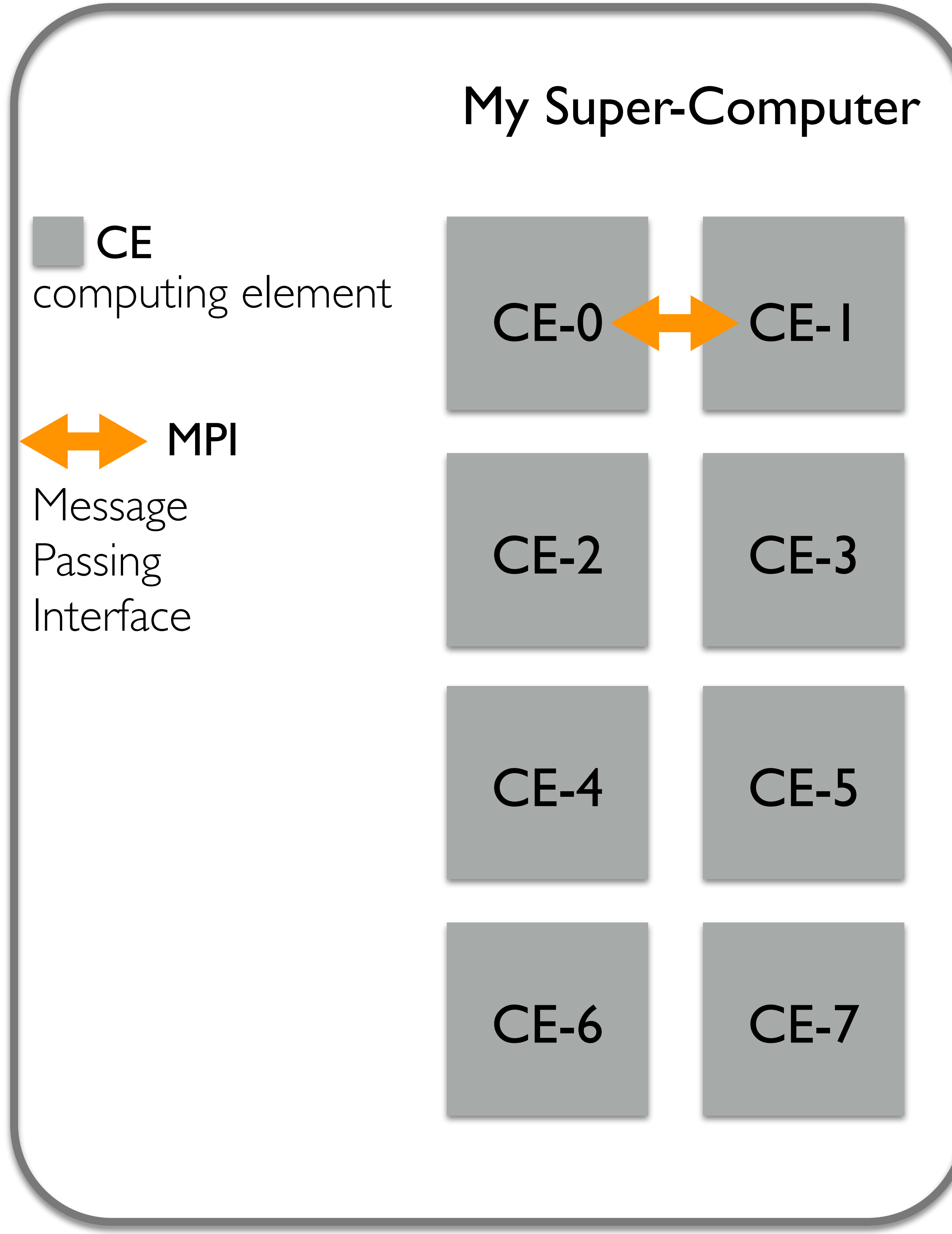


Domain Decomposition



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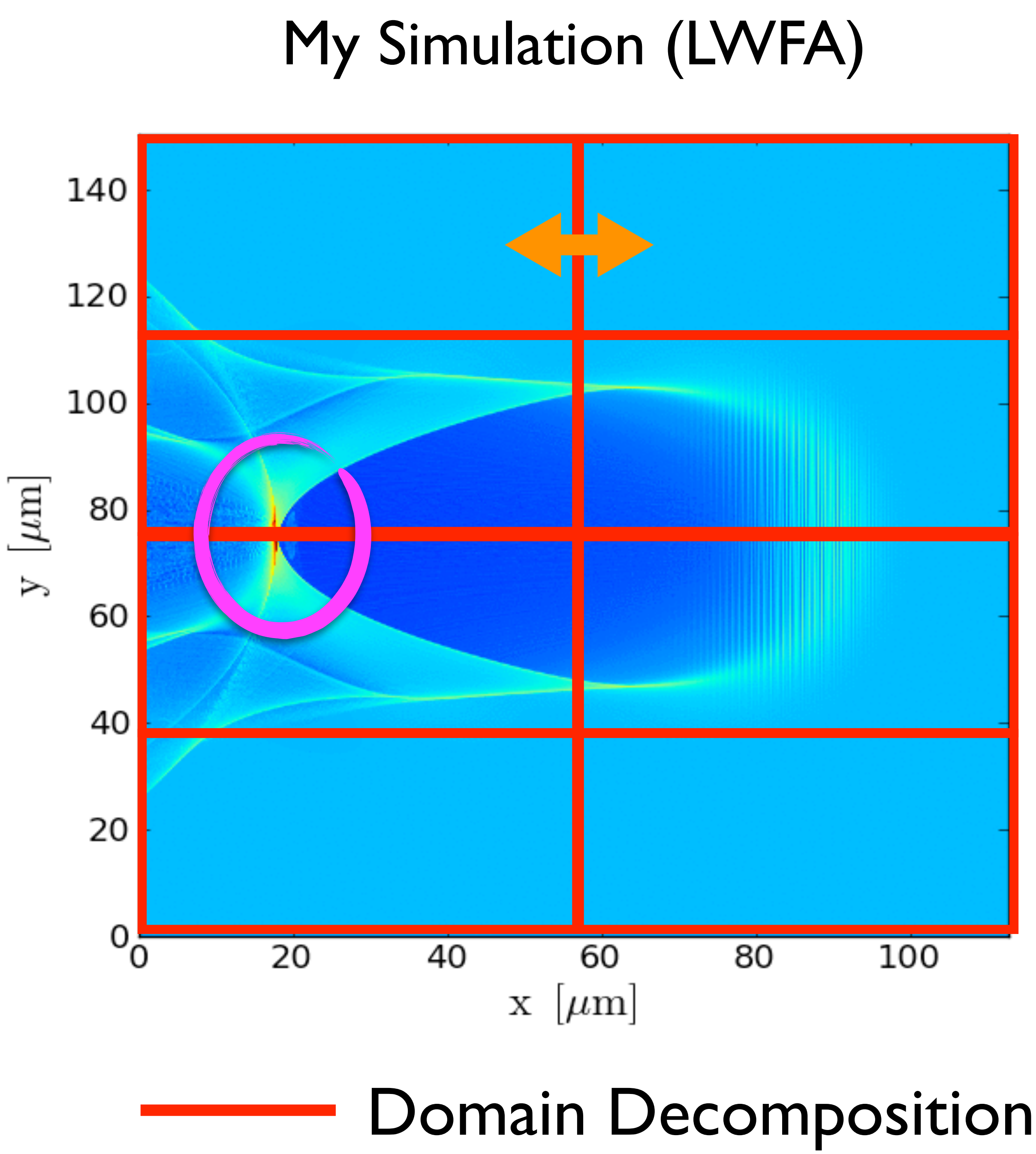

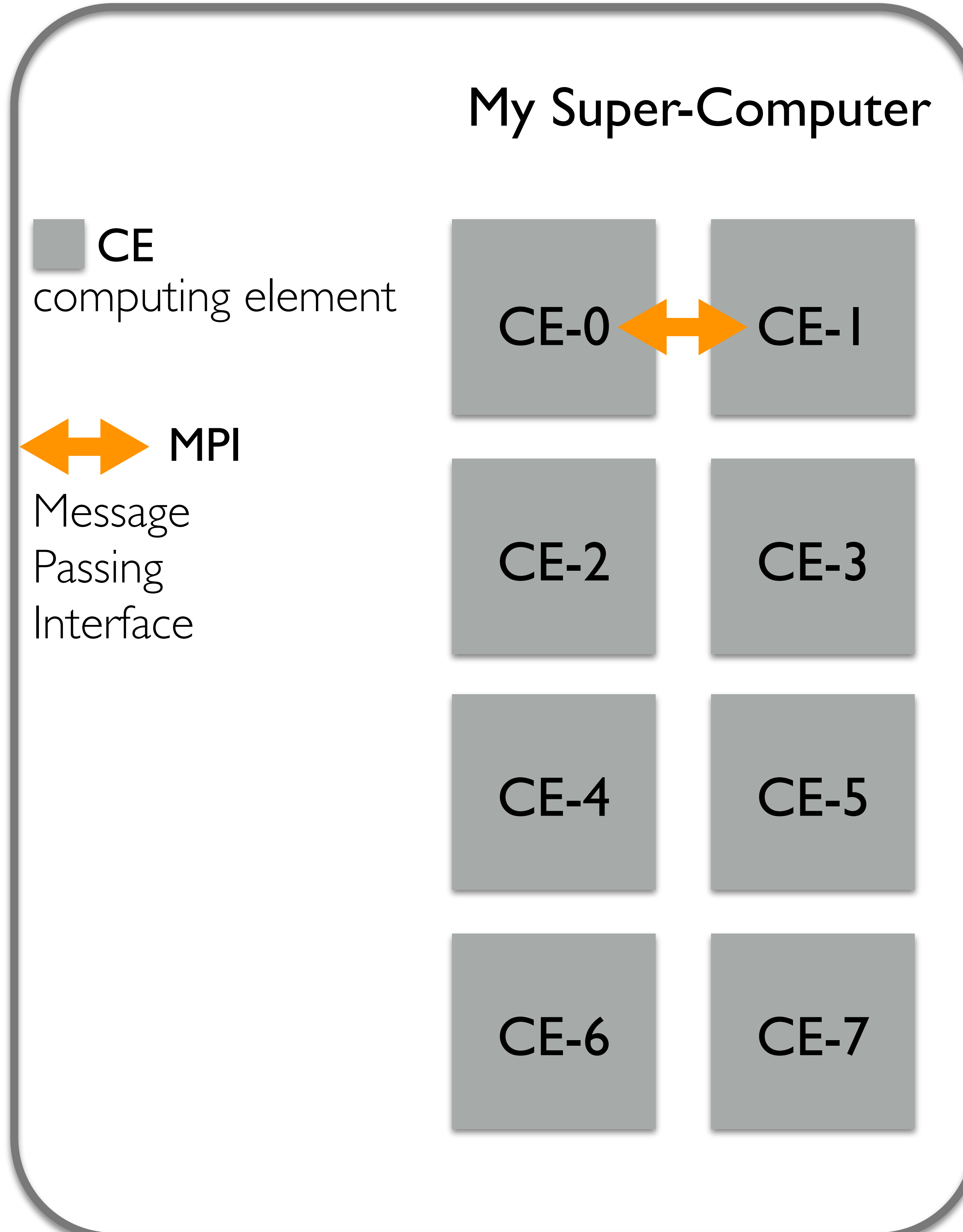

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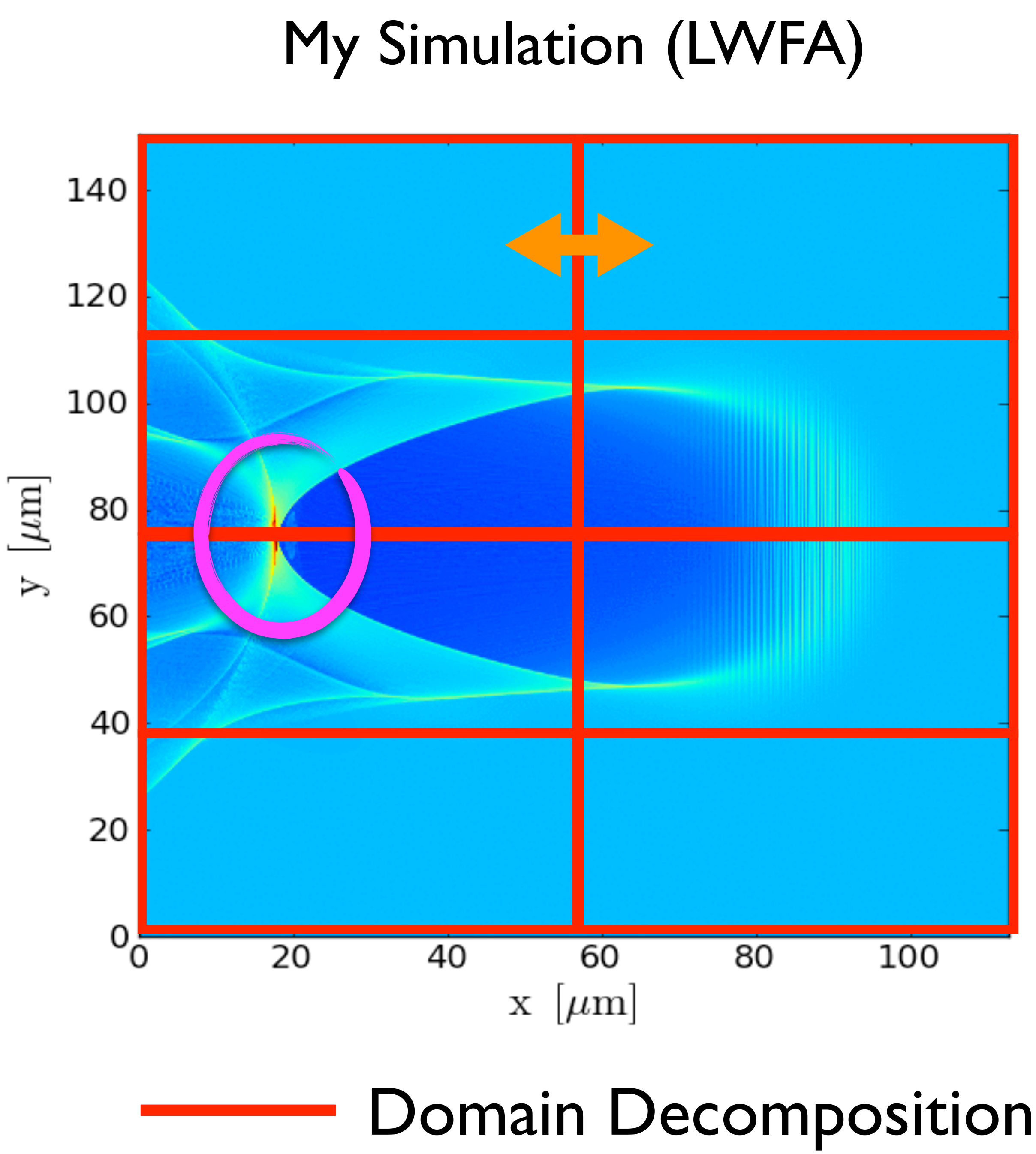

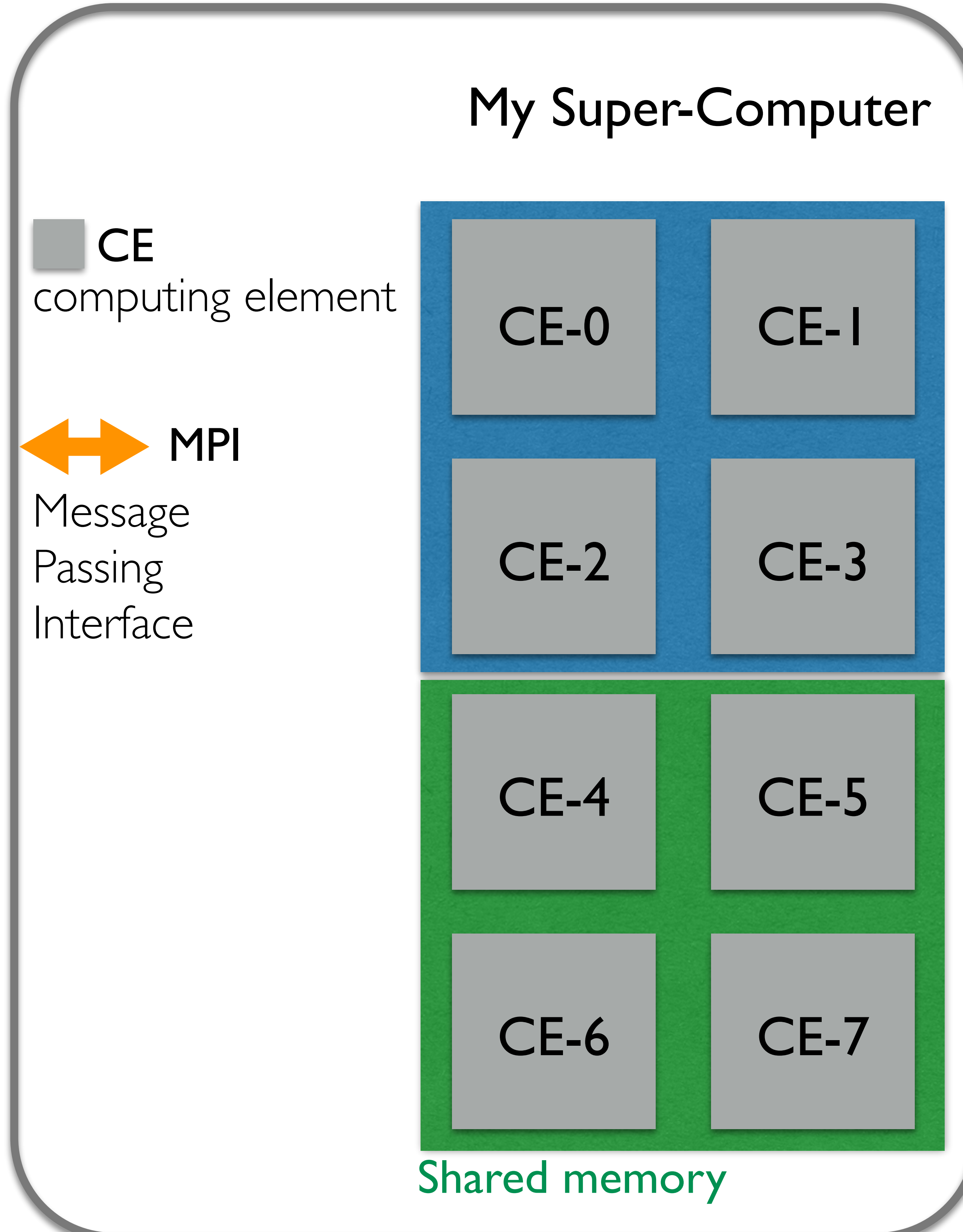

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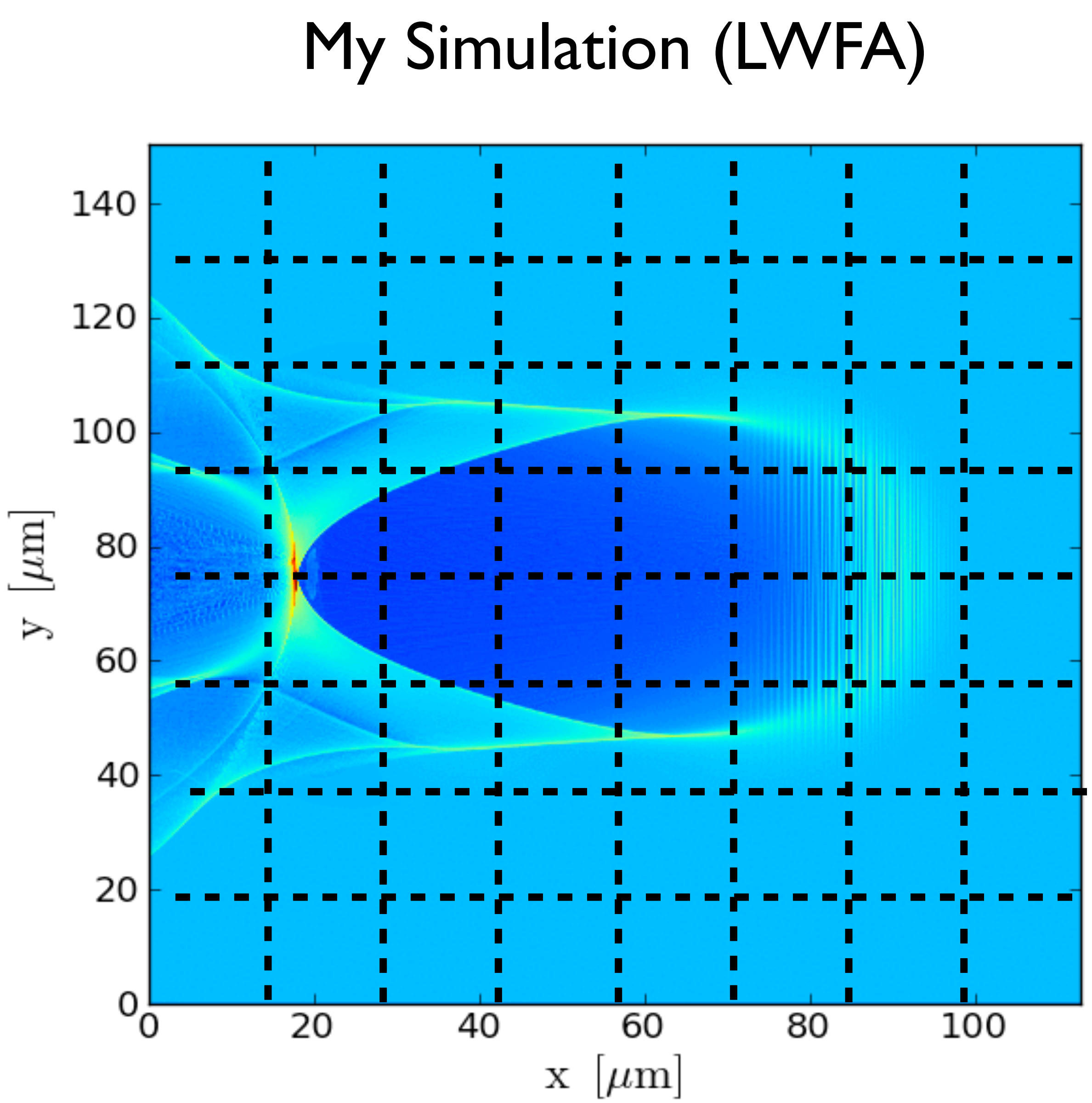

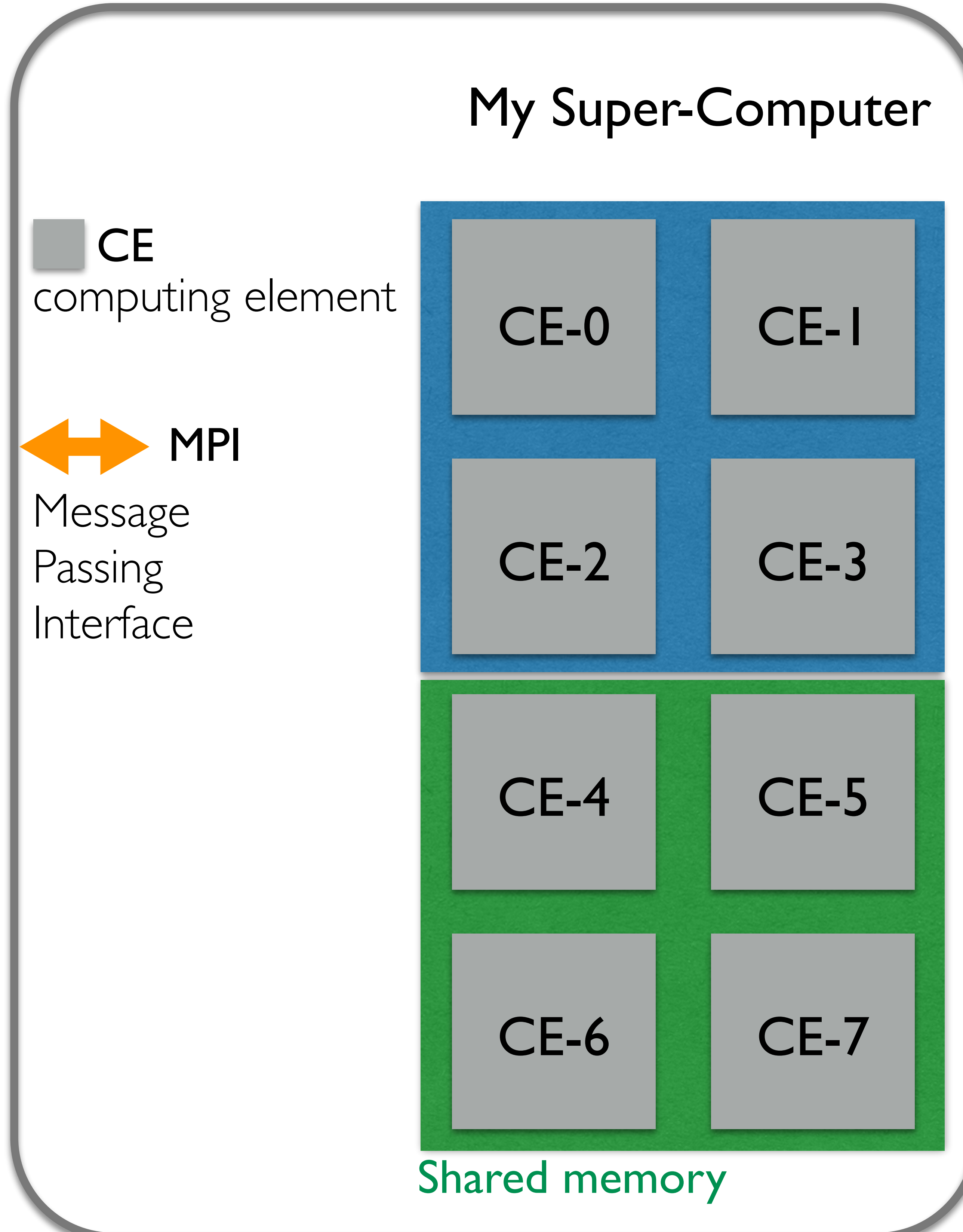

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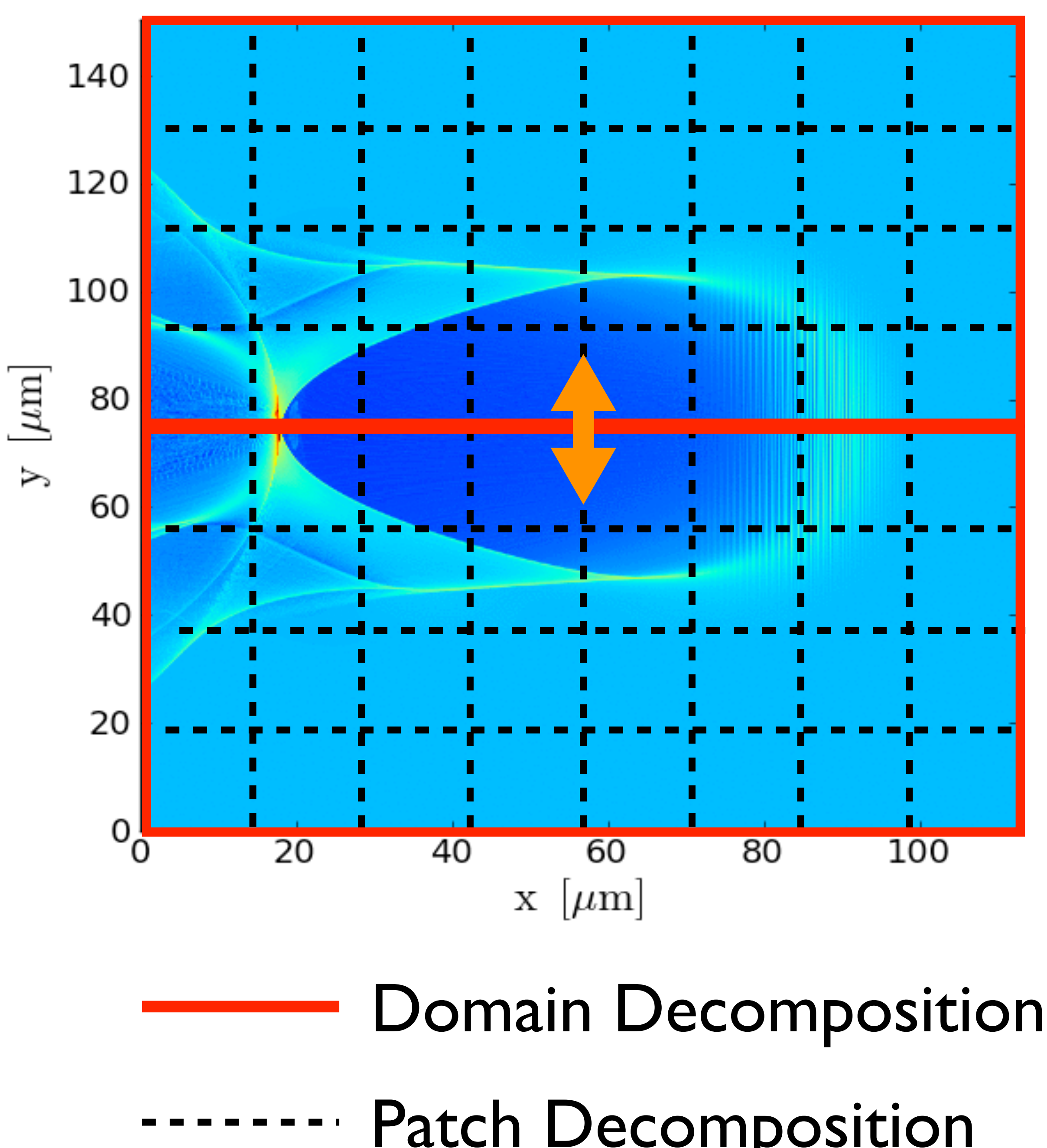

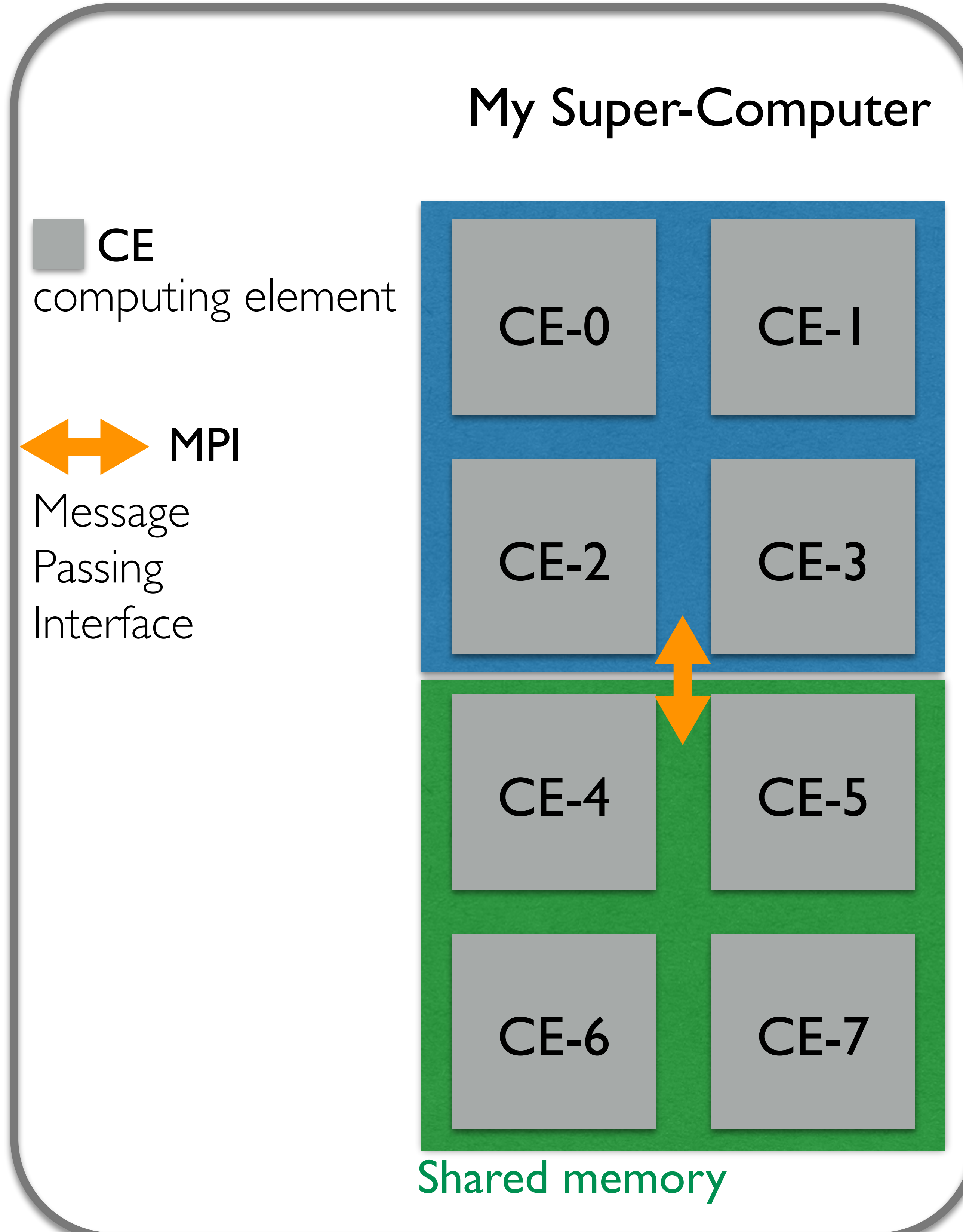

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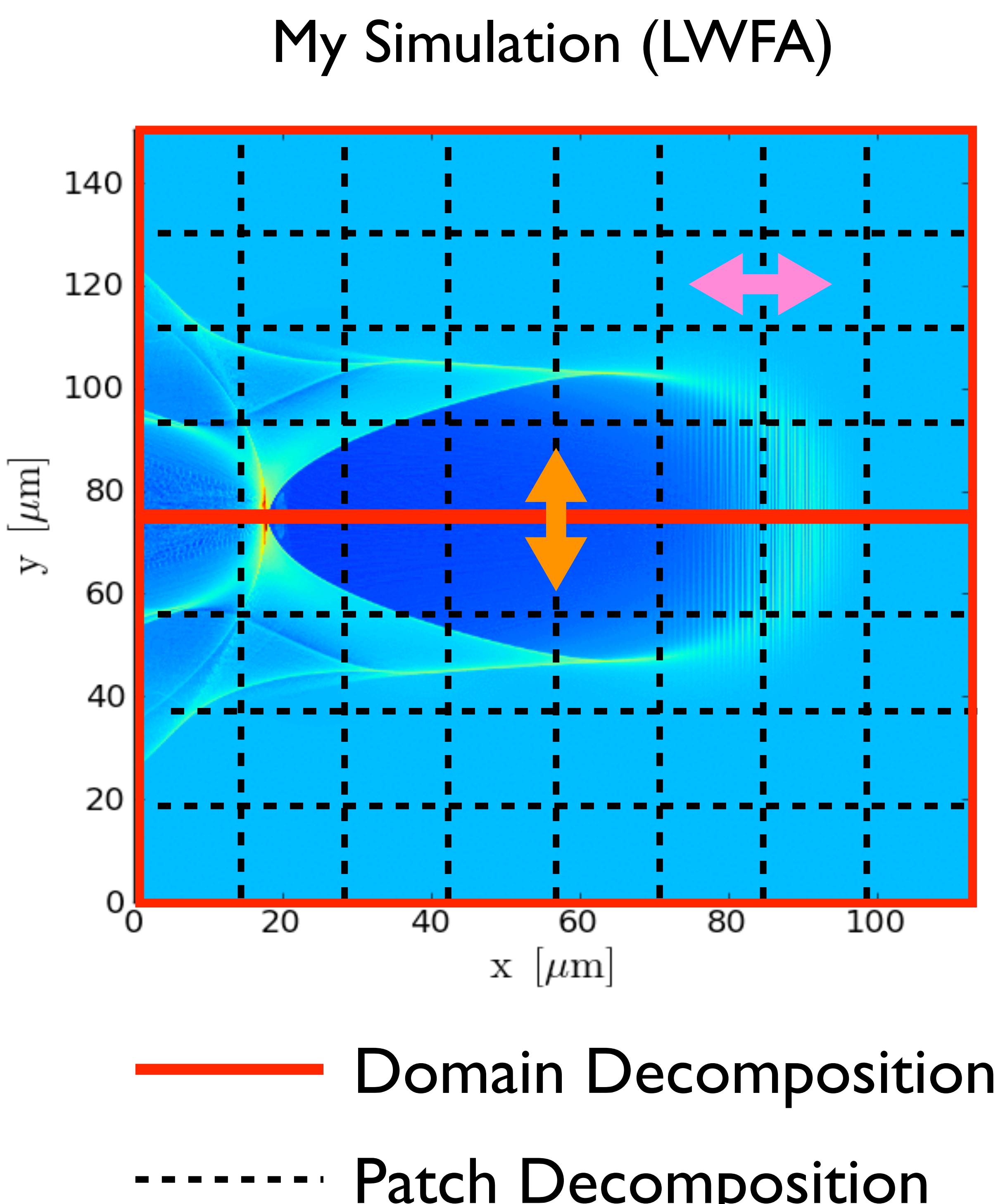

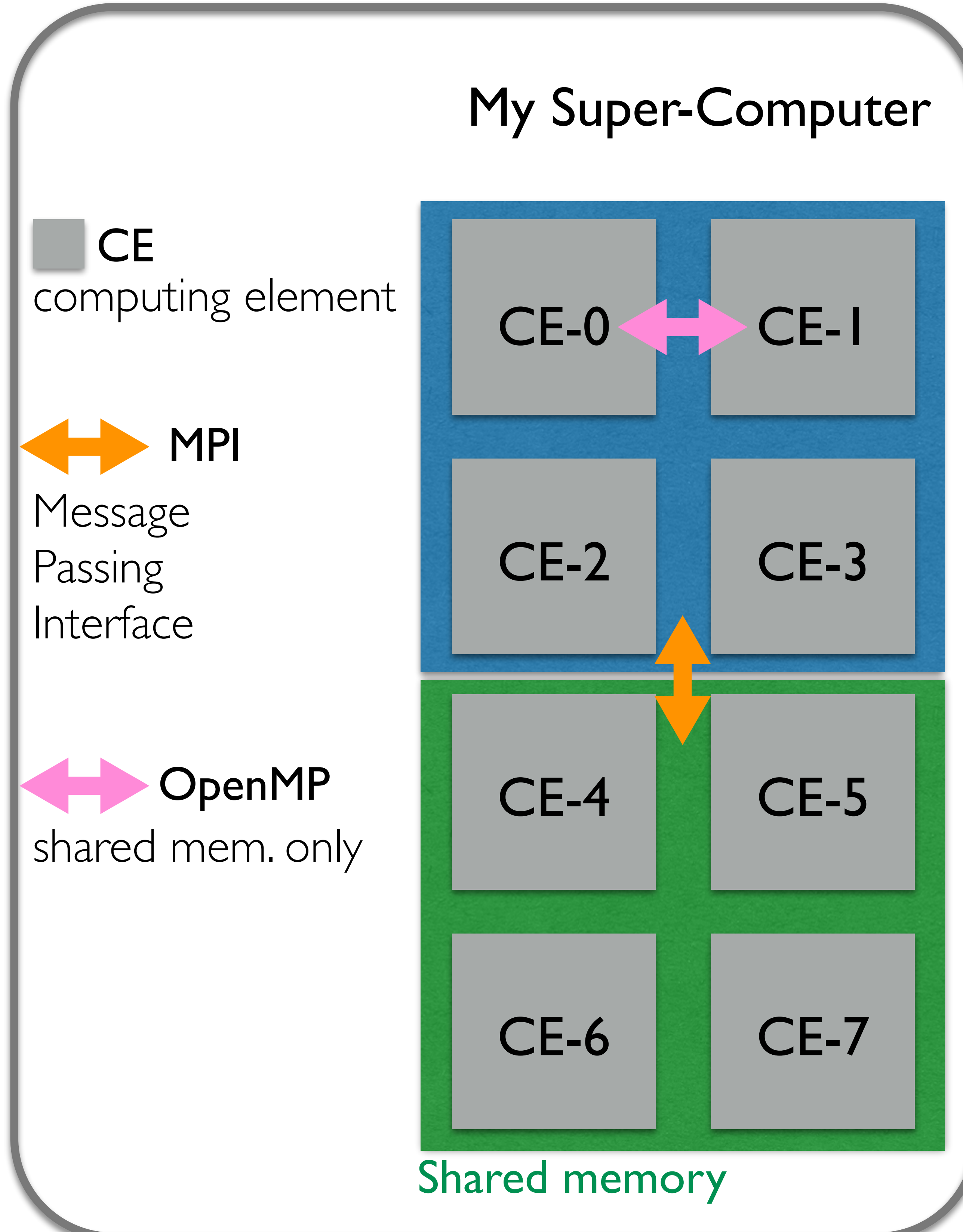

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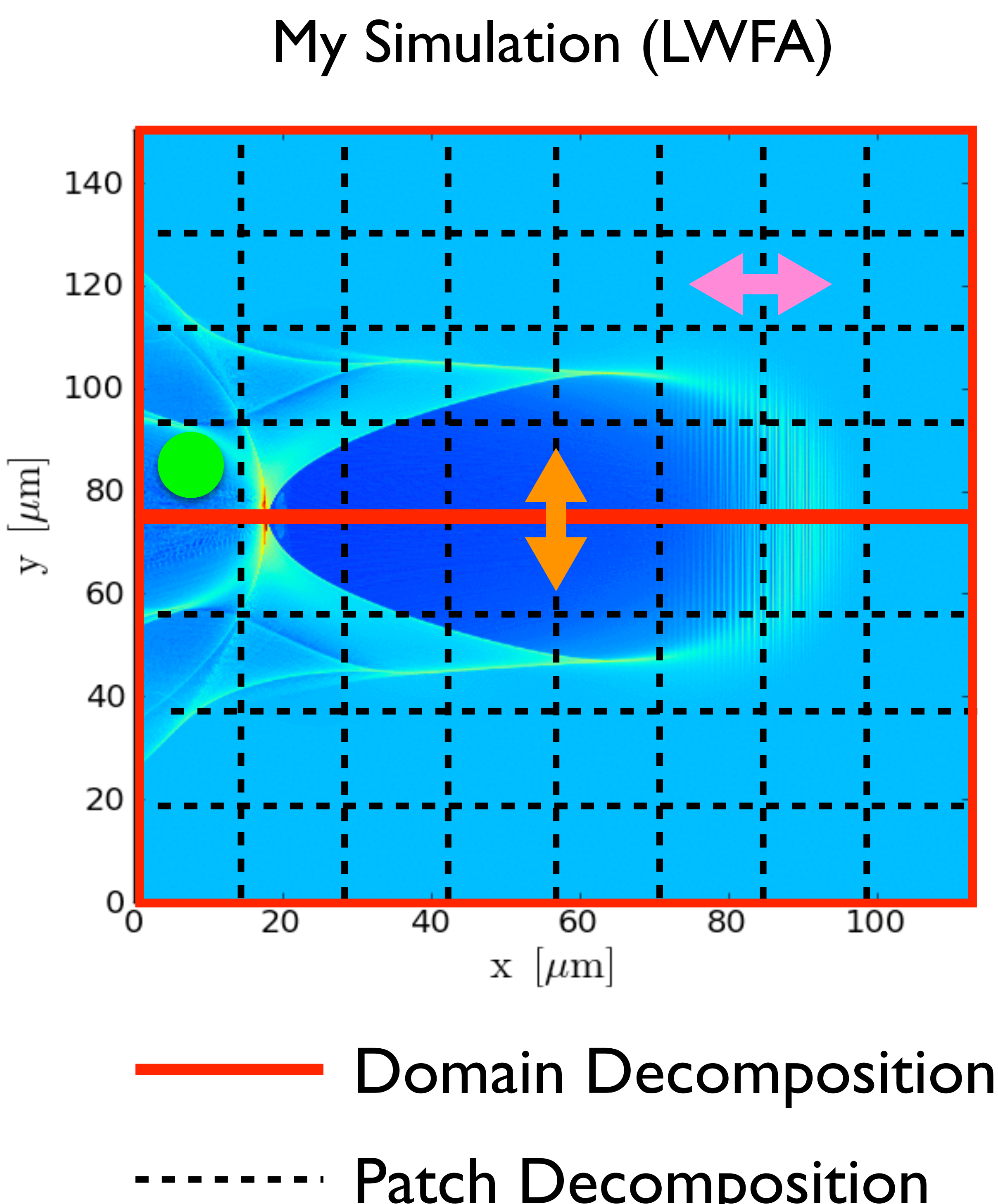

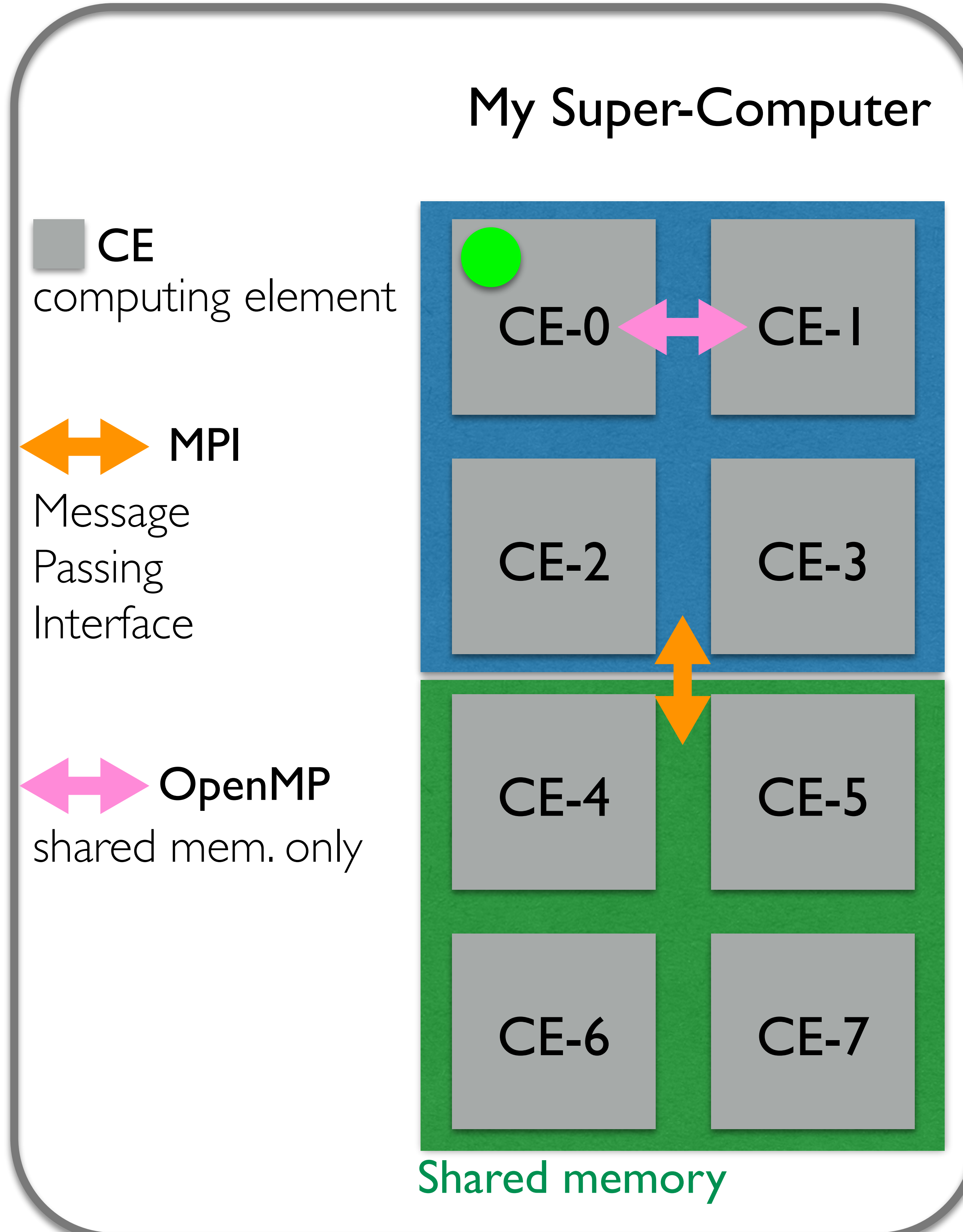

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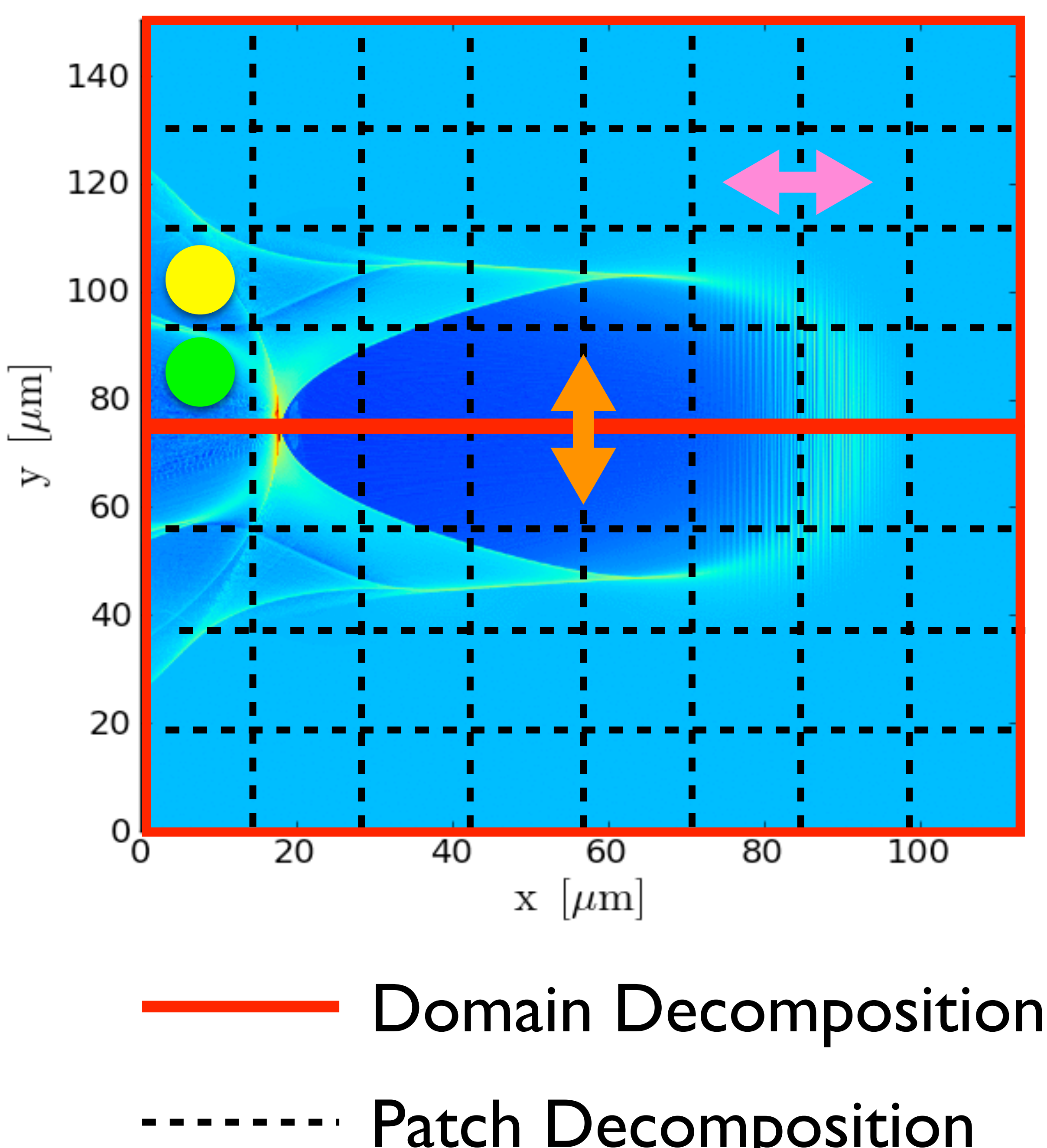
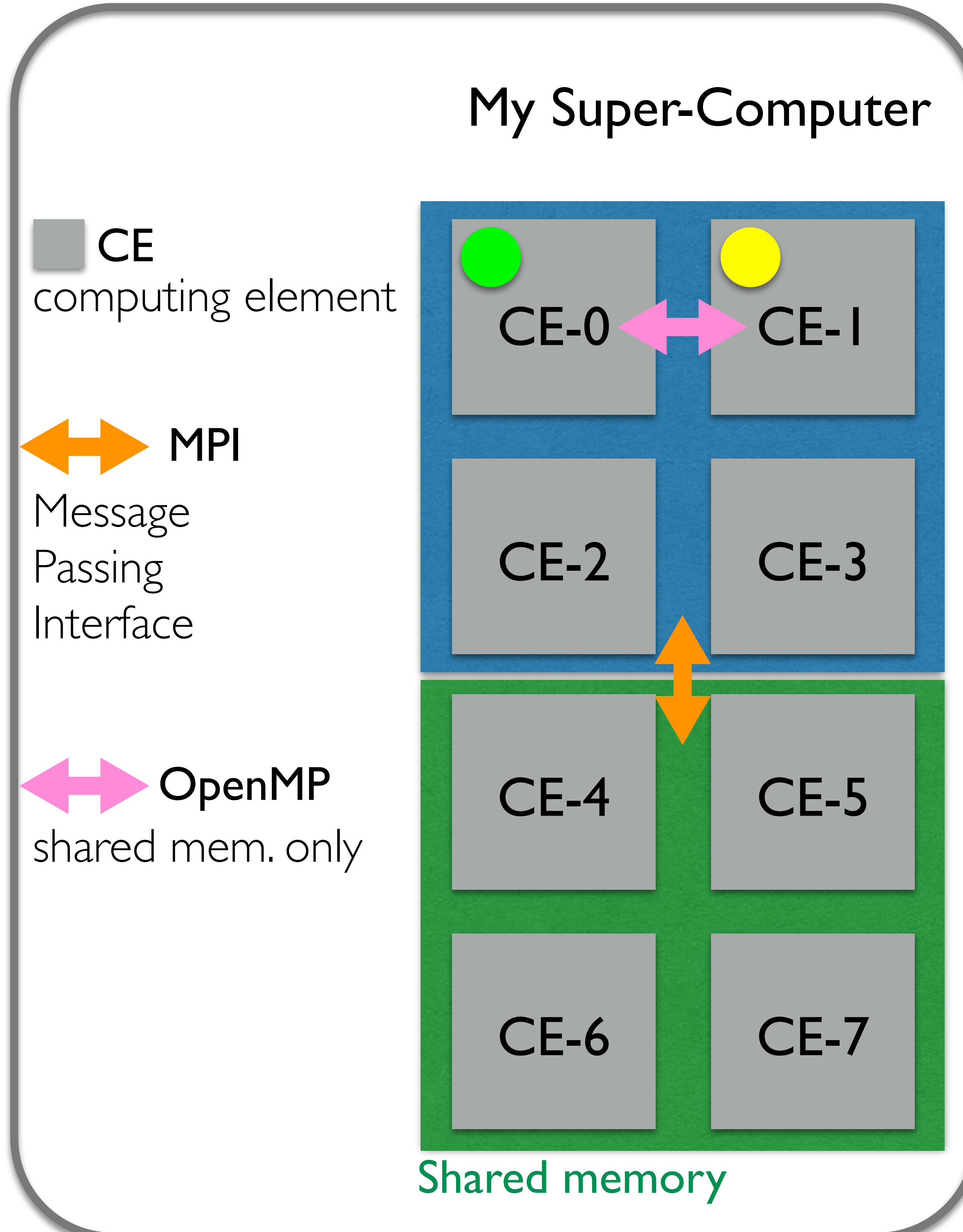
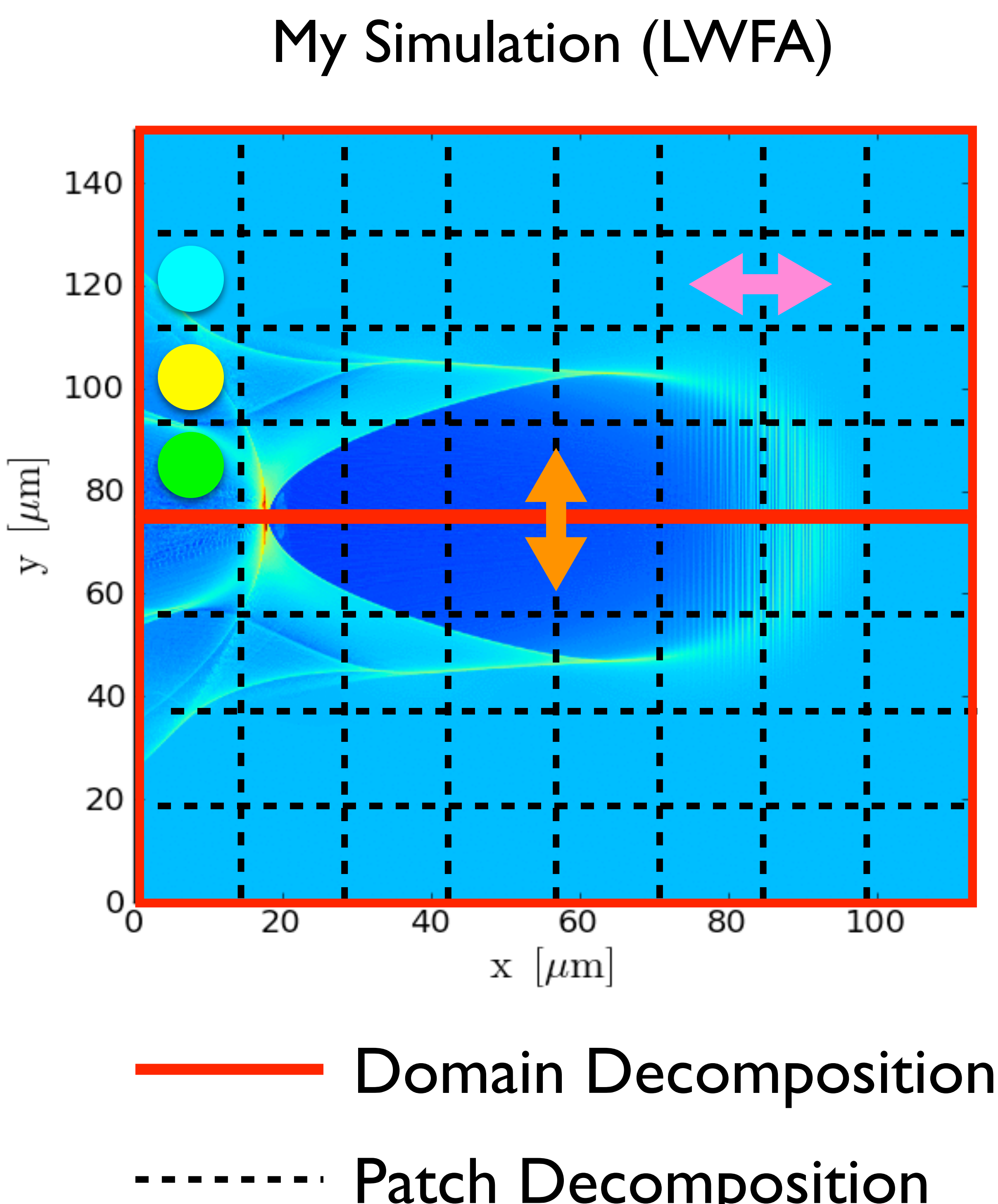
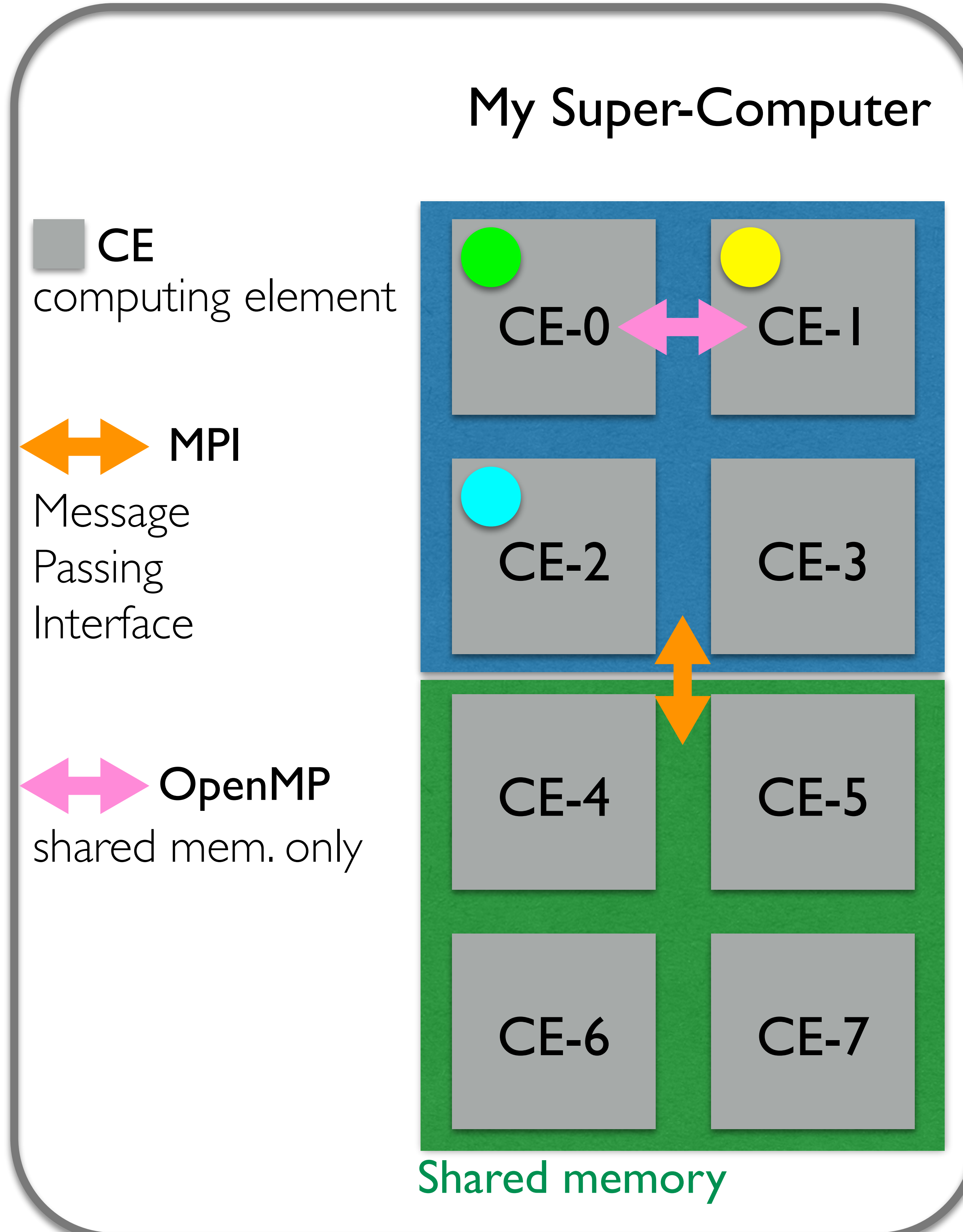




Step 1: Parallelization

PIC codes are well adapted to massive parallelism





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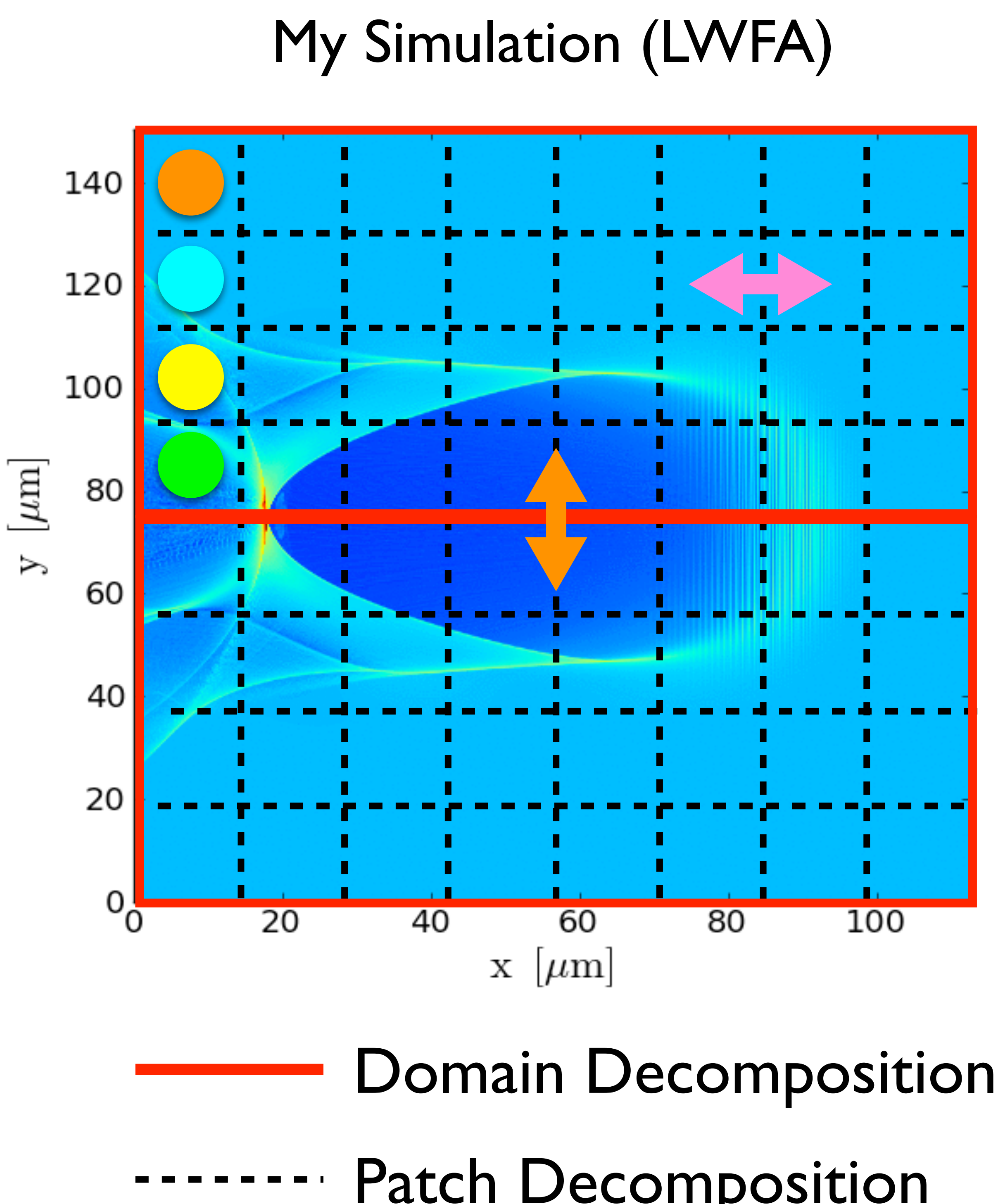

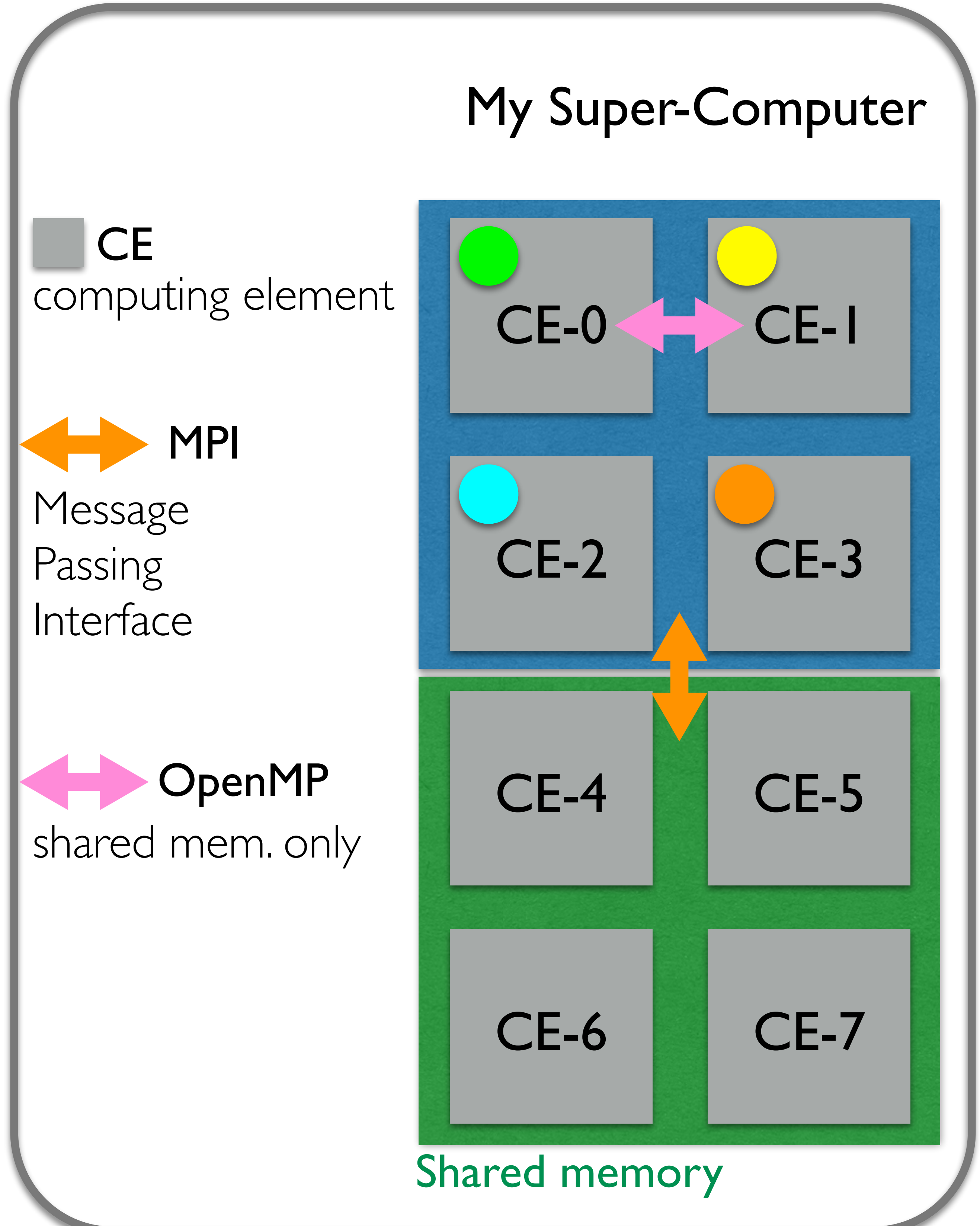

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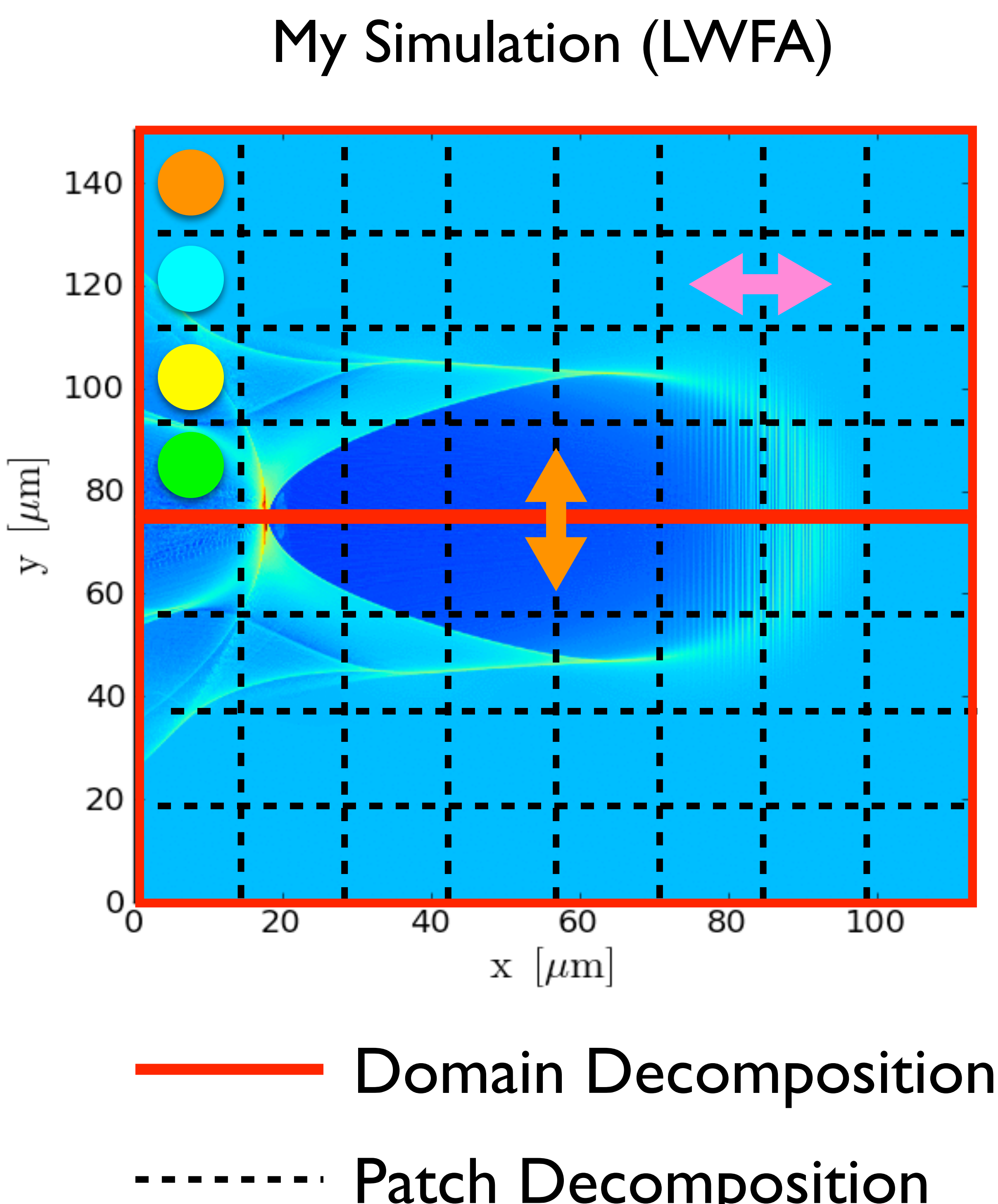

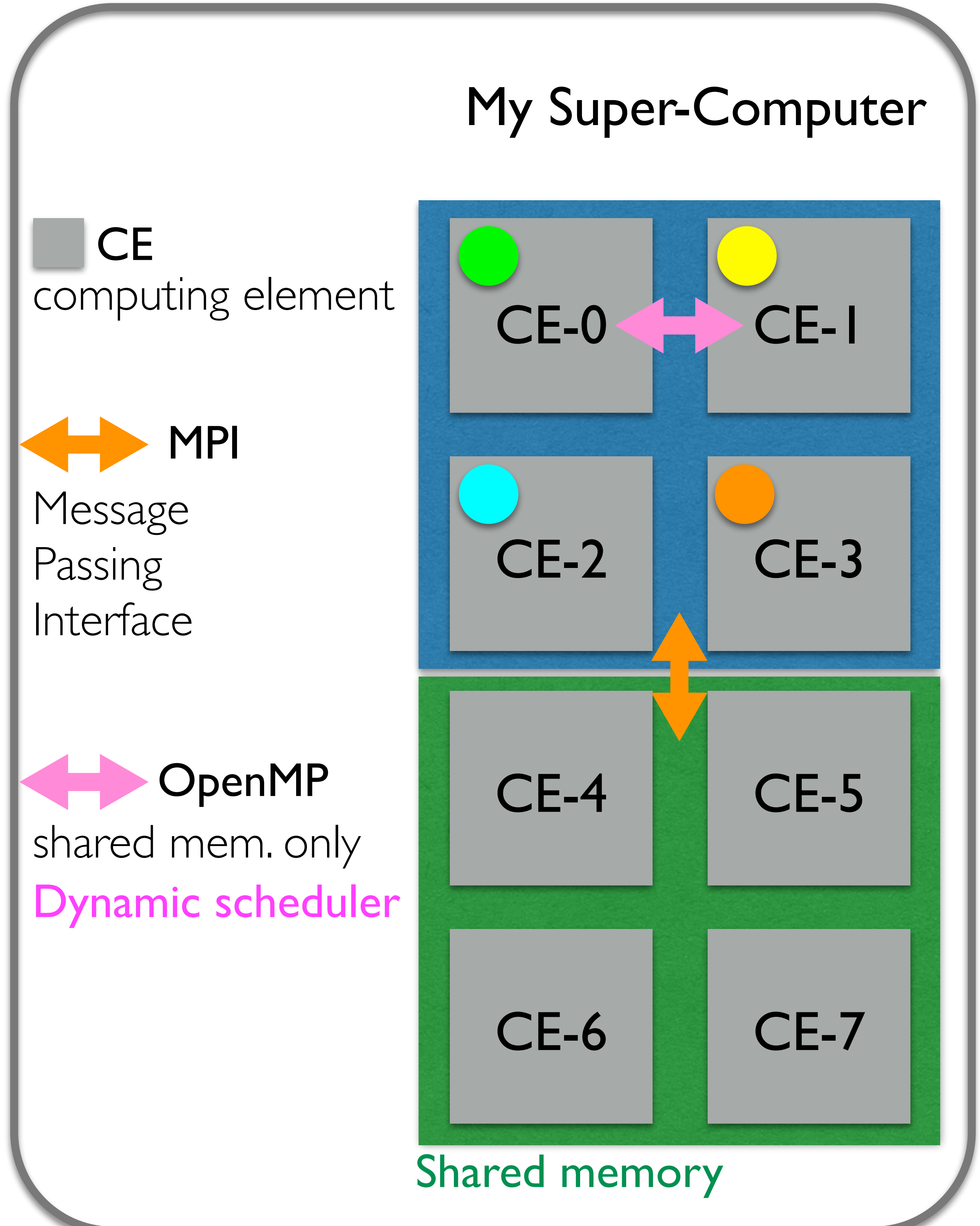

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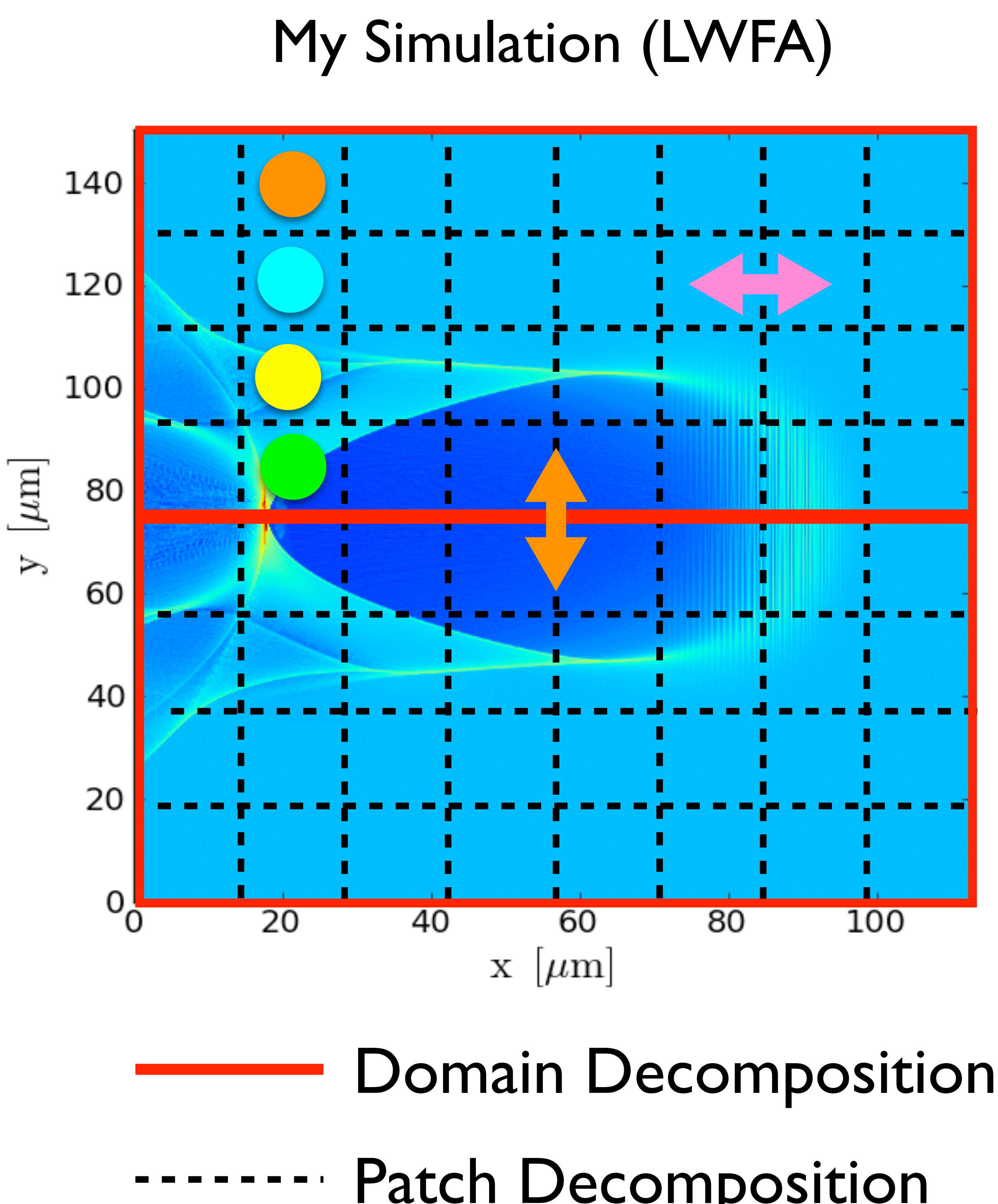

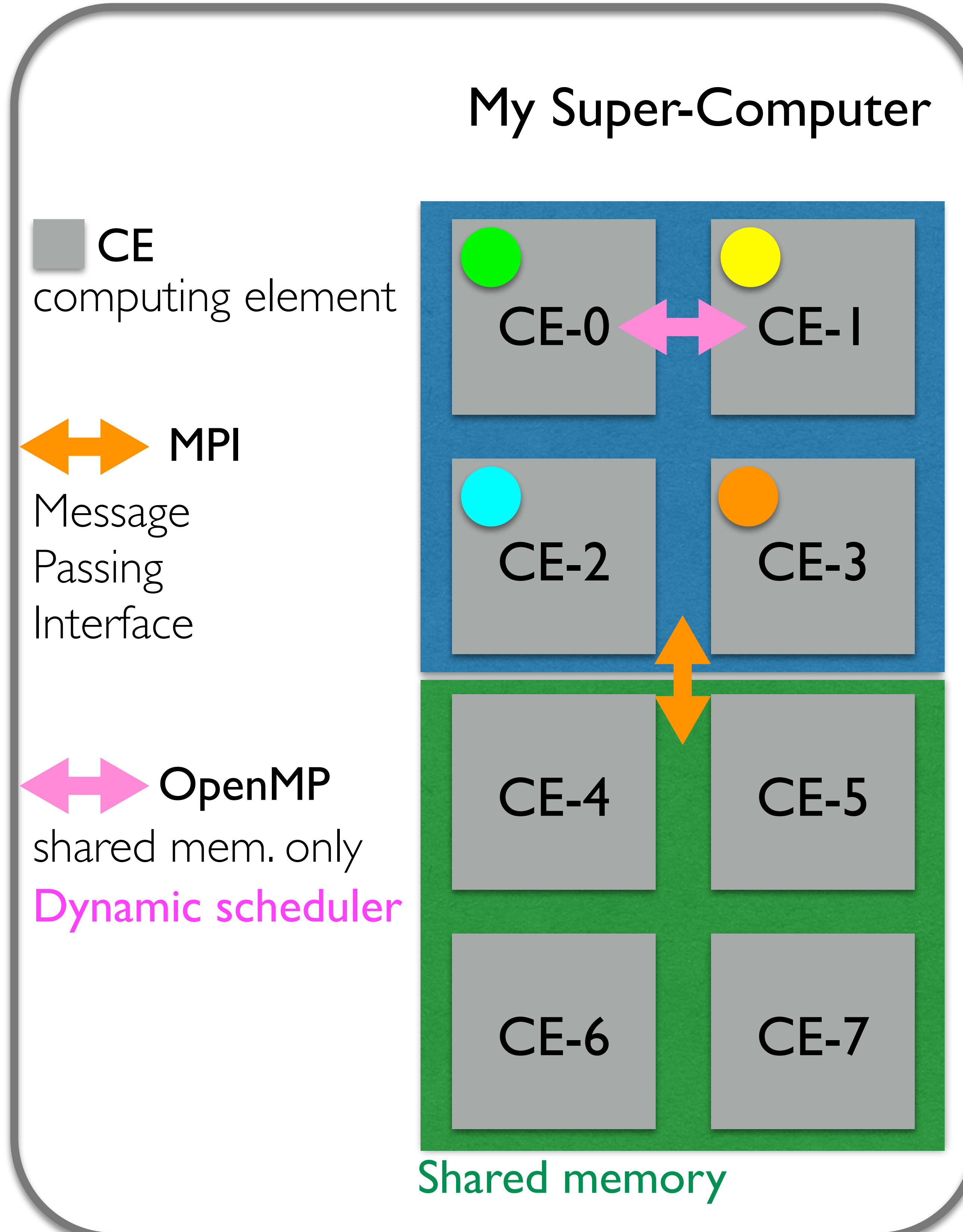

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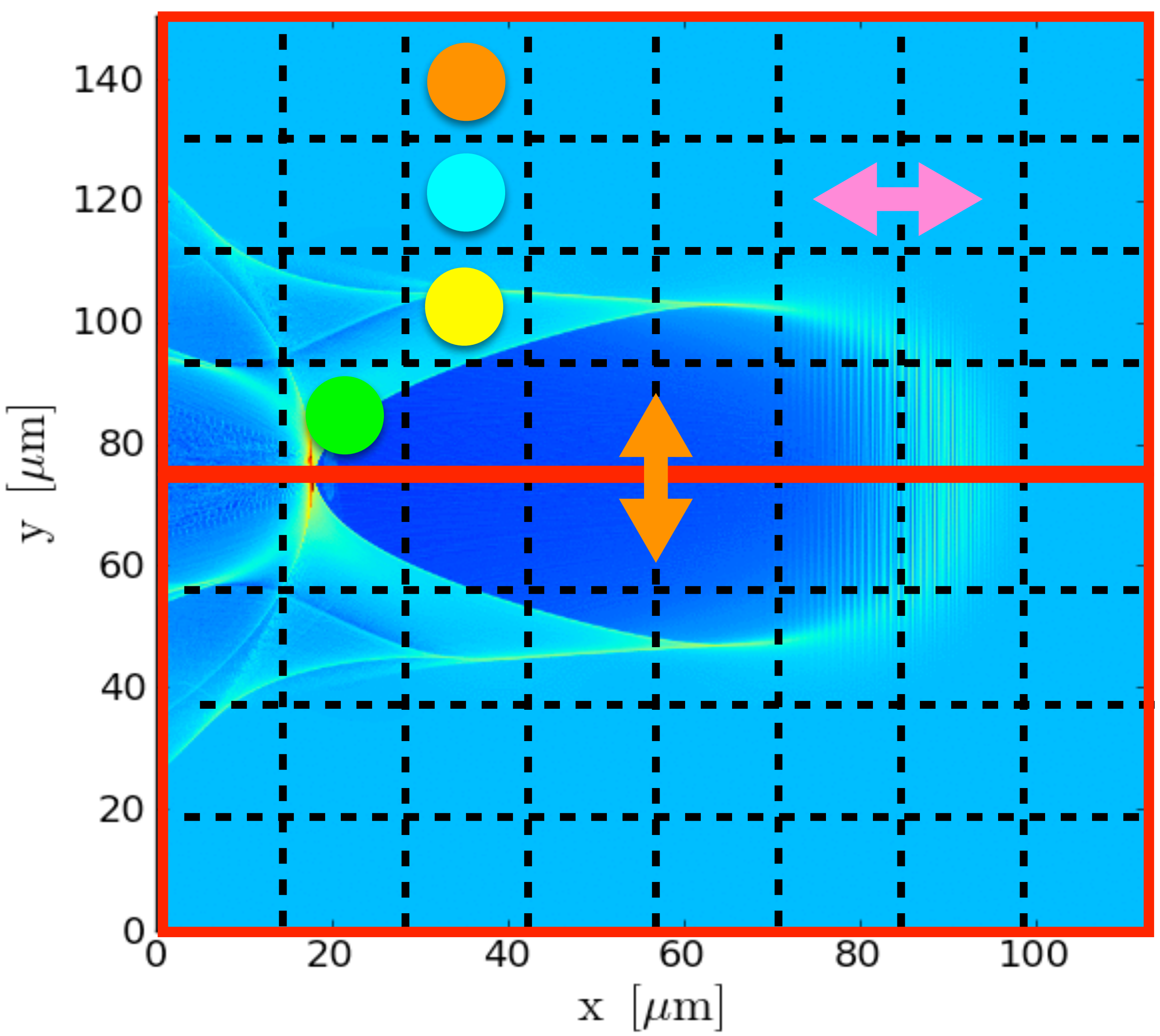
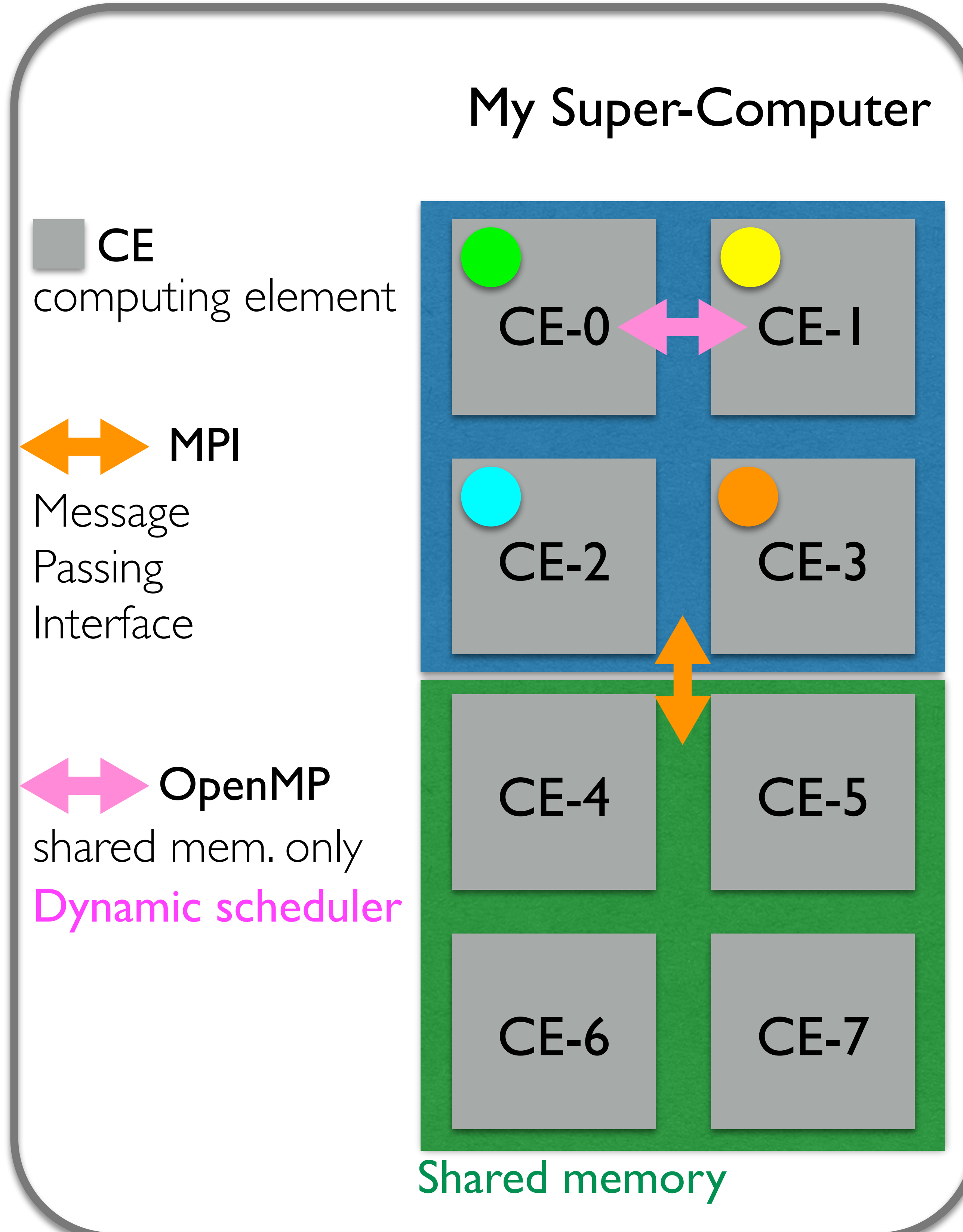


Domain Decomposition

Patch Decomposition



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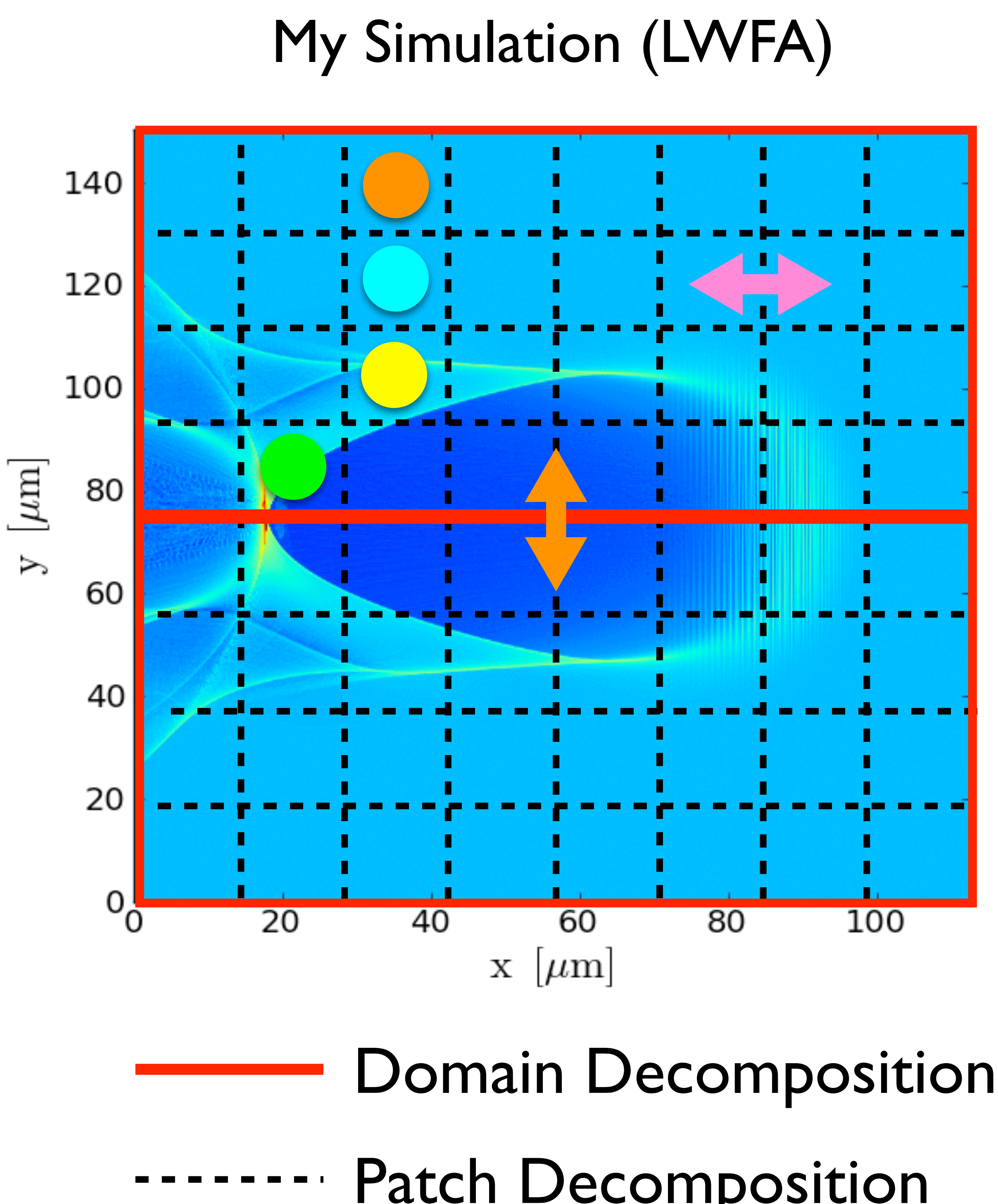
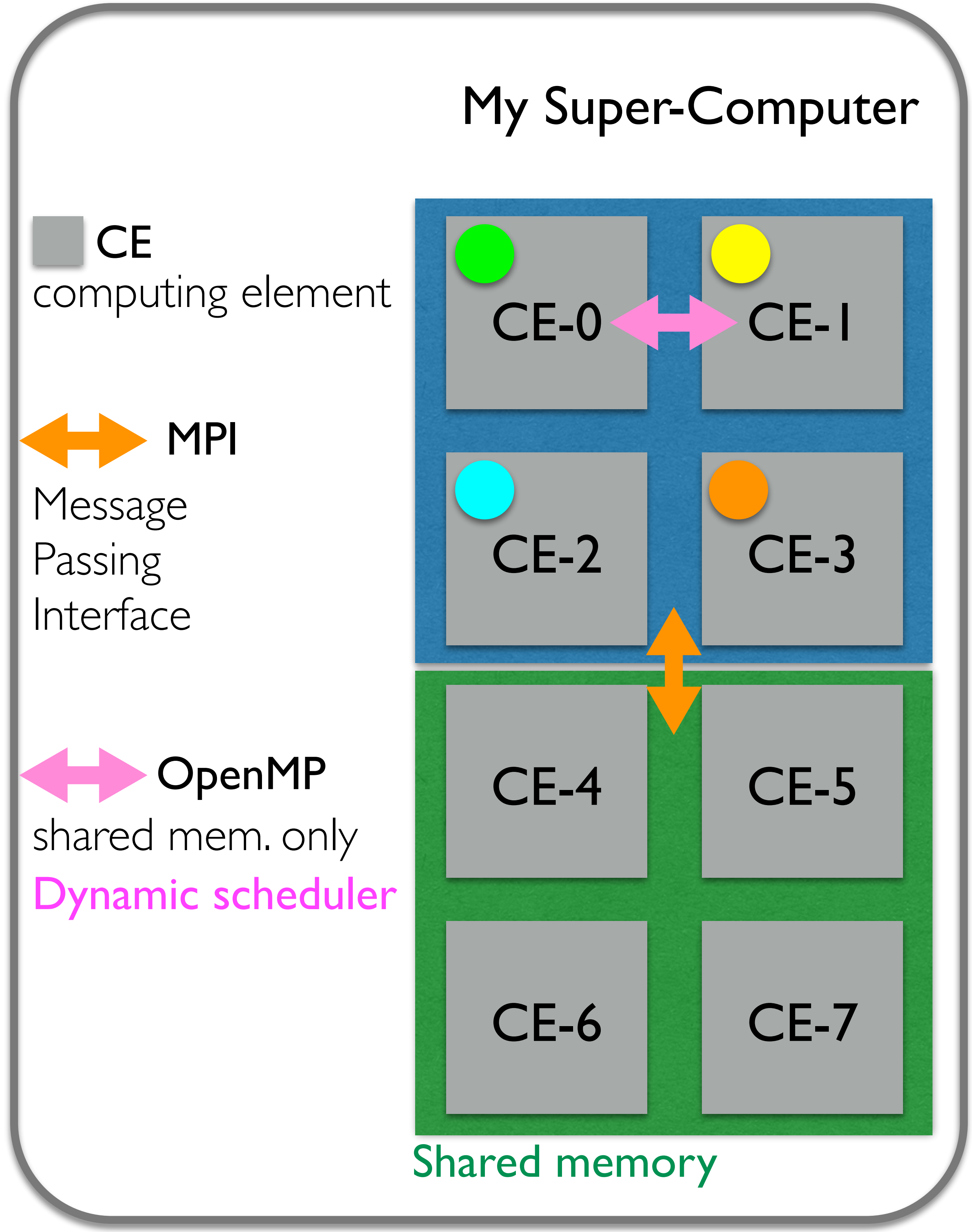
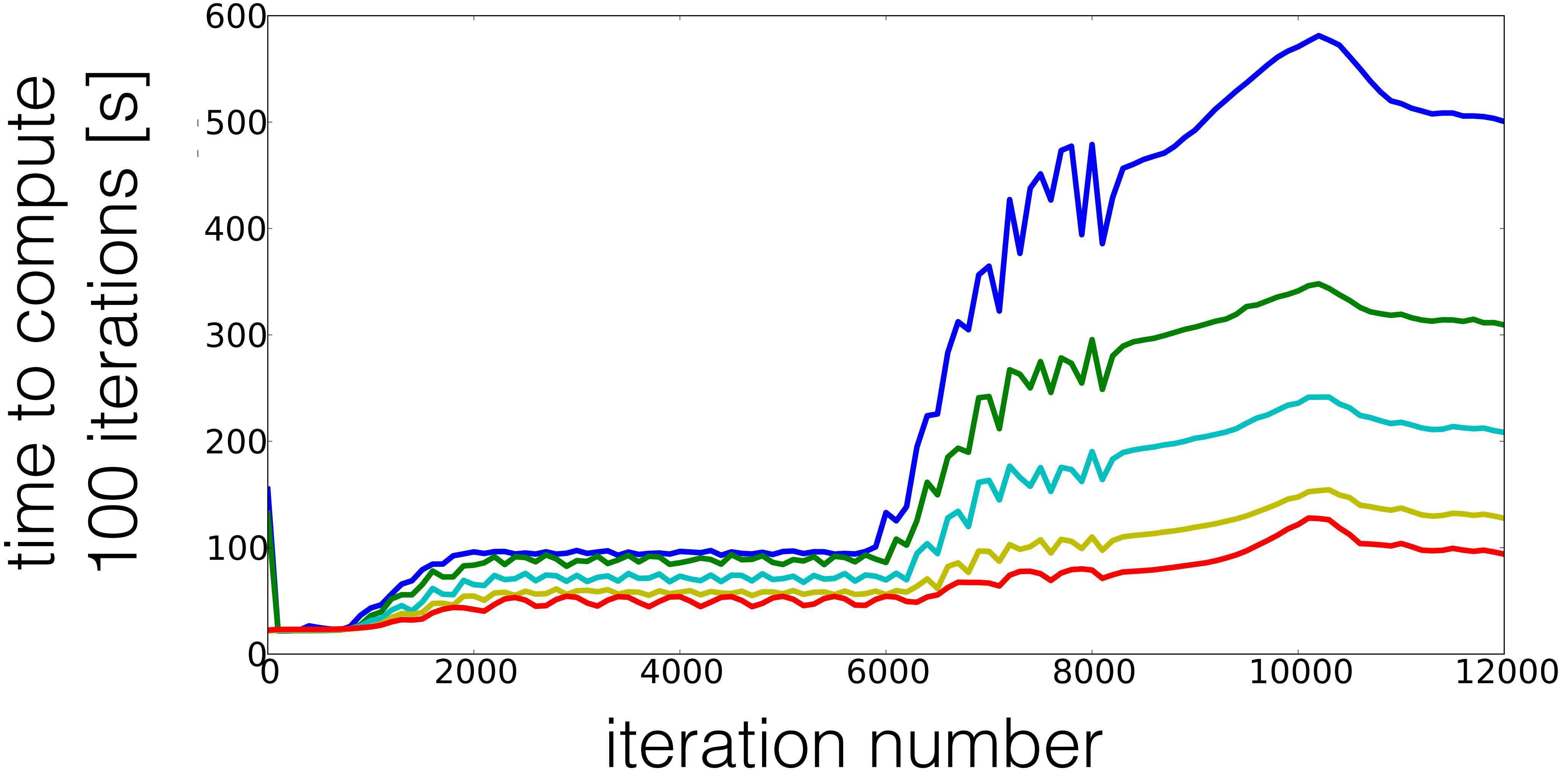




Step 1: Parallelization

Hybrid parallelization significantly improves performance



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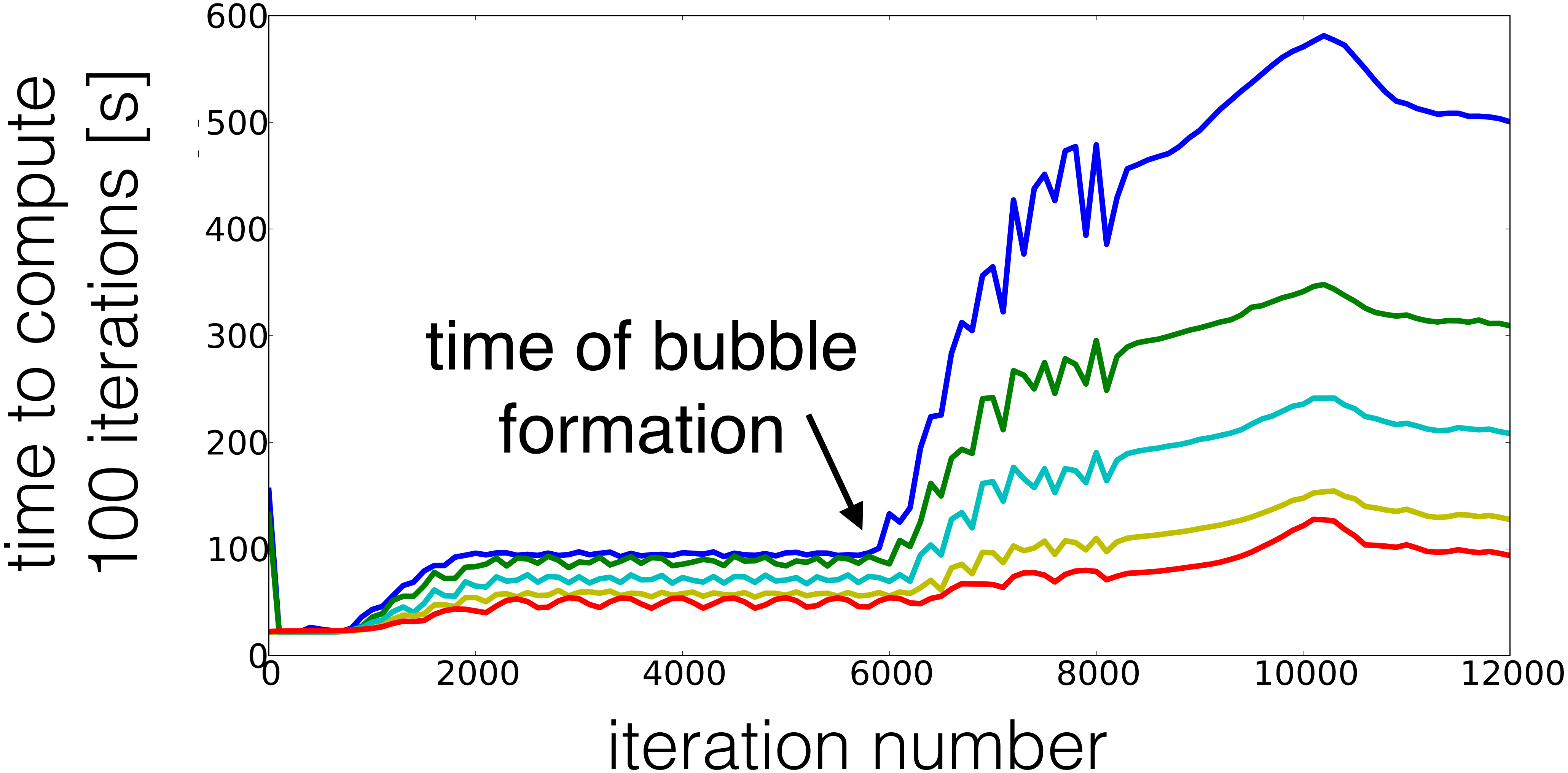

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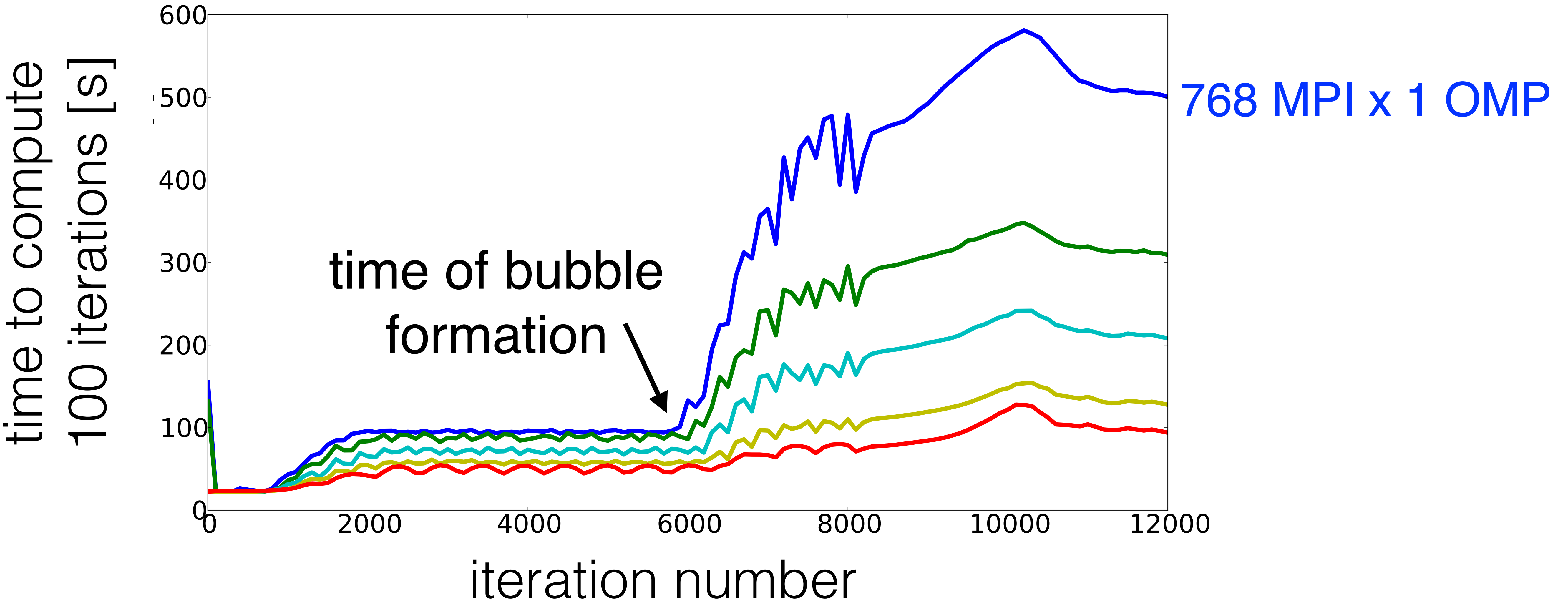

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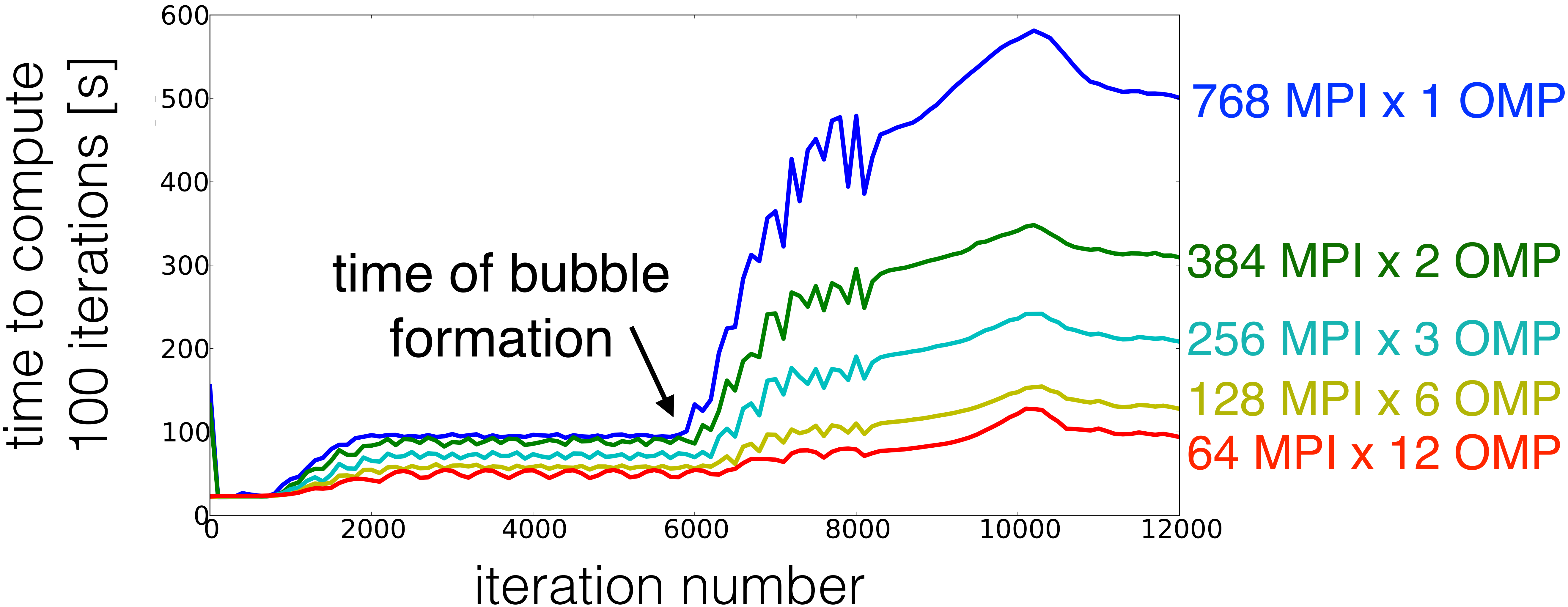

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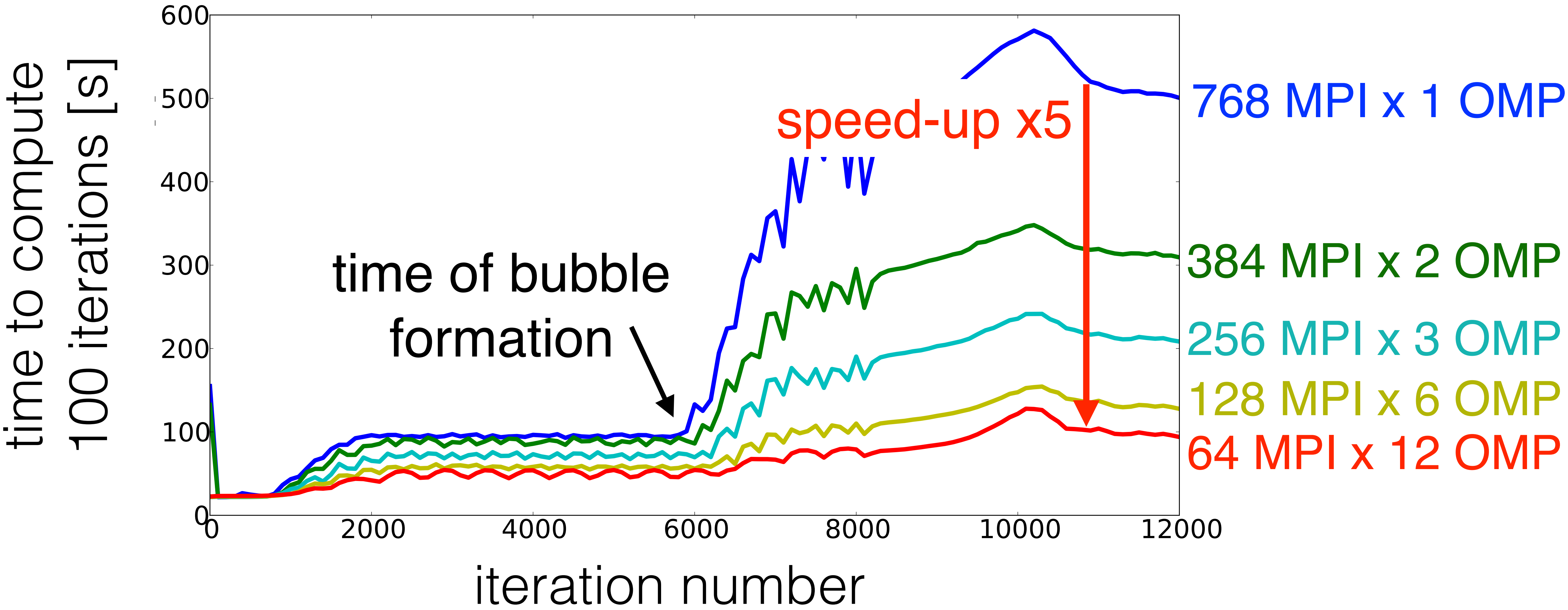

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Dynamic load balancing further improve performances

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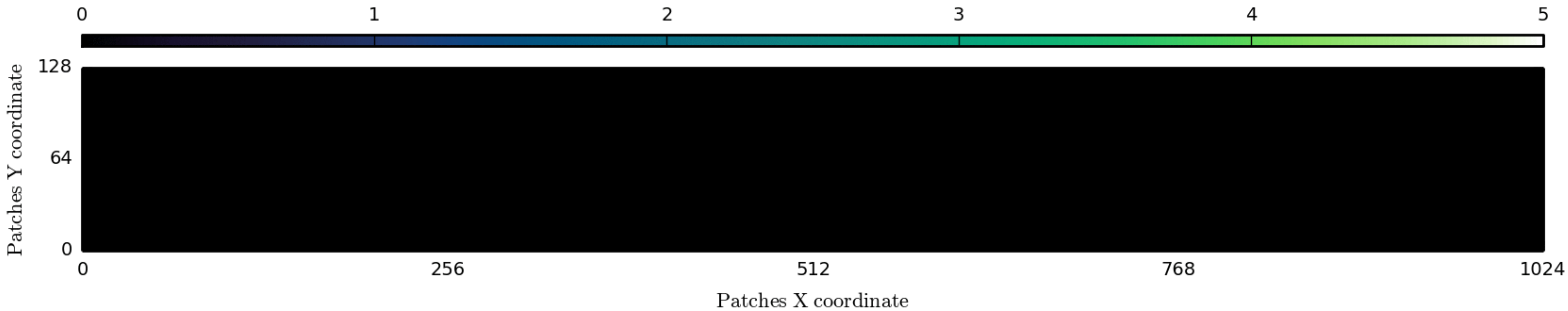

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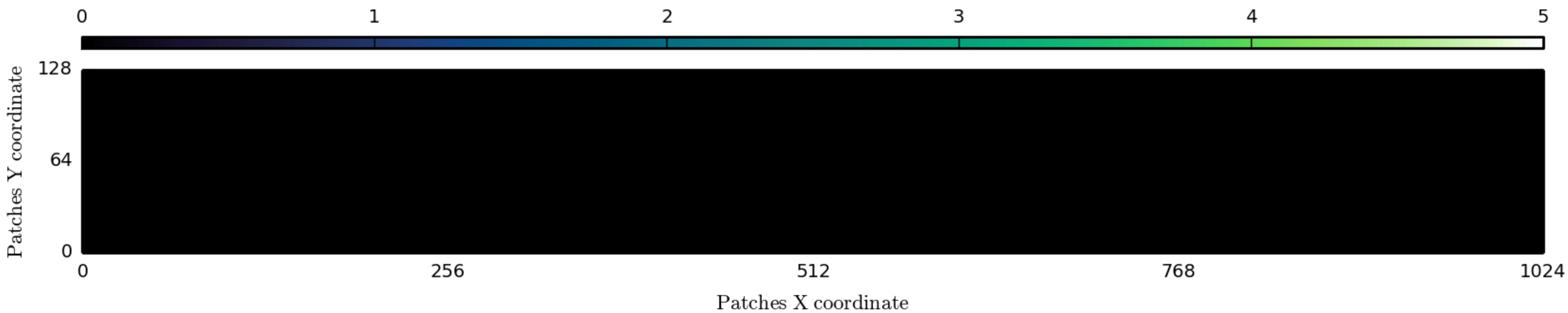

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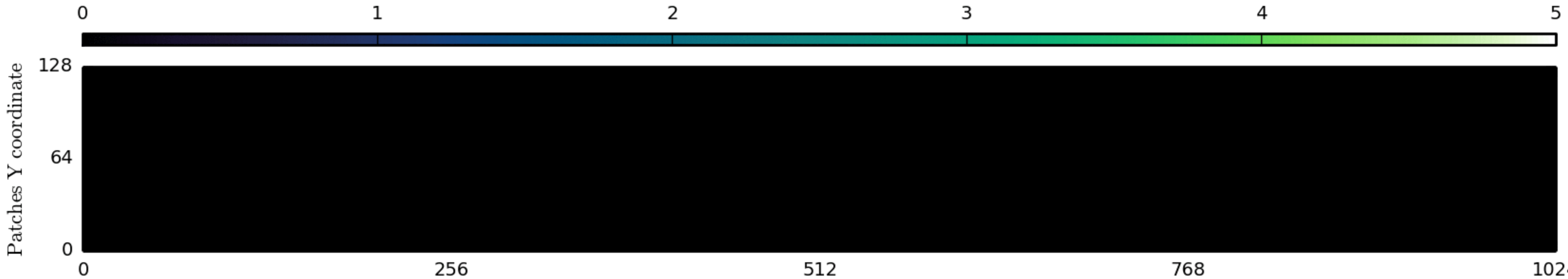

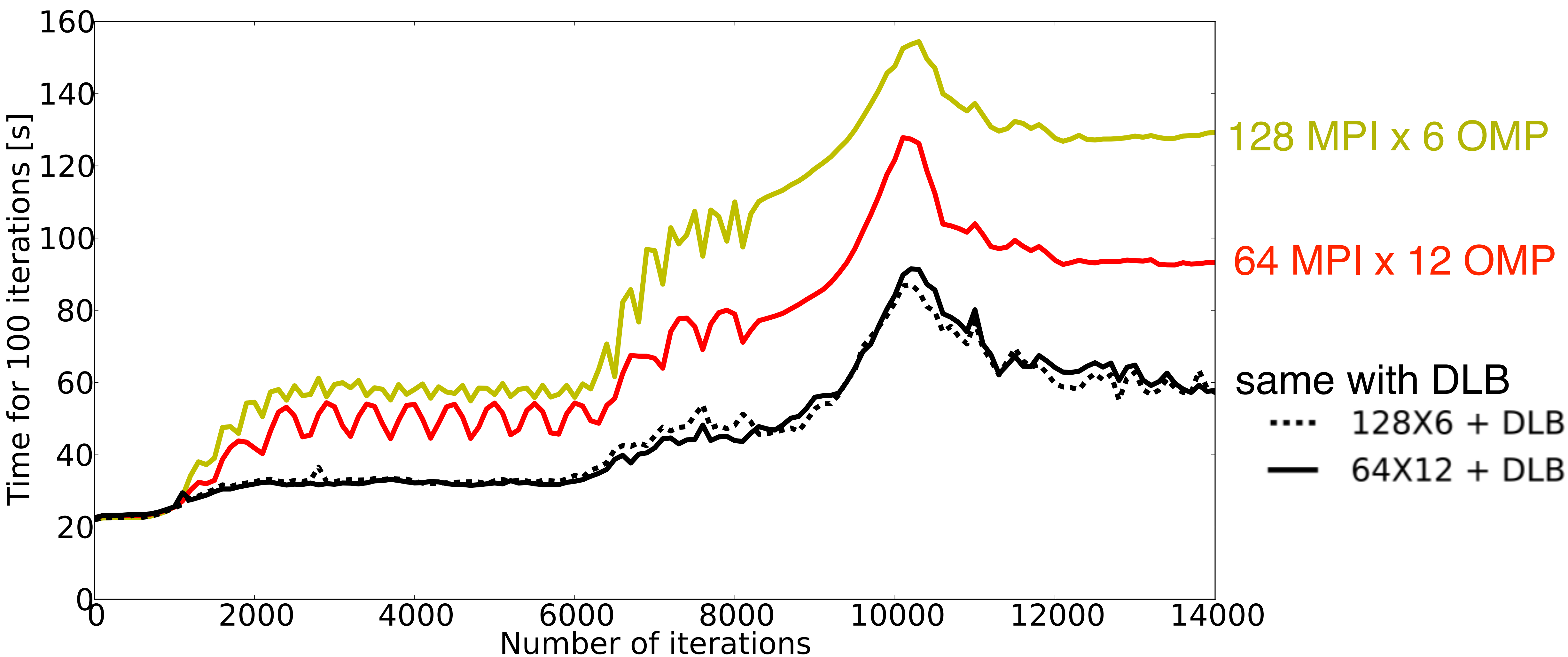

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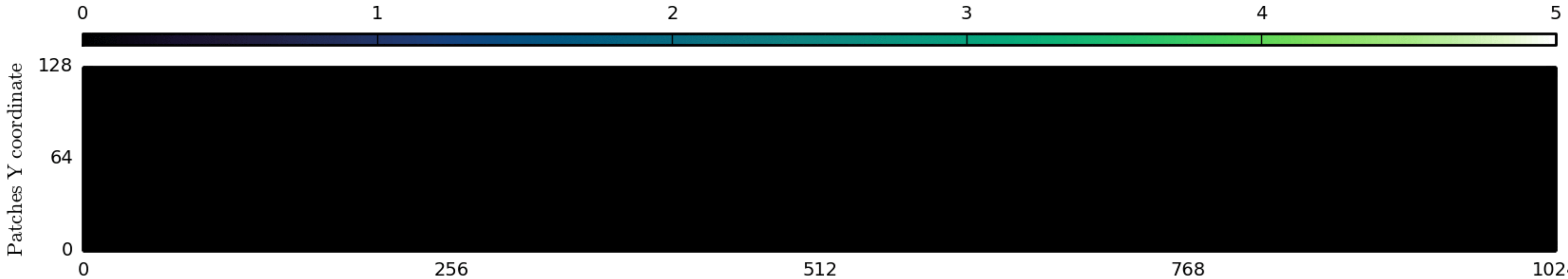

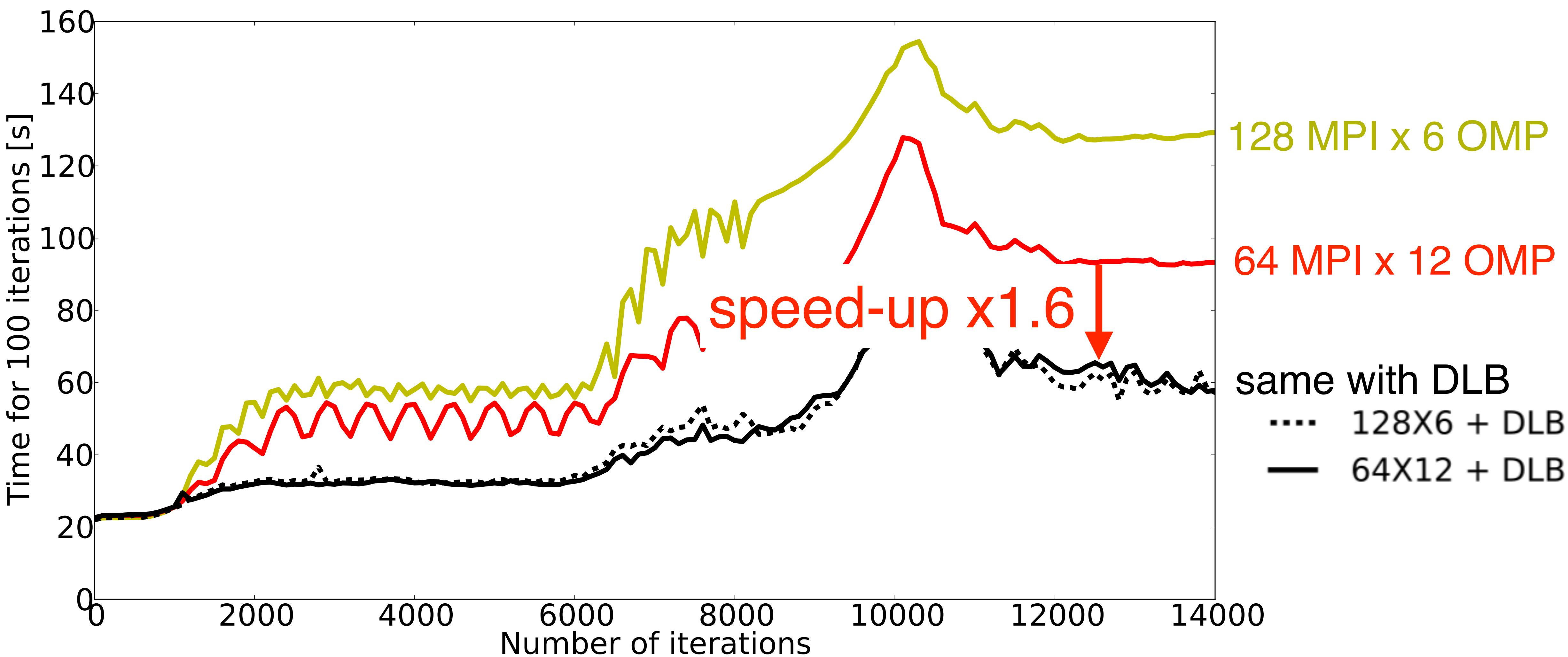

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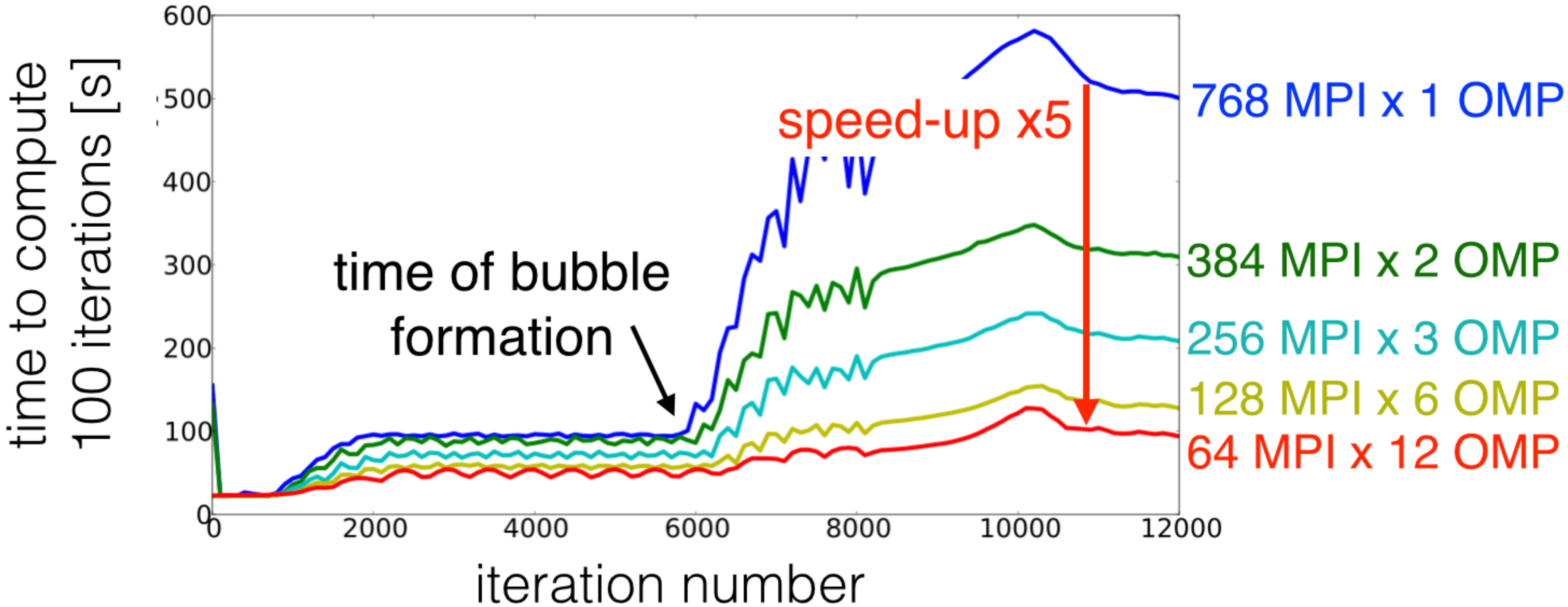
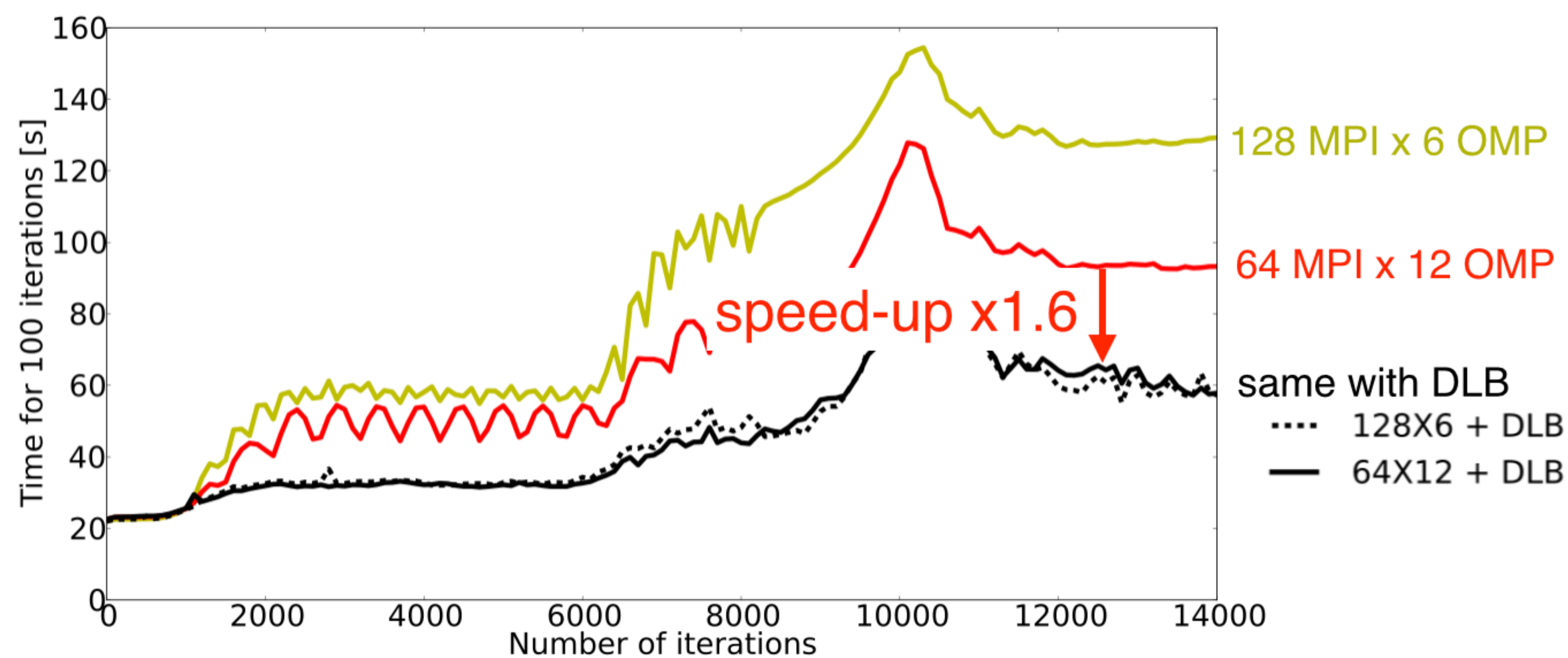
Hybrid + Dynamic Load Balancing





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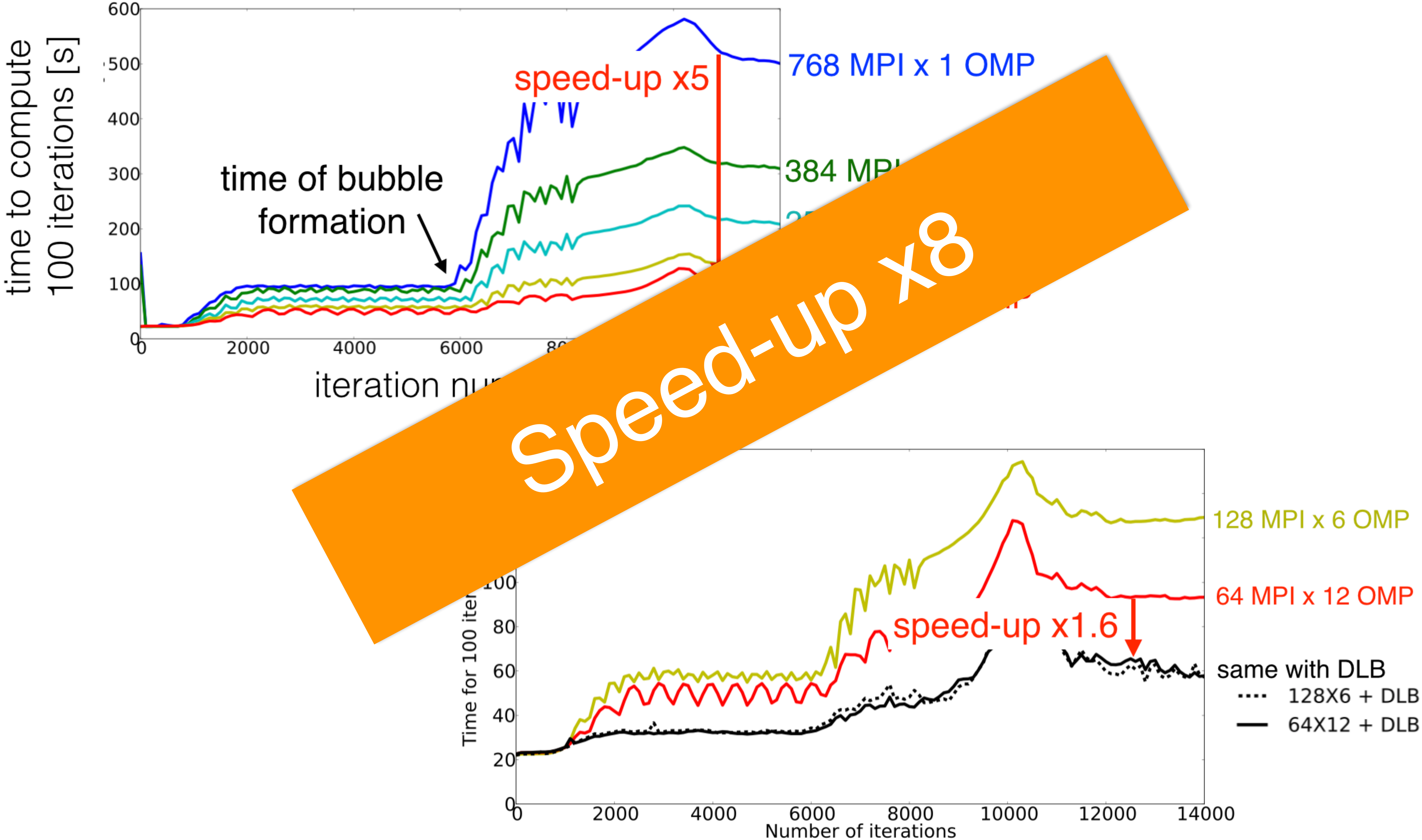
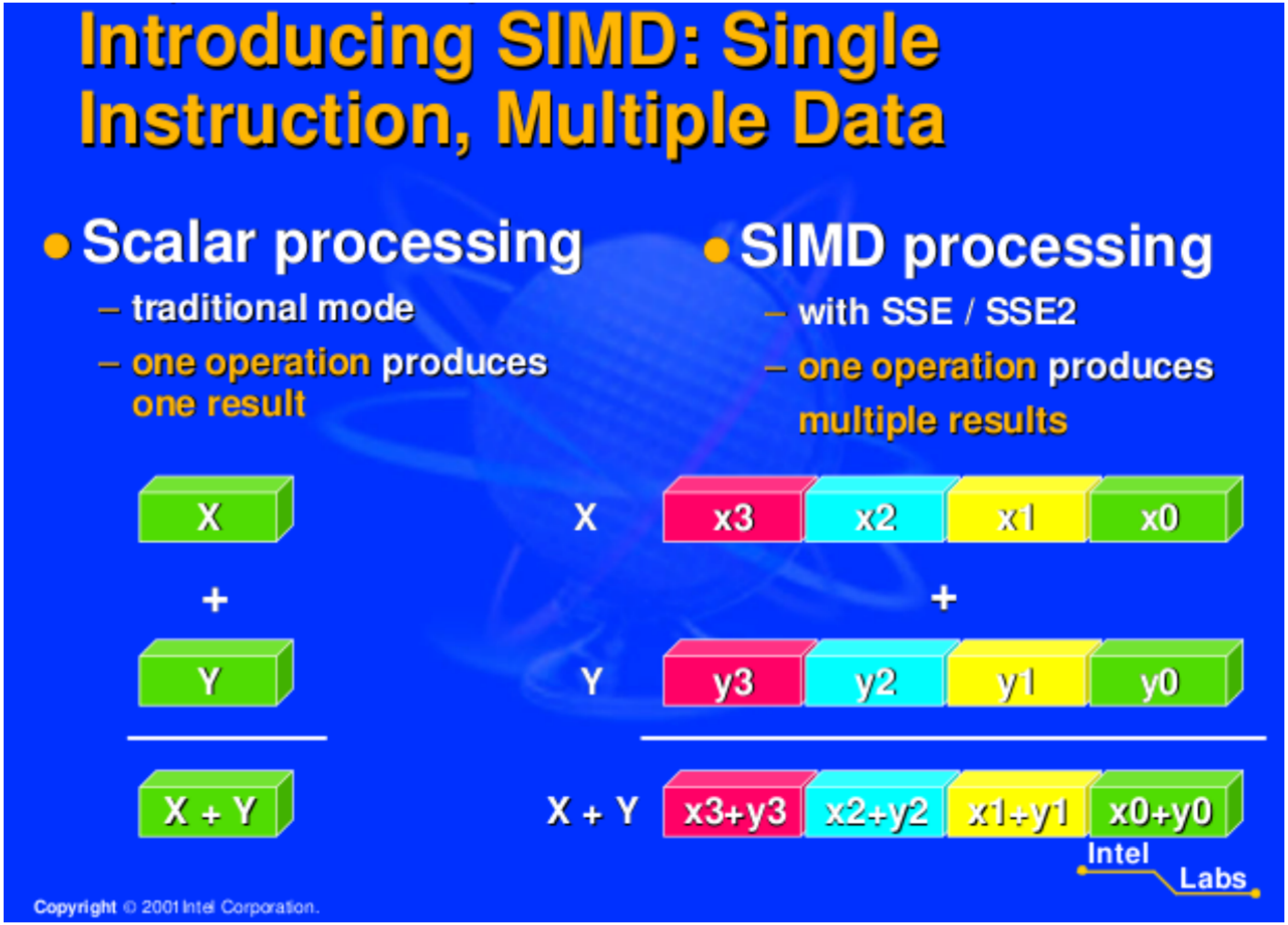


Step 2: Vectorization

Vectorization in a nutshell



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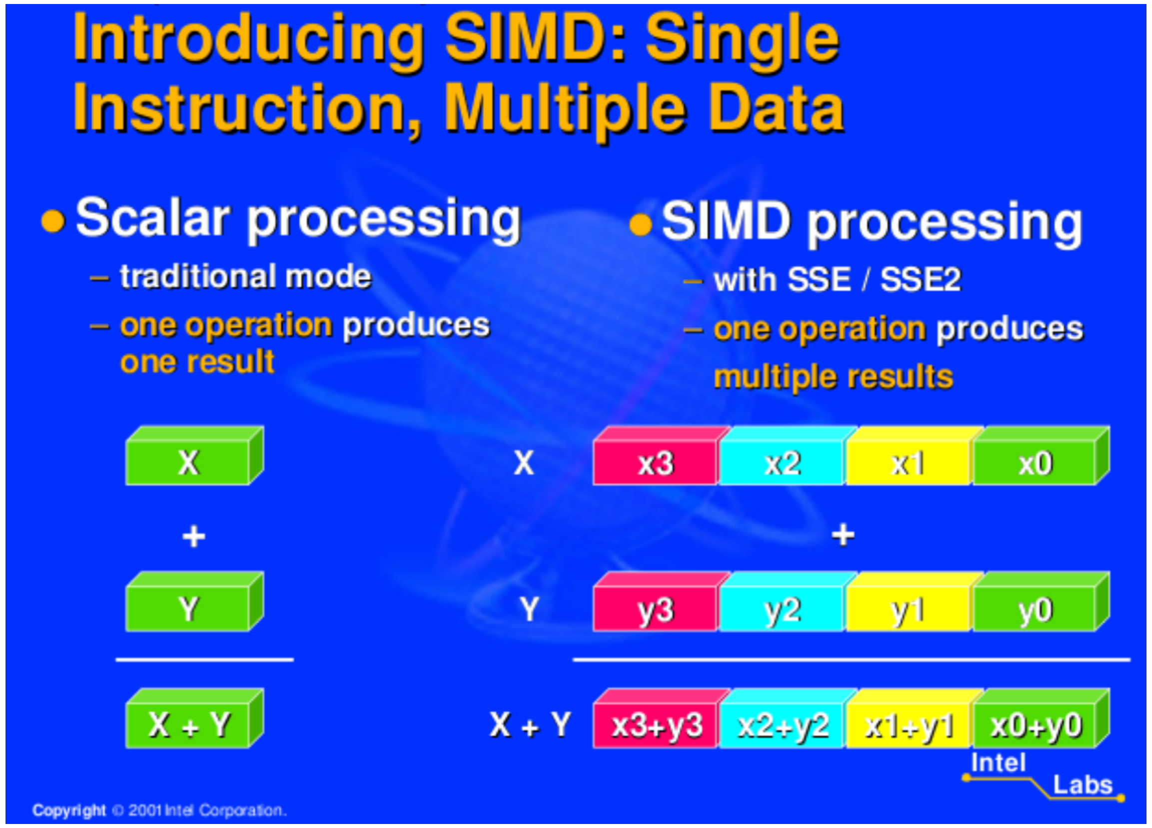

Beck *et al.*, arXiv:1810.03949

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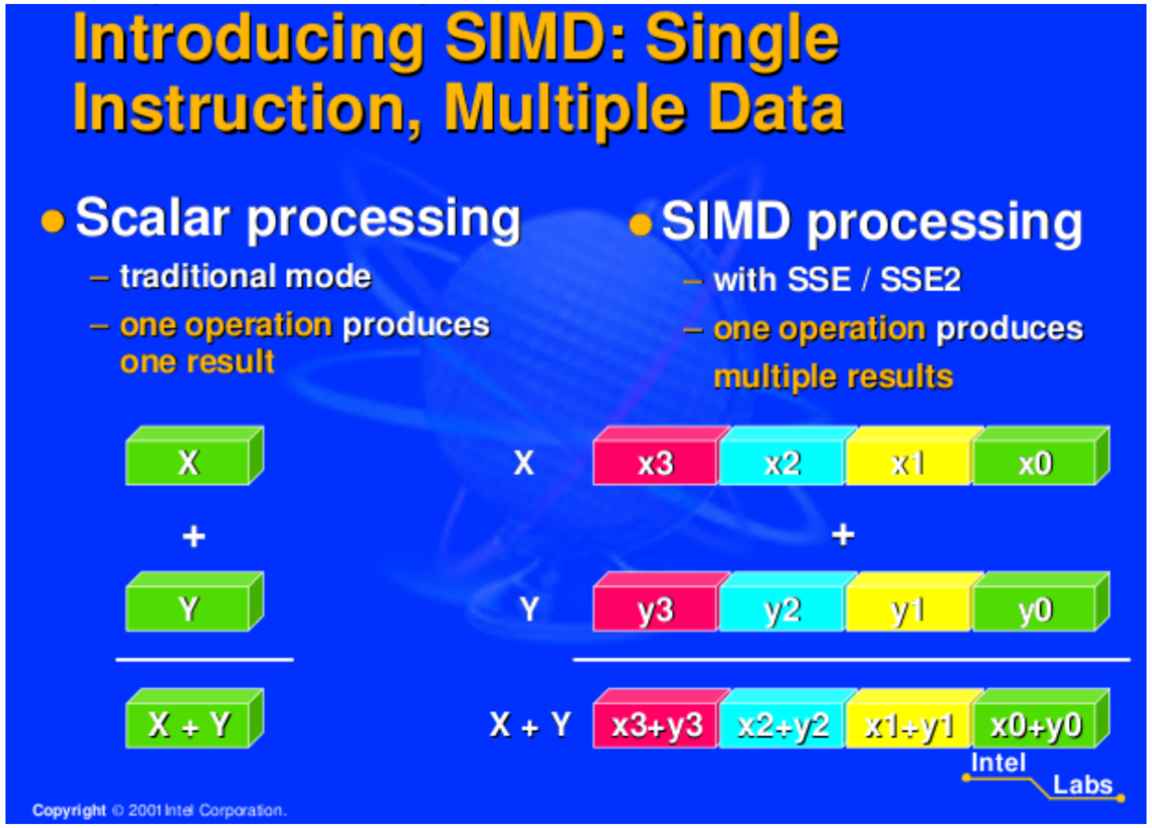

Smart (particles) operators:

- interpolator, pusher, projector

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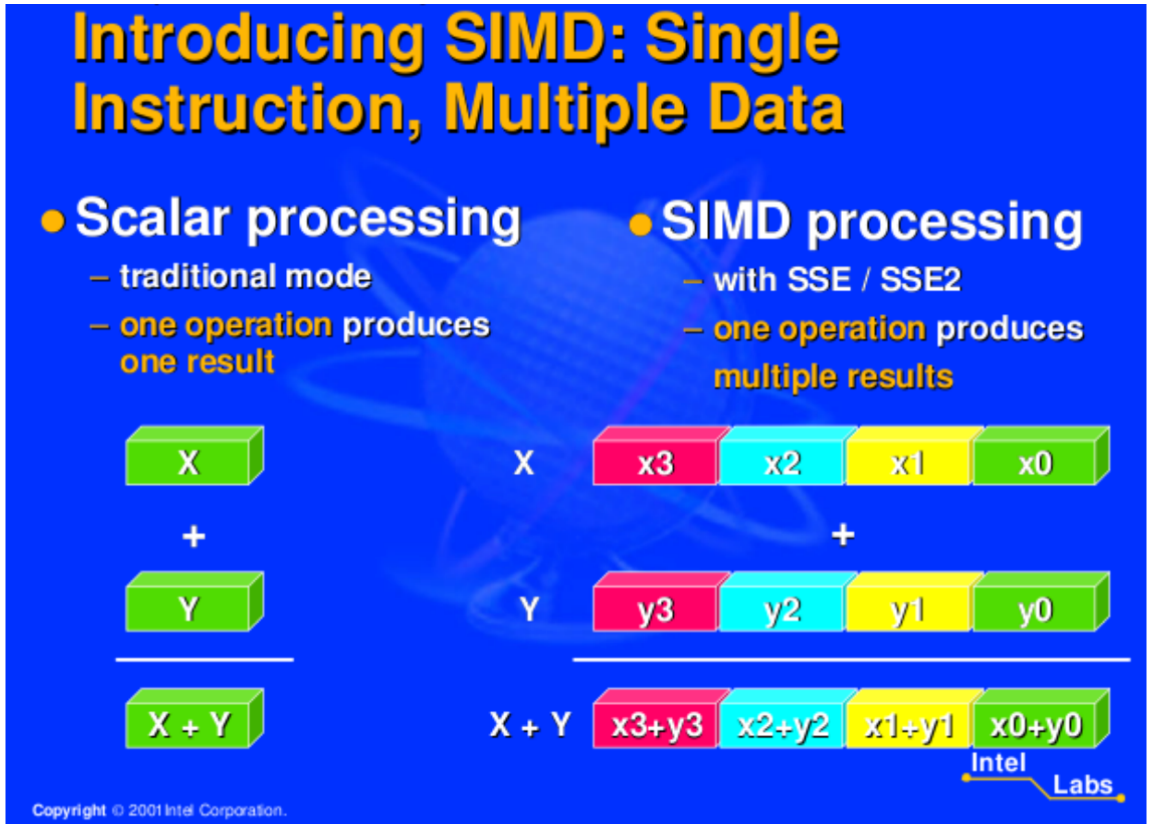


Smart (particles) operators:

- interpolator, pusher, projector

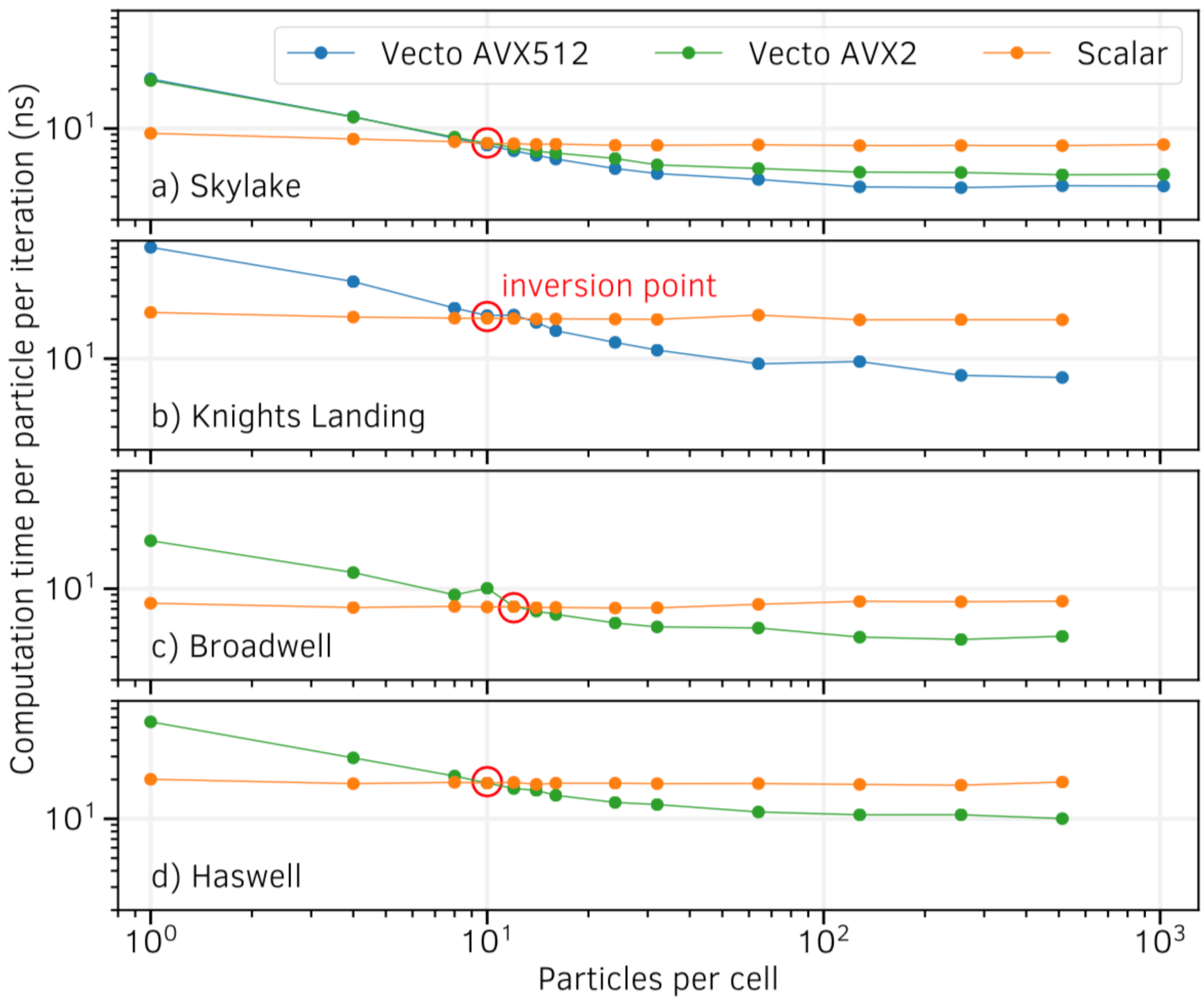
Smart (particles) data structures:

- beware random mem. access
- contiguous memory
- sort at all times!

Beck *et al*., arXiv:1810.03949

Step 2: Vectorization

SMILEI uses an adaptive vectorization approach



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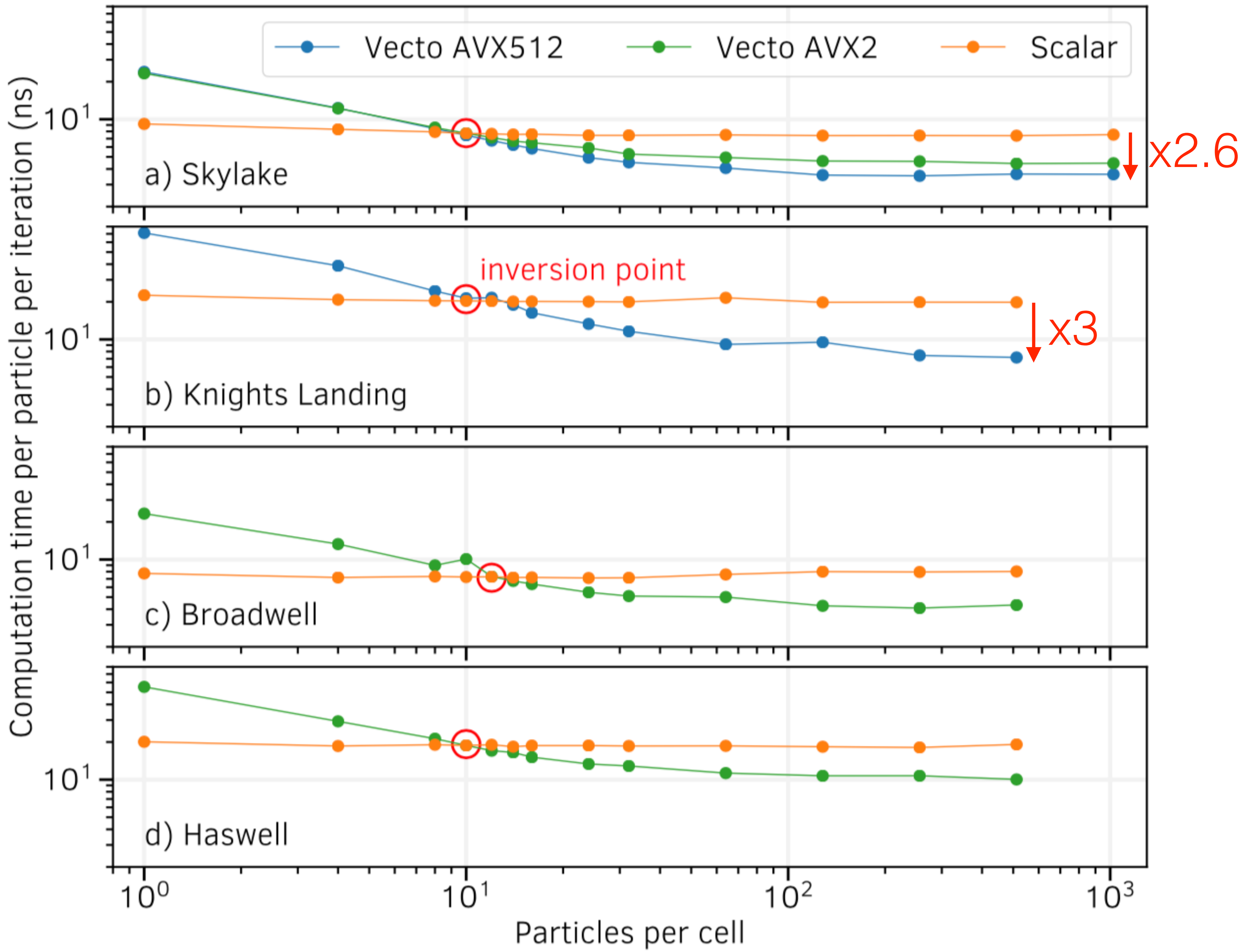
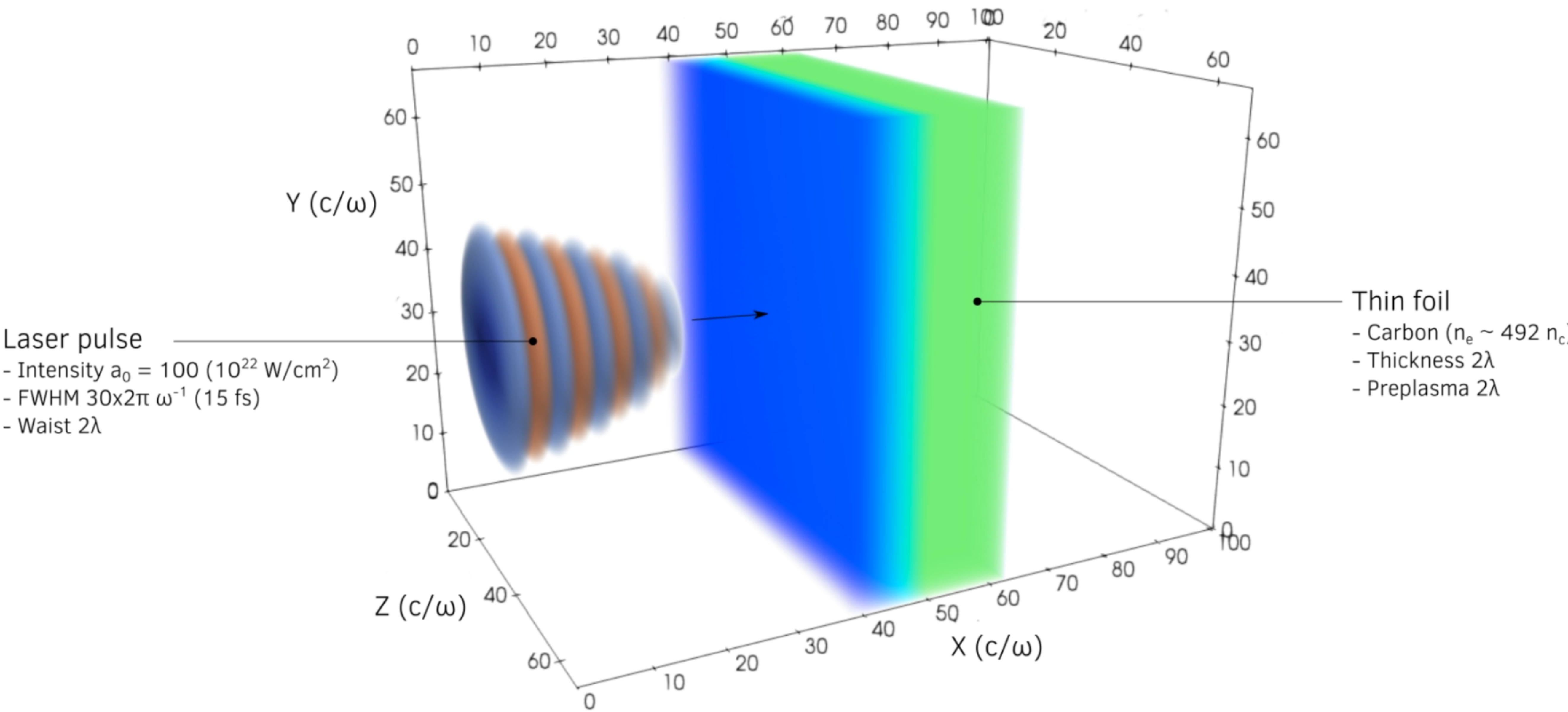


Step 2: Vectorization

Laser-driven hole-boring



@ 32 PPC : speed-up x 1.5

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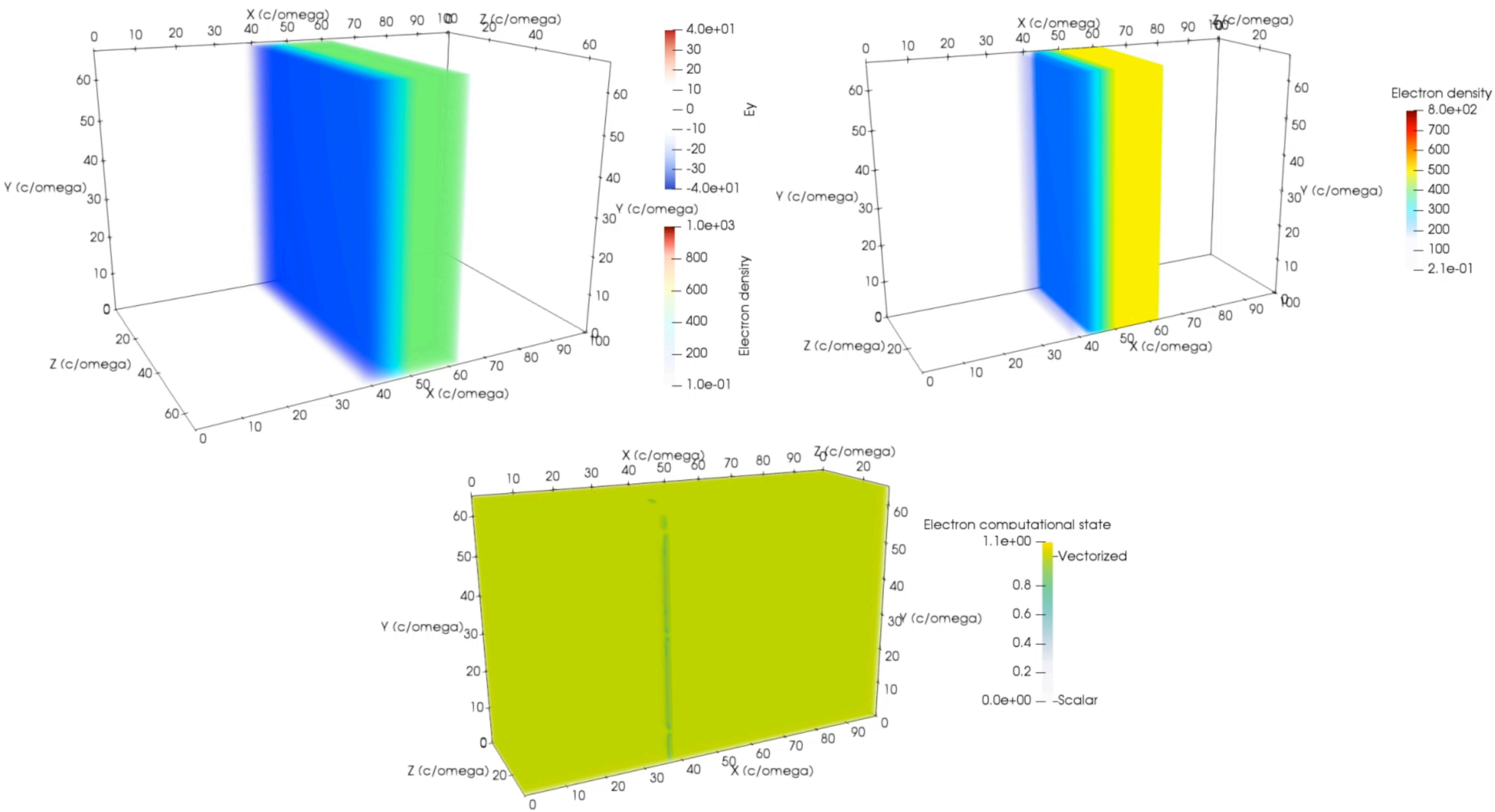

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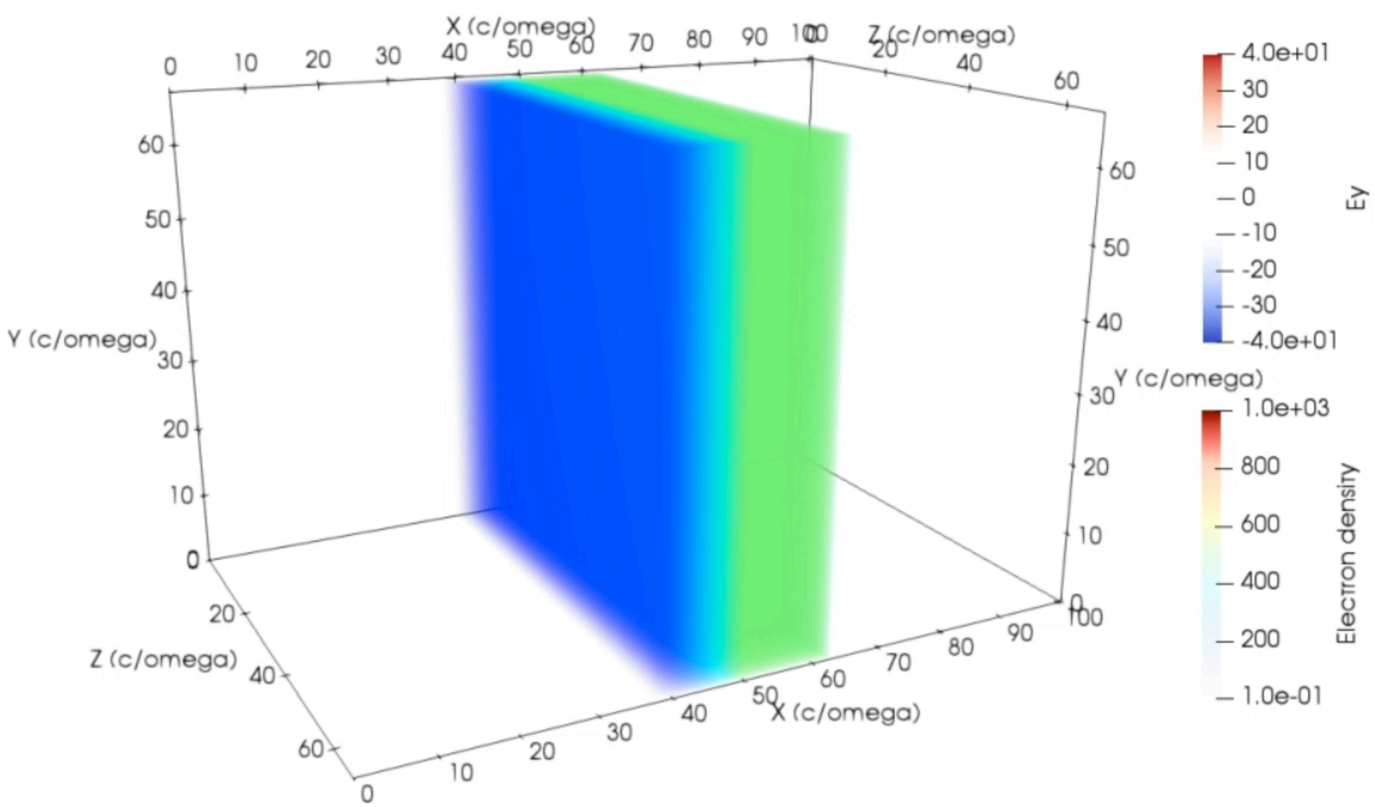
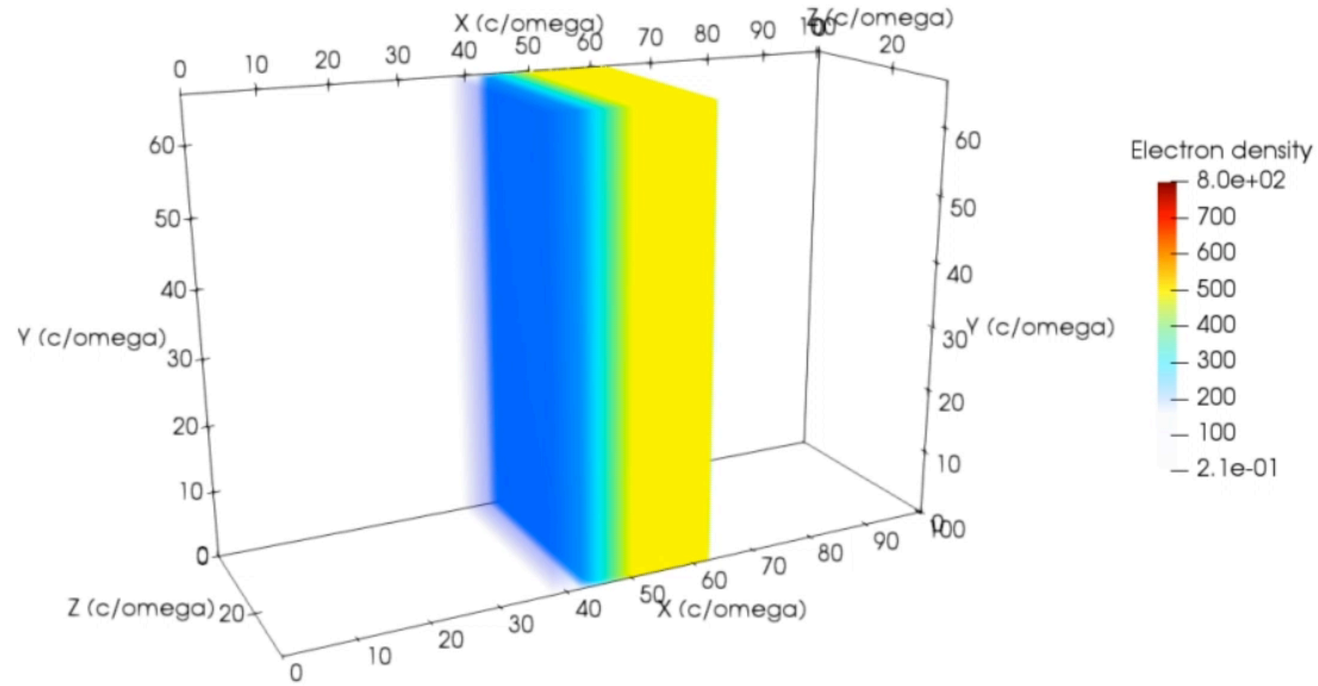
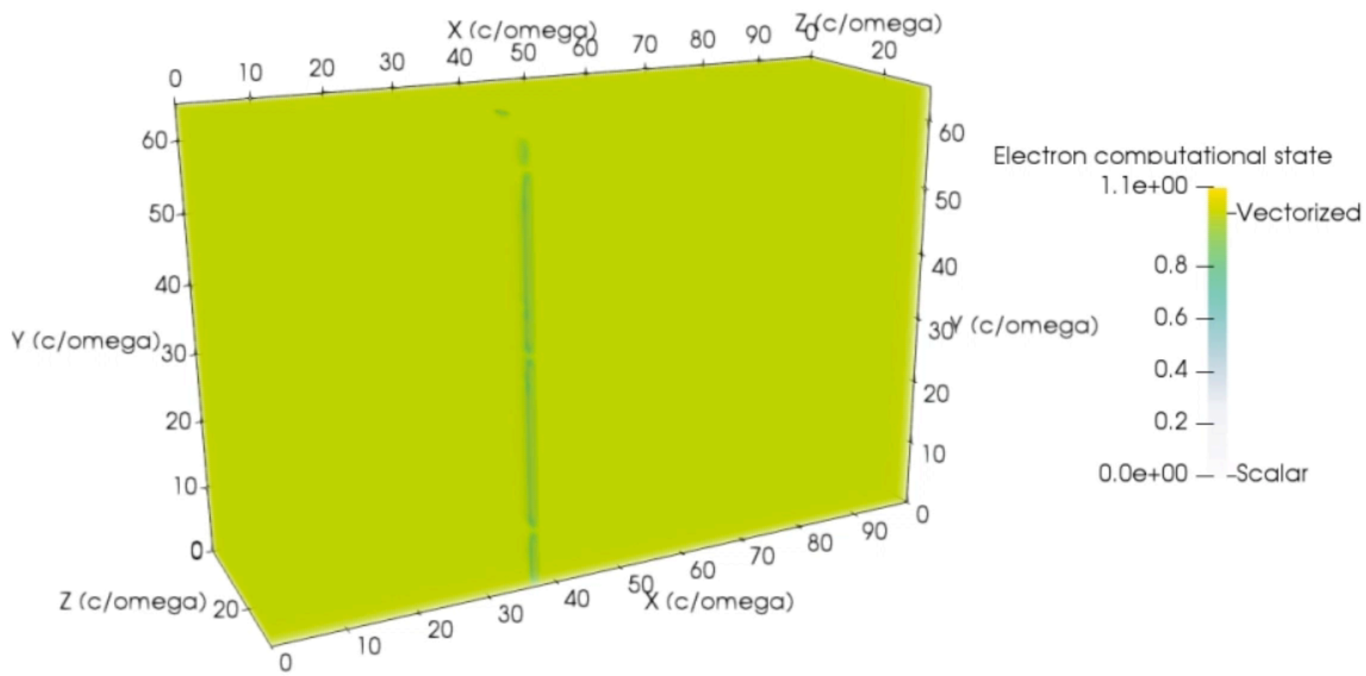
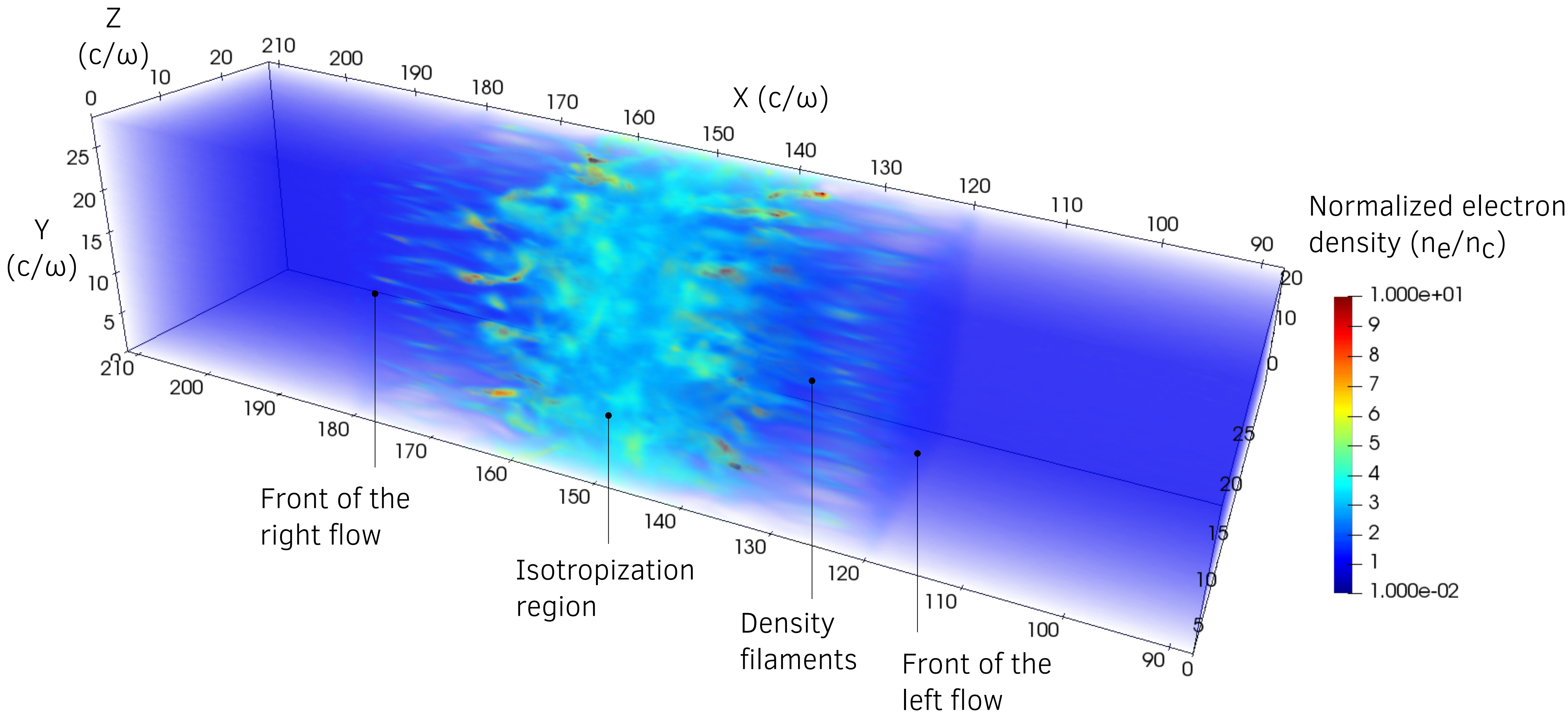






Step 2: Vectorization

Weibel-mediated collisionless shocks



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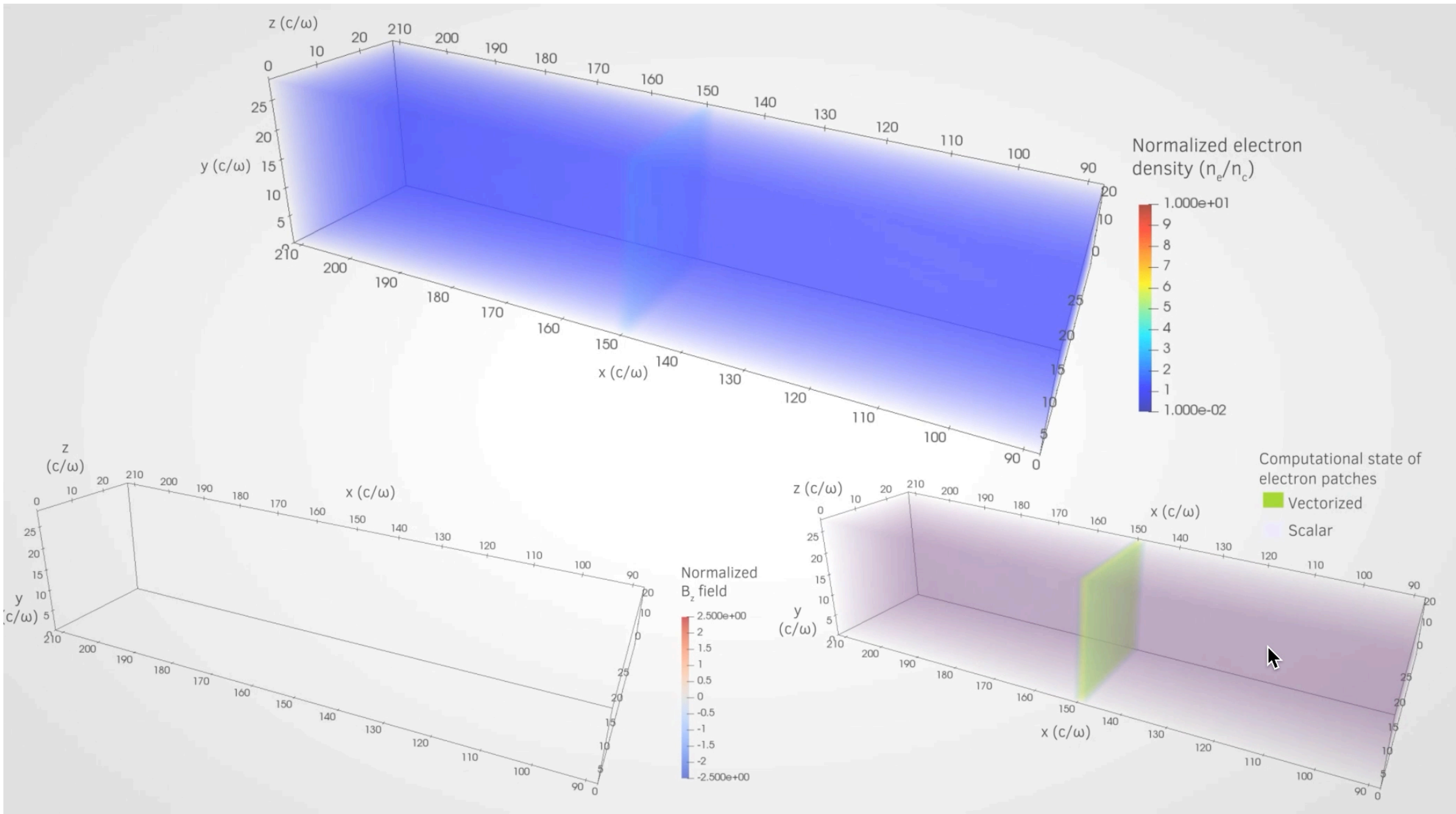

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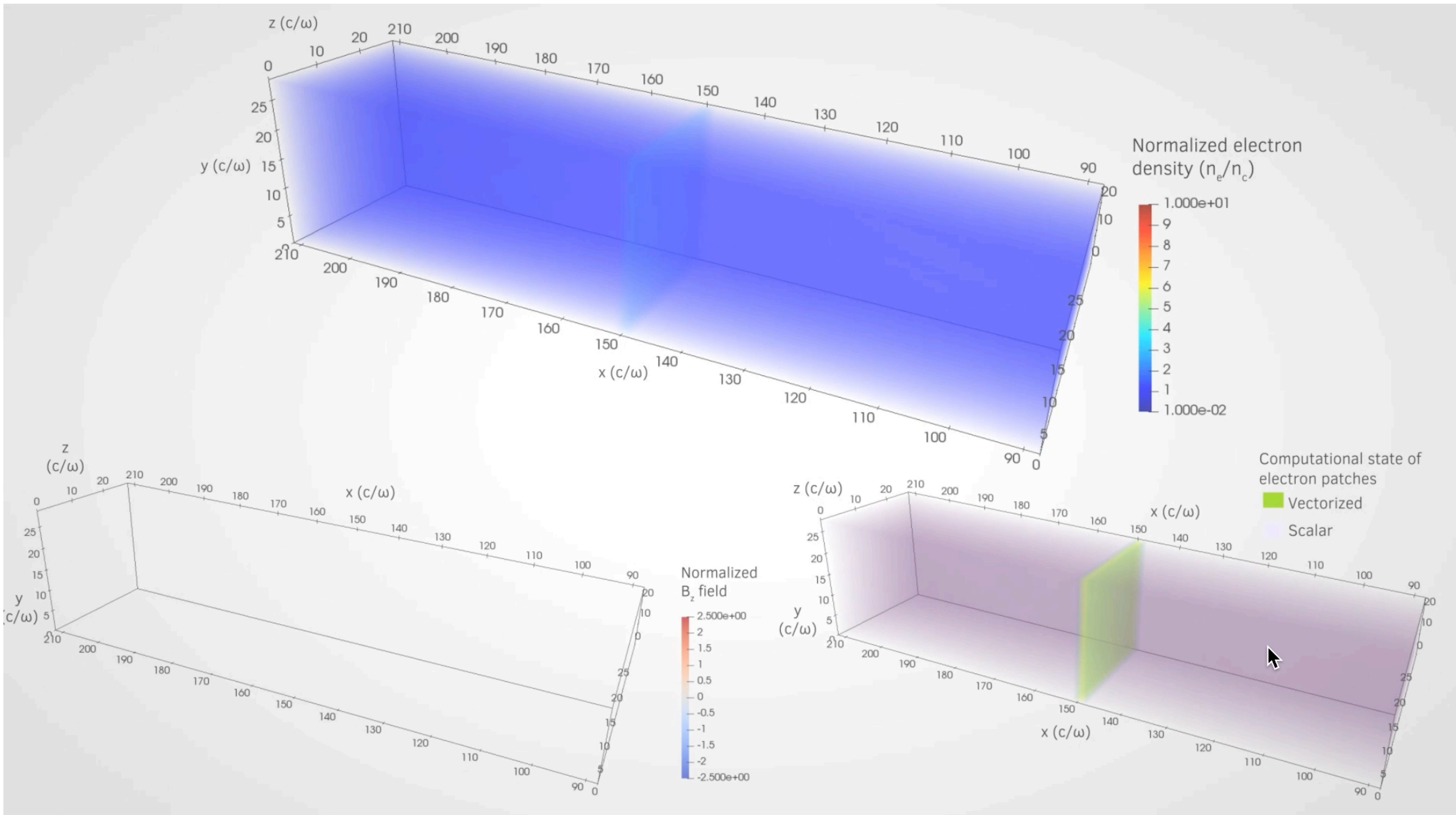

Outlines

- Numerical approach: how to build your PIC code
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- Additional modules: beyond the *collisionless* plasma
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Collisions can be introduced using an *ad-hoc* Monte-Carlo module

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Collisions are computed inside the cell

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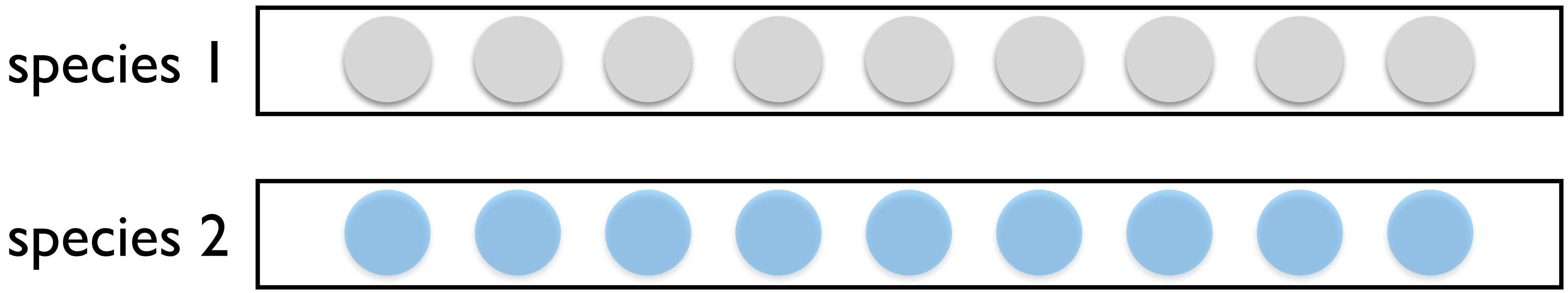
To avoid the N-body problem, quasi-particles in the cell are randomly "paired"



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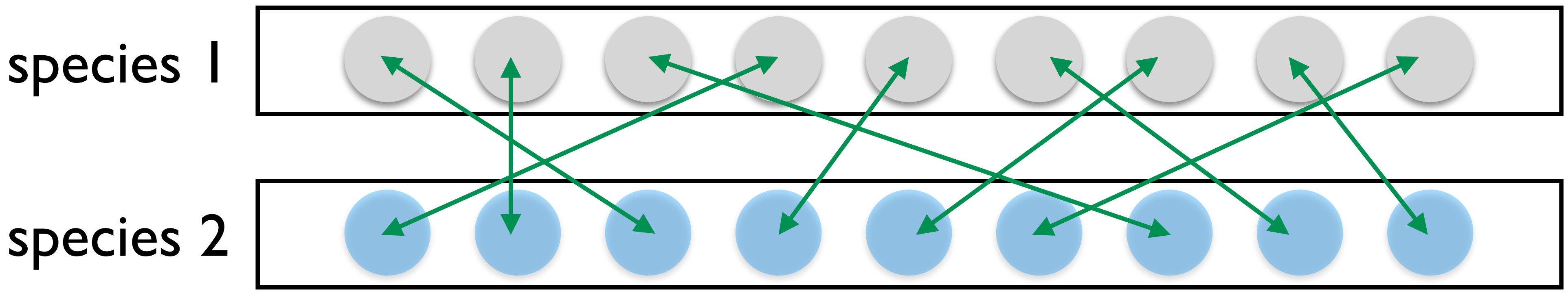

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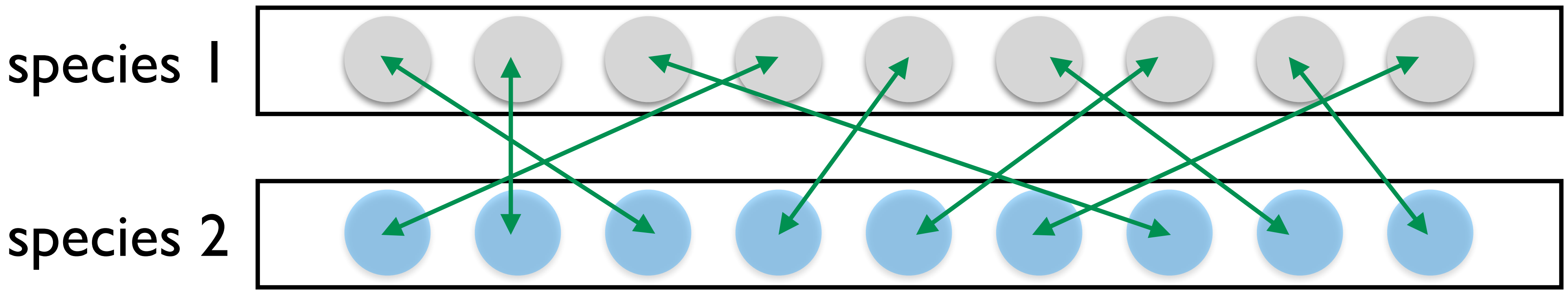
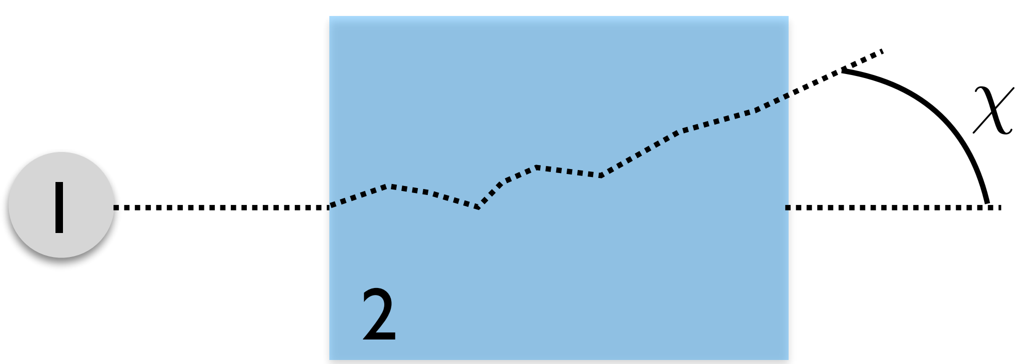


A single particle goes through many $(N \gg 1)$ collisions at small angle θ which translates in a total deflection angle χ (not necessarily small)



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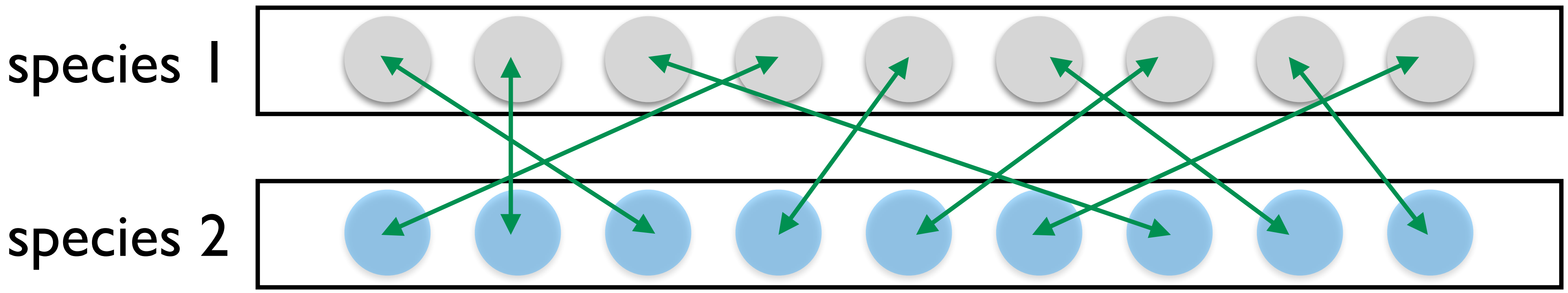

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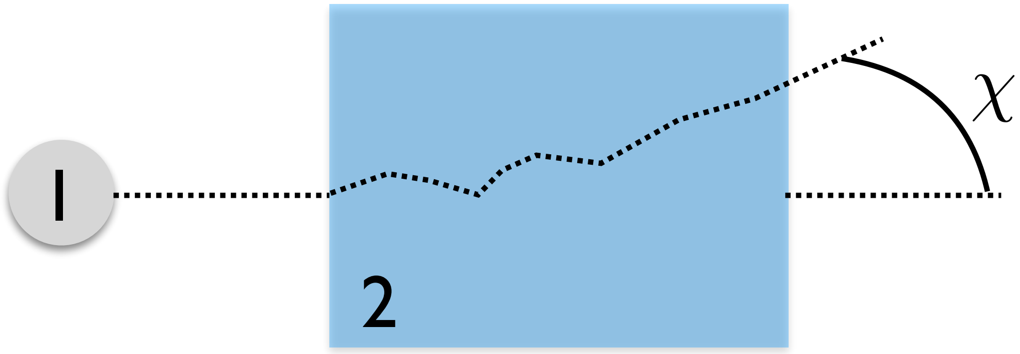


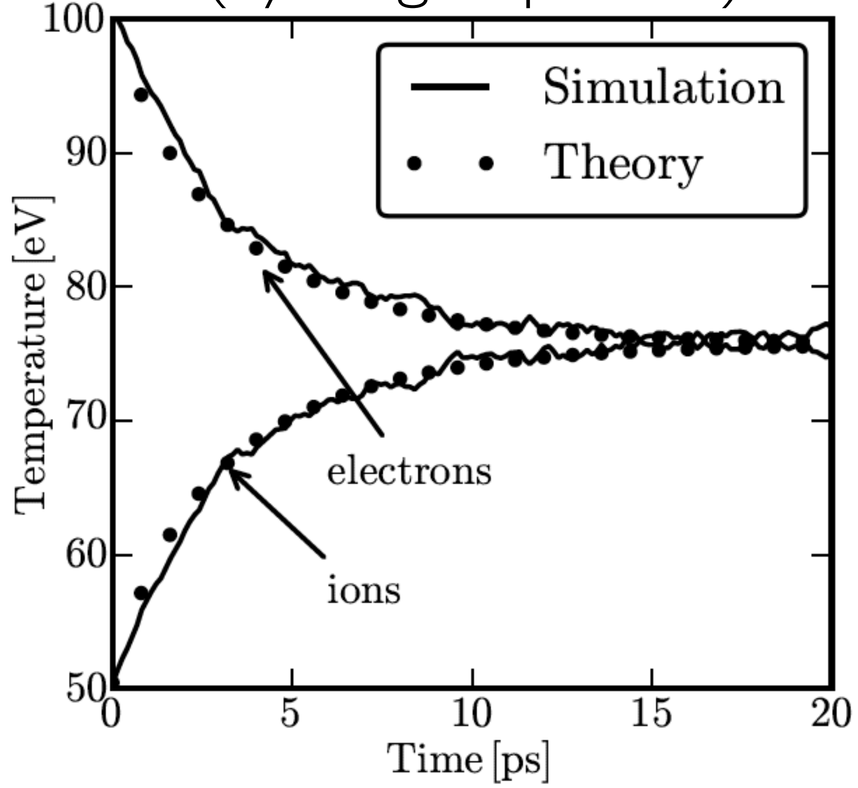
for each pair (Monte-Carlo)

- compute the collision rate
- compute the deflection angle
- deflect one or both particles

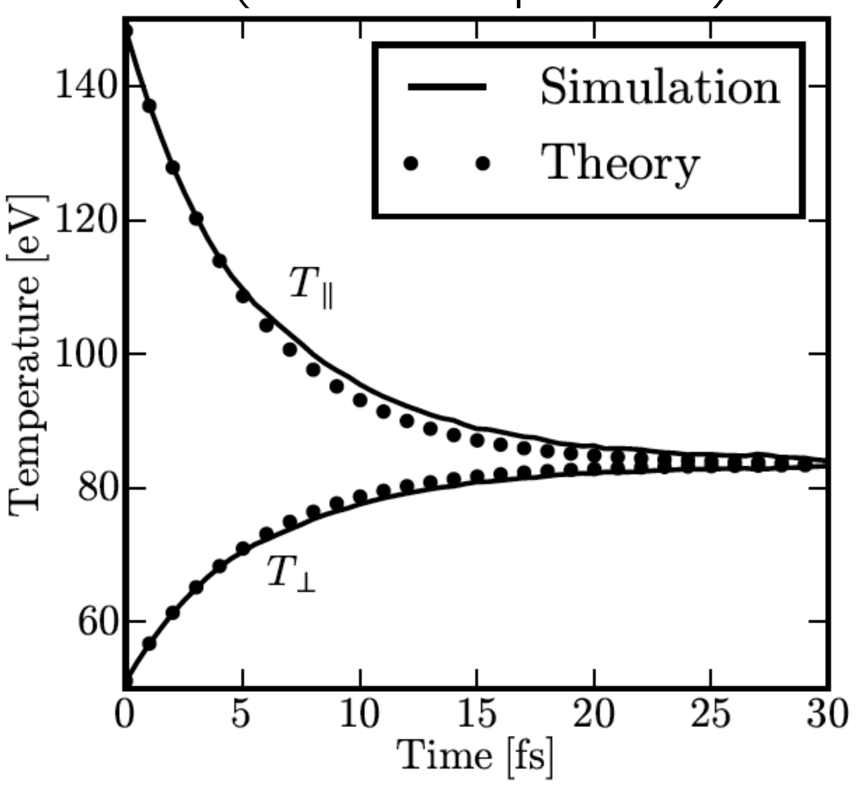
Nanbu, Phys. Rev. E **55**, 4642 (1997); J. Comp. Phys. **145**, 639 (1998)
F. Pérez et al., Phys. Plasmas **19**, 083104 (2012)

PIC codes are then able to treat purely collisional processes

Thermalization
(hydrogen plasma)



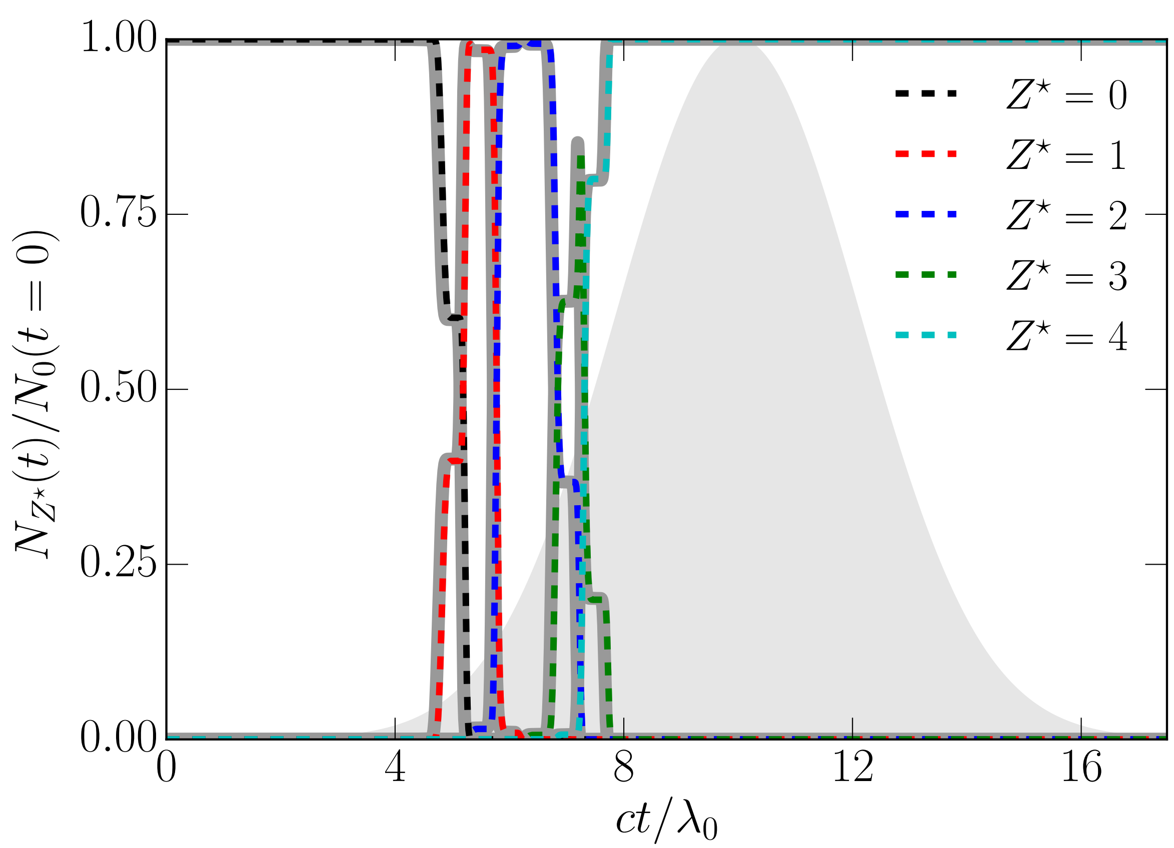
Isotropization
(electron plasma)



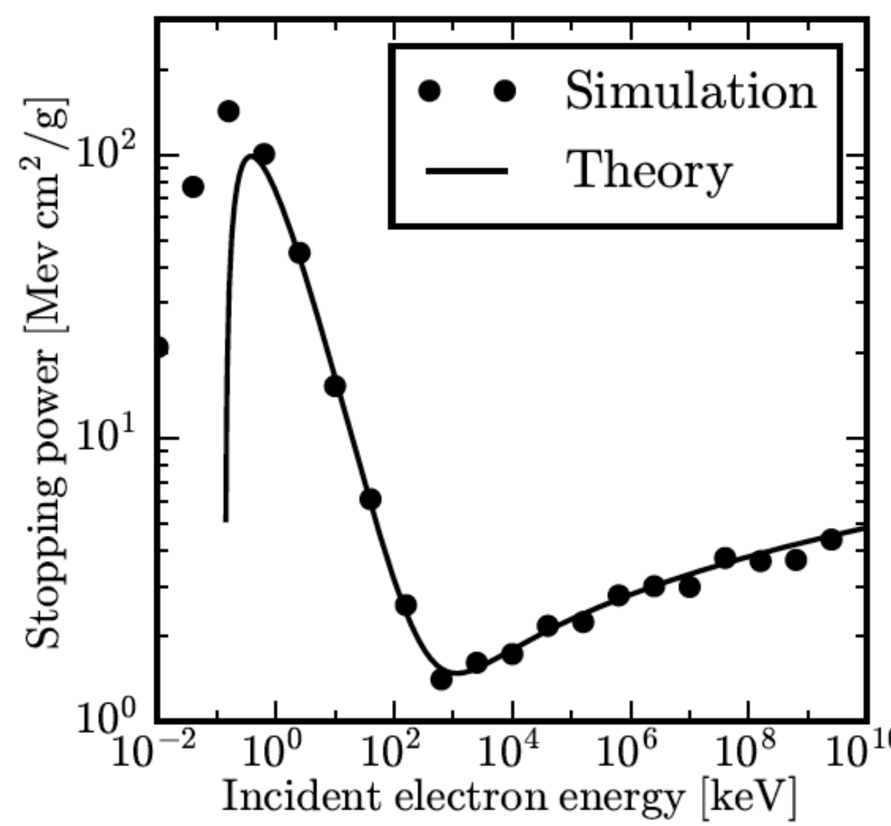
J. Derouillat et al., *SMILEI: a collaborative, open-source, multi-purpose PIC code for plasma simulation,* to be submitted (available upon request)

Similarly field and collisional ionization can be treated using a Monte-Carlo approach

Field ionization of Carbon by a 5x10^{16} W/cm^2 20 fs laser pulse



Stopping power of a cold aluminium plasma of density 10^{21} cm^{-3}



R. Nuter *et al.*, Phys. Plasmas **18**, 033107 (2011); F. Pérez et al., Phys. Plasmas **19**, 083104 (2012)
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Adding Quantum Electrodynamics (QED) effect is also very interesting for forthcoming multi-petawatt facilities

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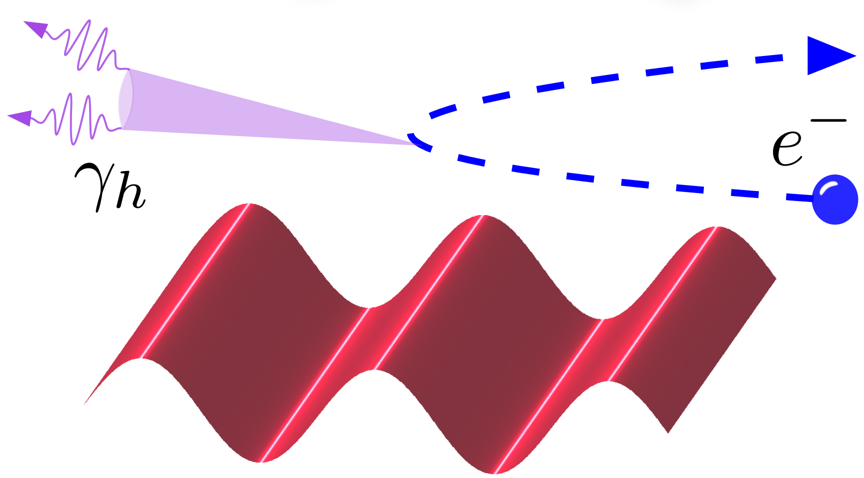
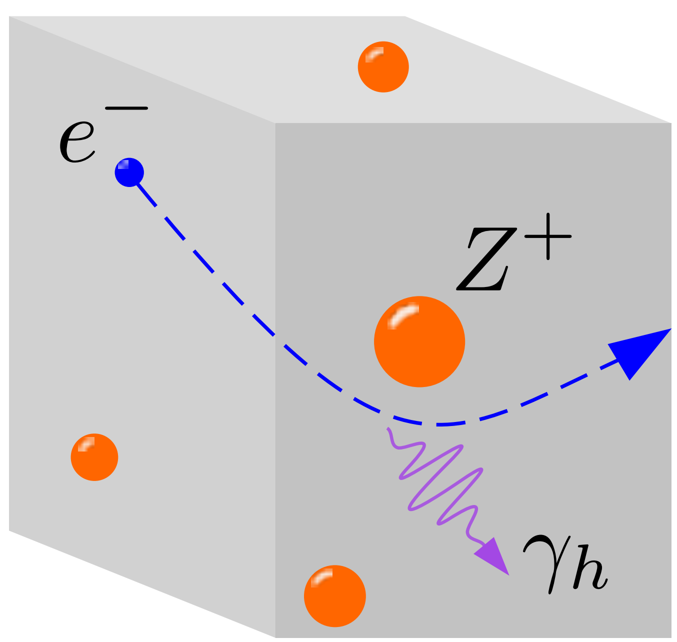
High-Energy Photon Production

Nonlinear Thomson and Compton scattering



$$e^- + n\,\gamma_L \rightarrow e^- + \gamma_h$$

Bremsstrahlung



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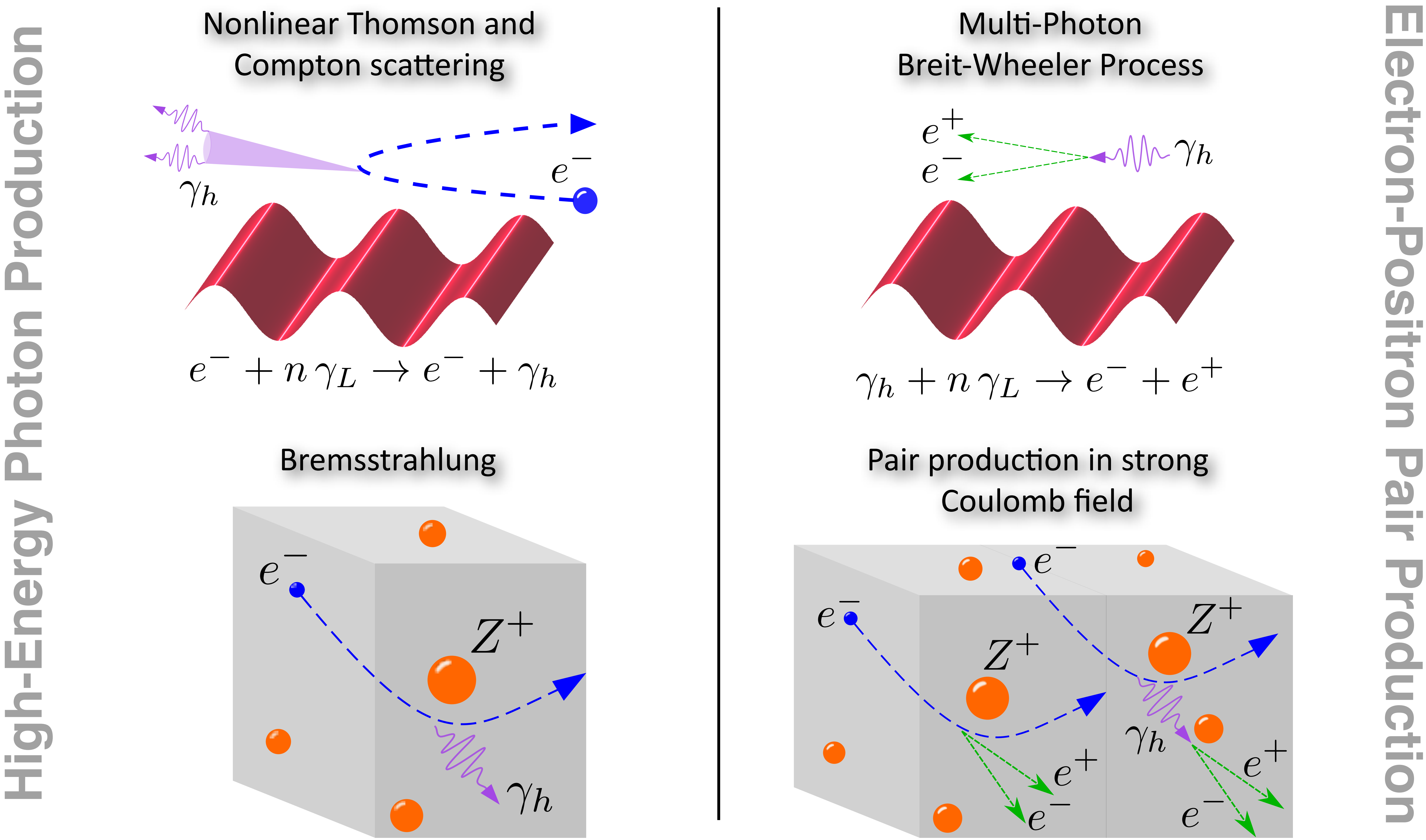

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Even 1D simulation can bring a deep insight into the physics at play

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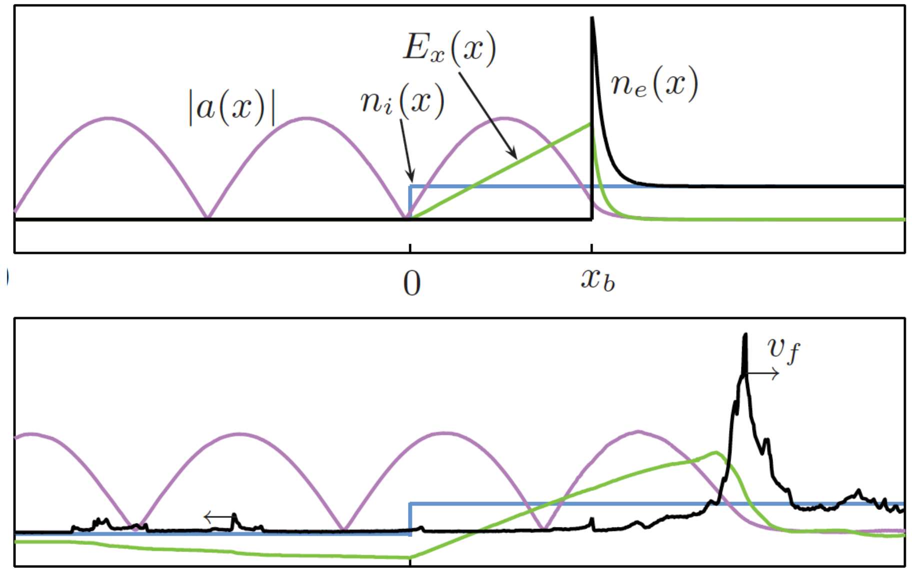
Relativistically-Induced Transparency



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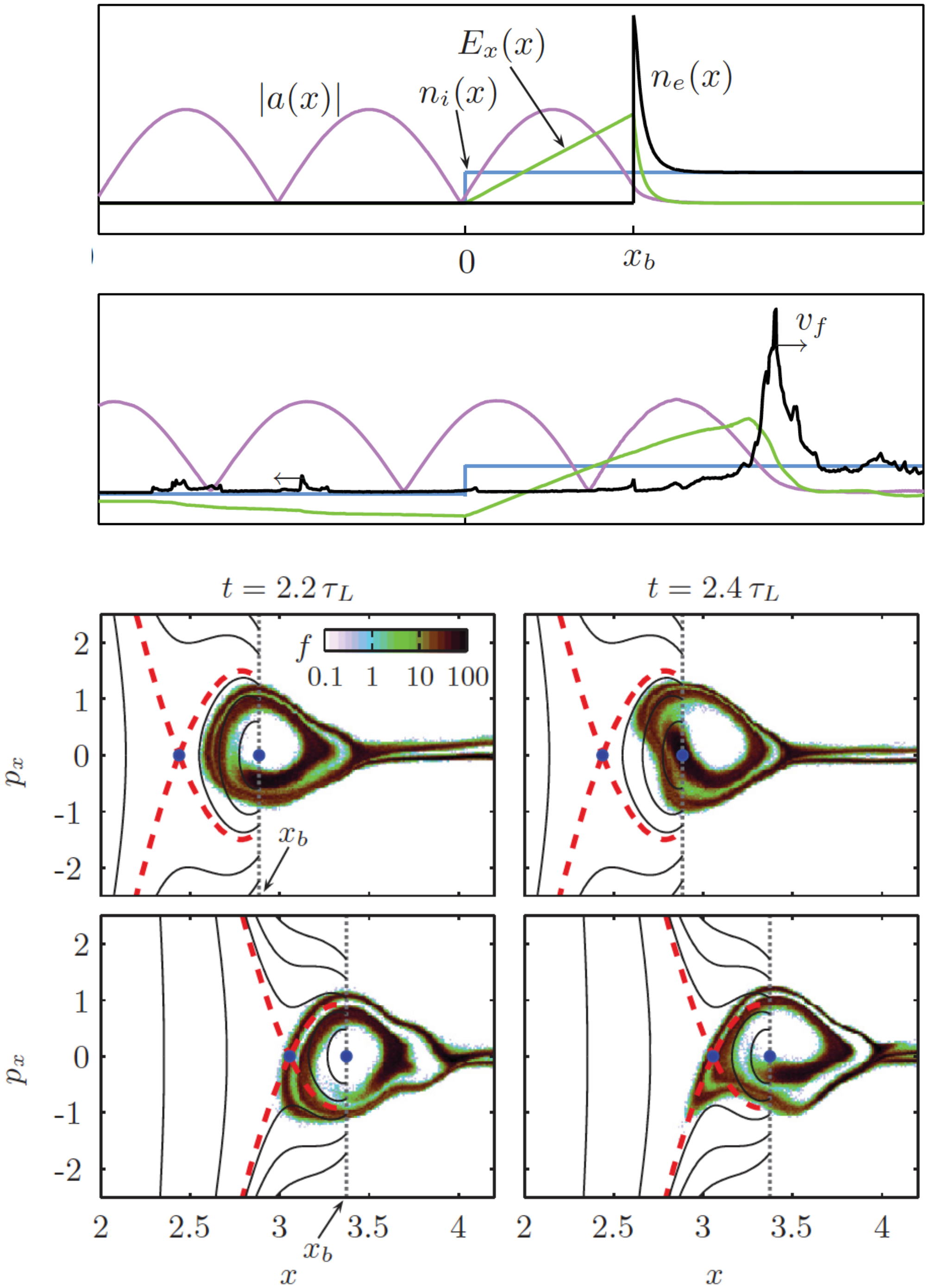

E. Siminos *et al.*, Phys. Rev. E **86**, 056404 (2012)

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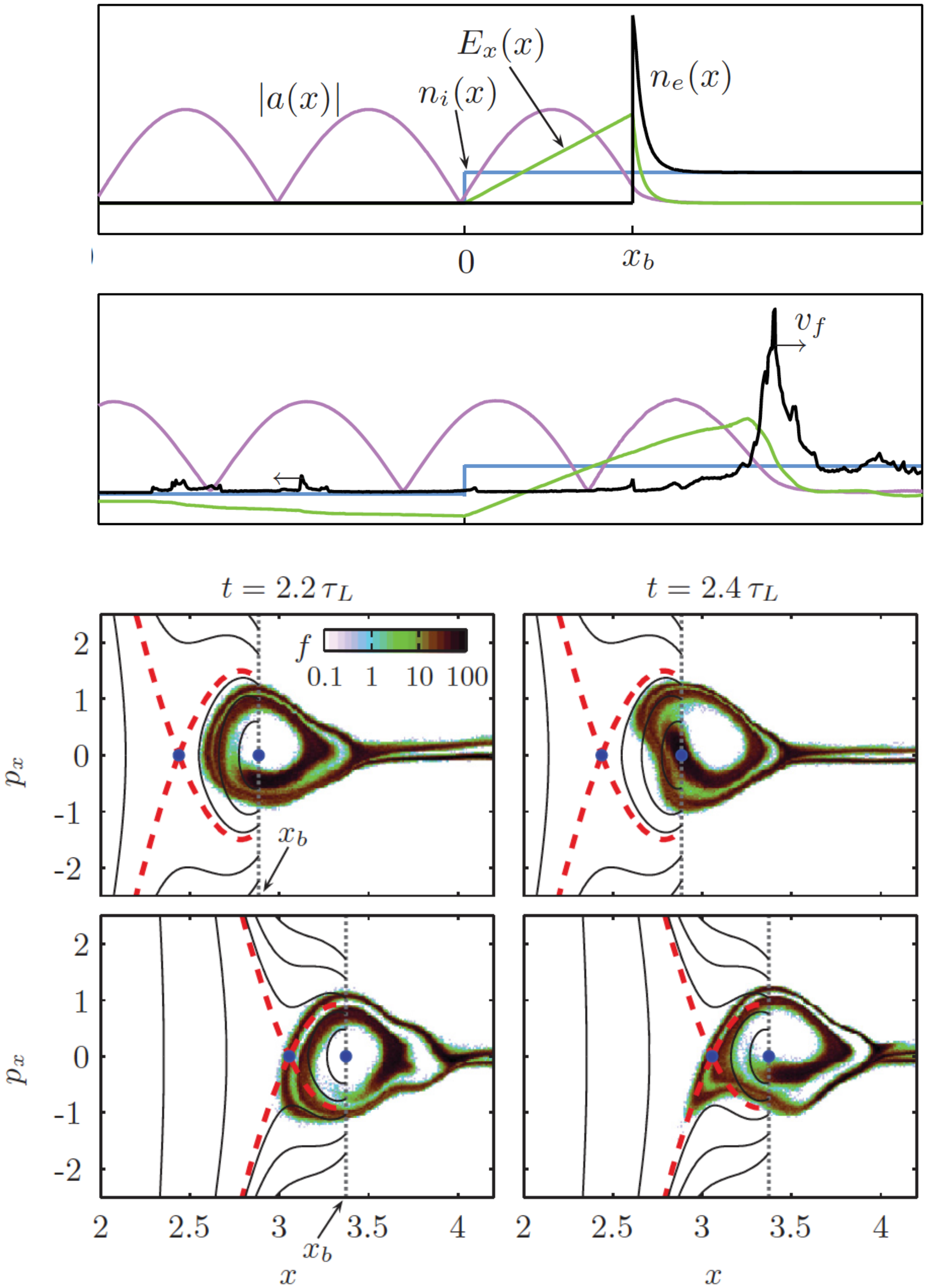
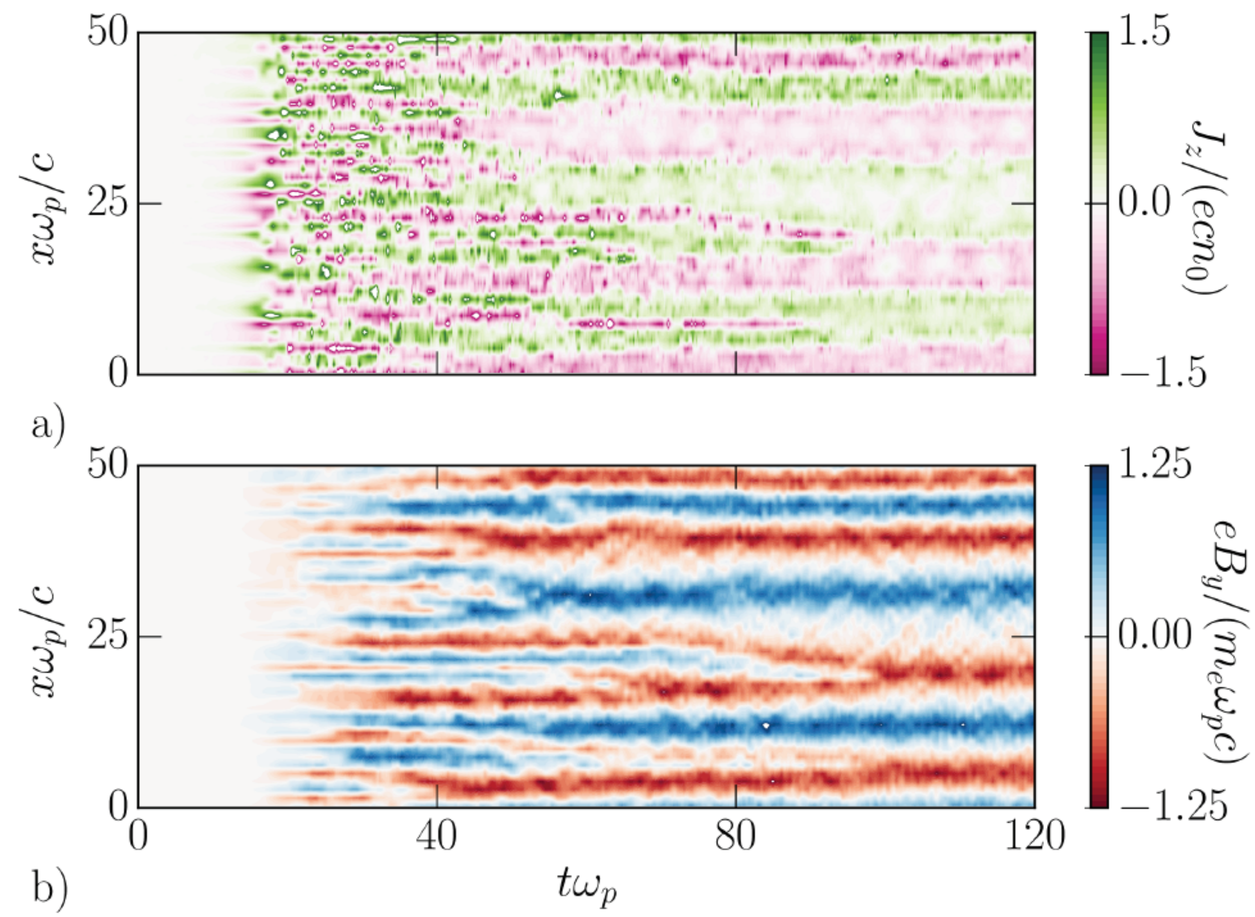


E. Siminos *et al.*, Phys. Rev. E **86**, 056404 (2012)

Weibel instability in the presence of an external magnetic field



A. Grassi *et al.*, Phys. Rev. E (in press)

PIC codes can help design & interpret experimental campaigns

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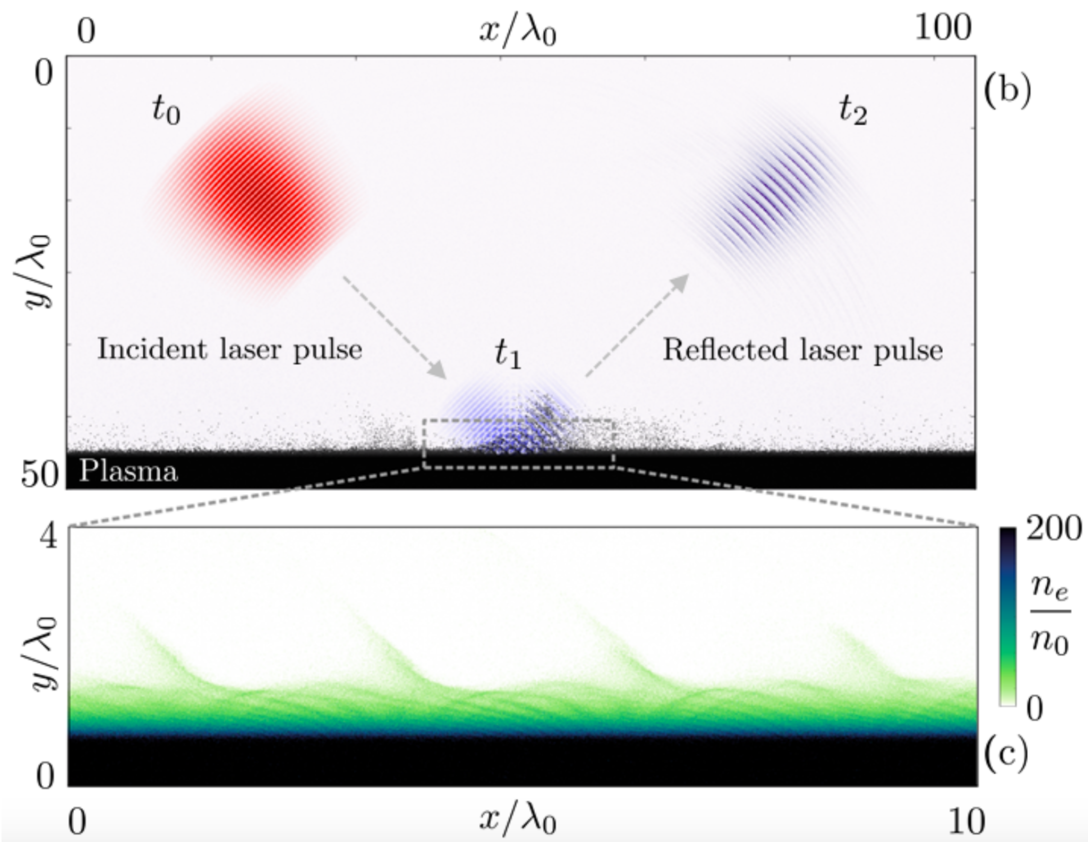
High-harmonic generation & electron acceleration from laser-solid interaction



G. Bouchard, F. Quéré, CEA/IRAMIS

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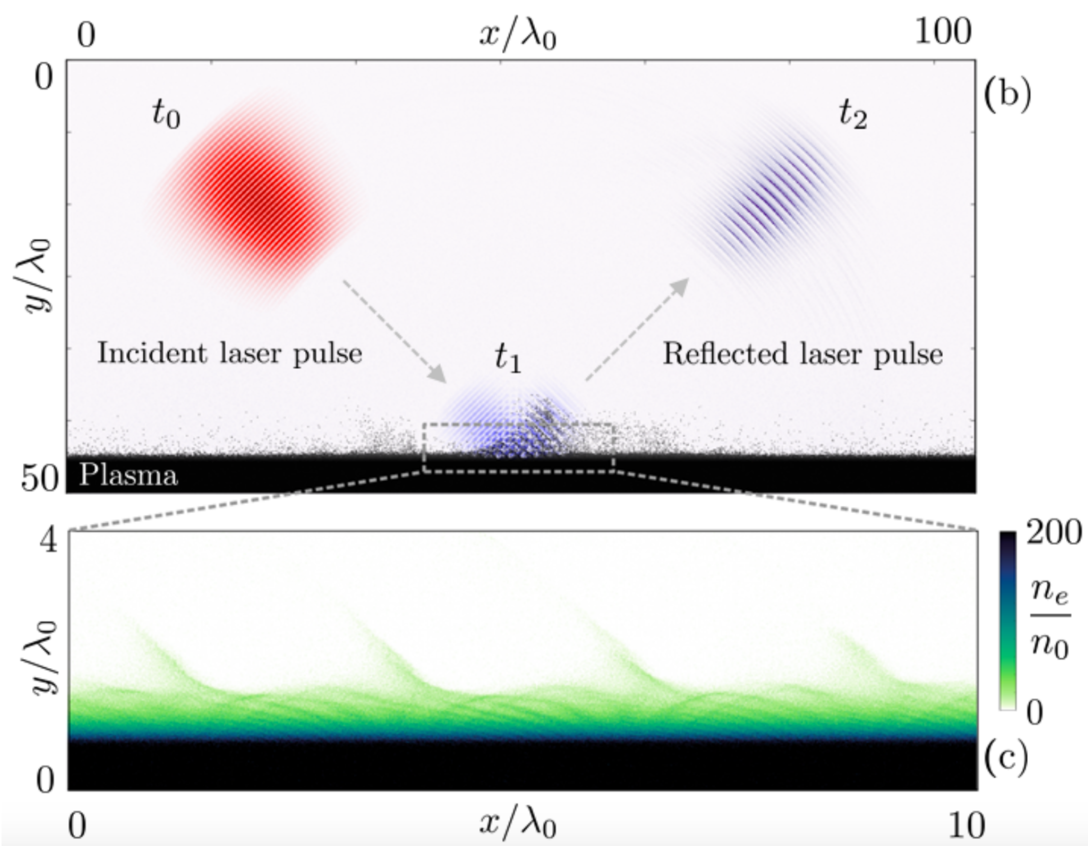

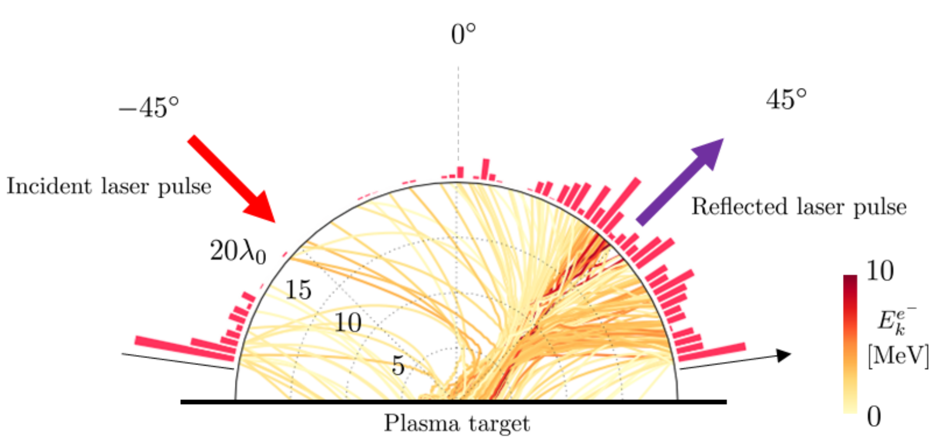

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PIC codes can help design & interpret experimental campaigns

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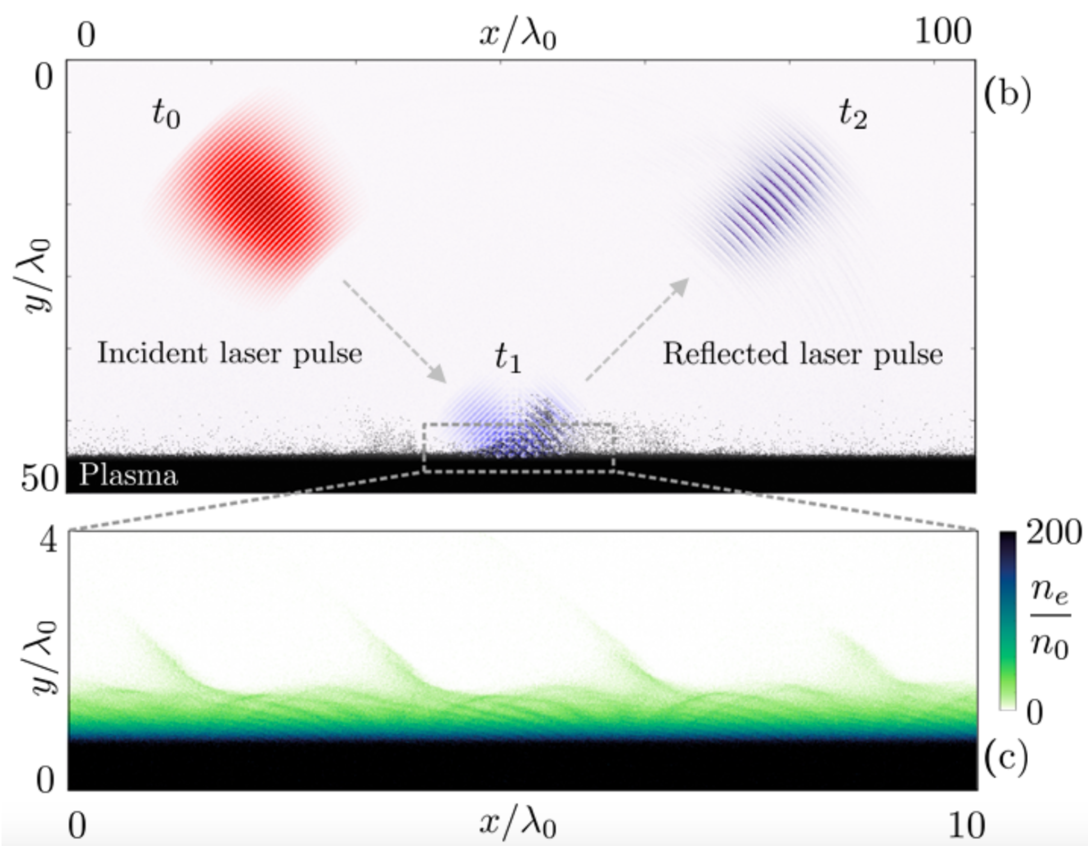

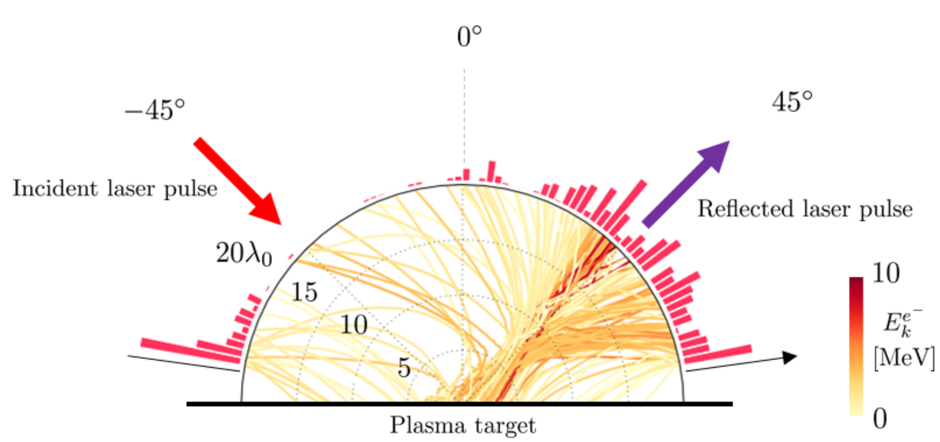

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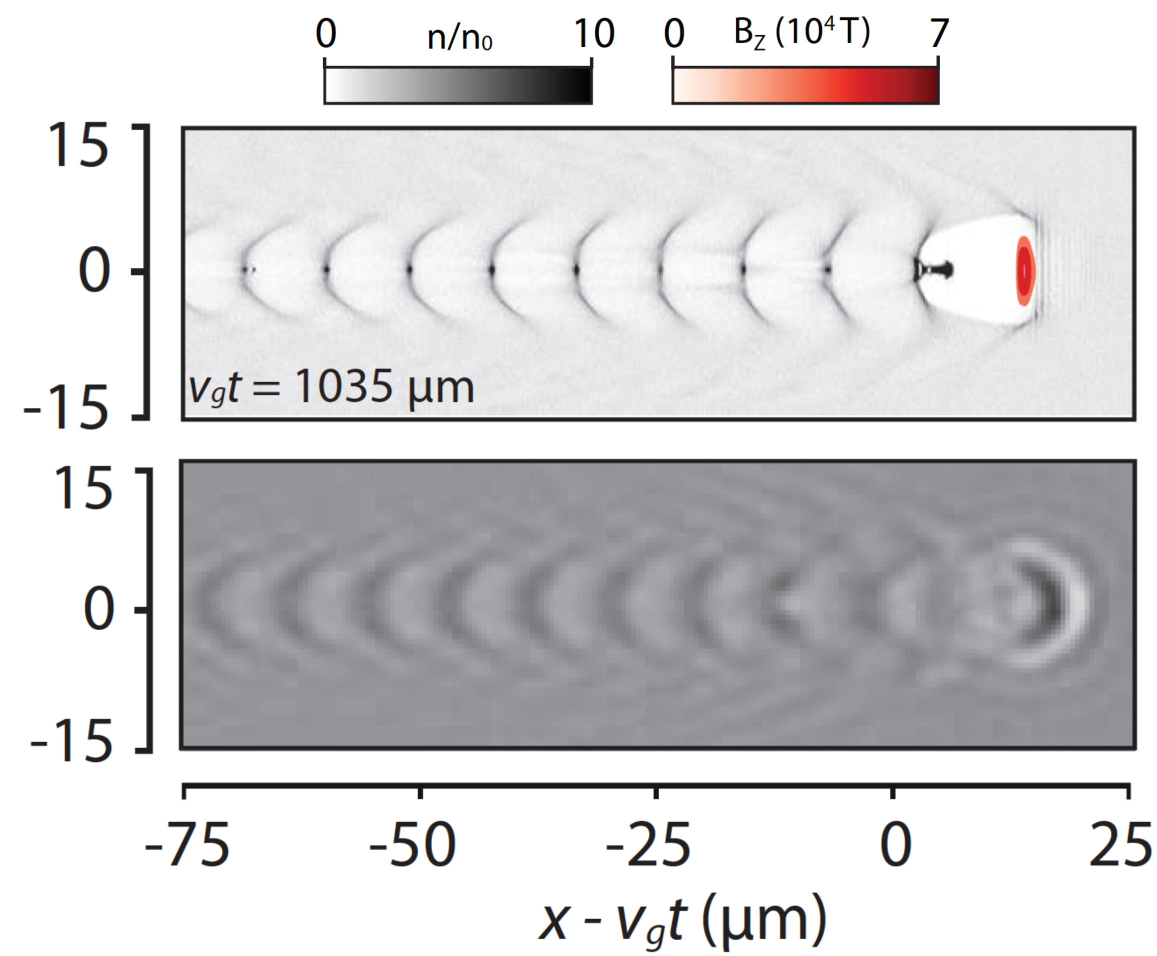
Laser wakefield acceleration of electrons



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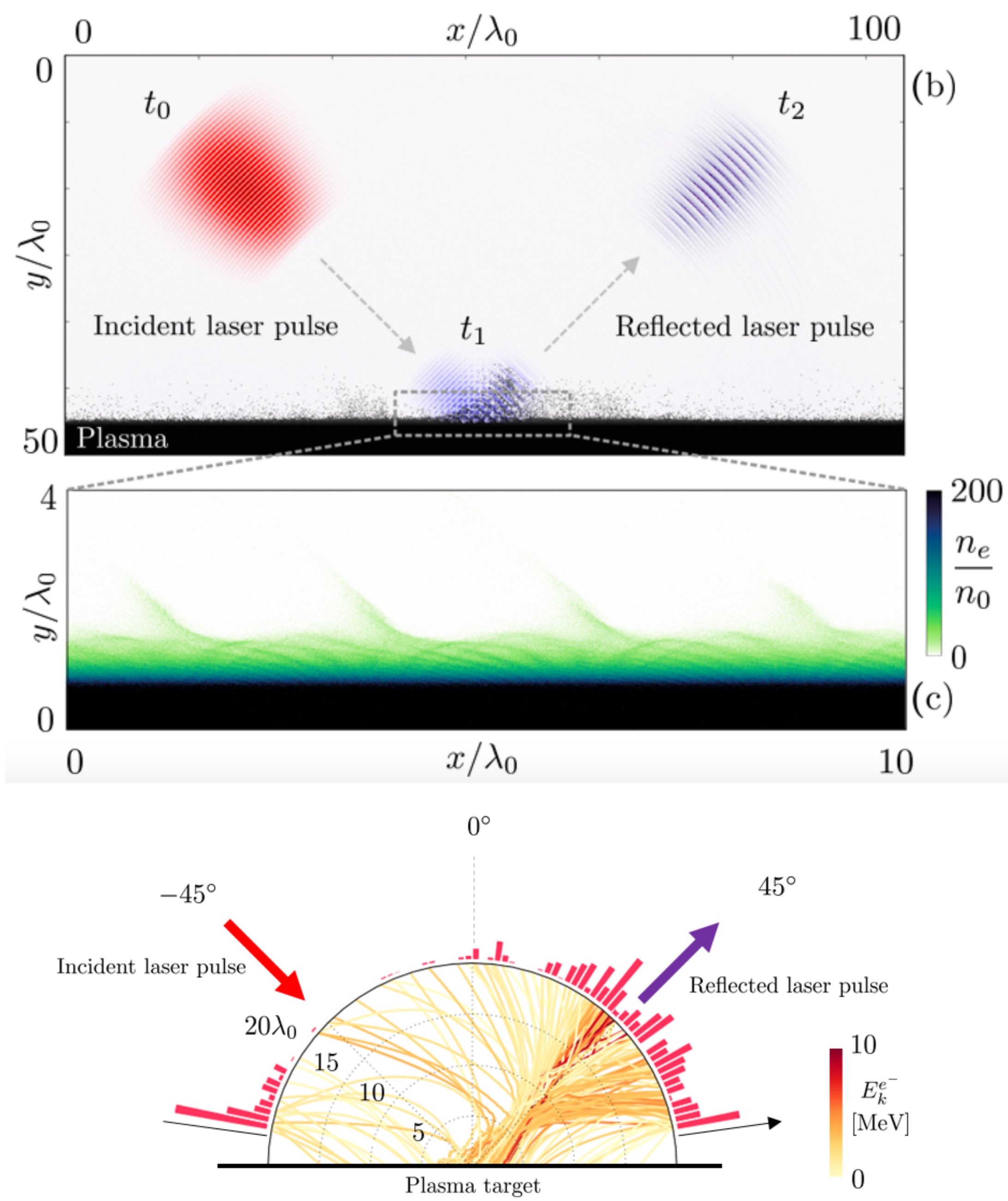

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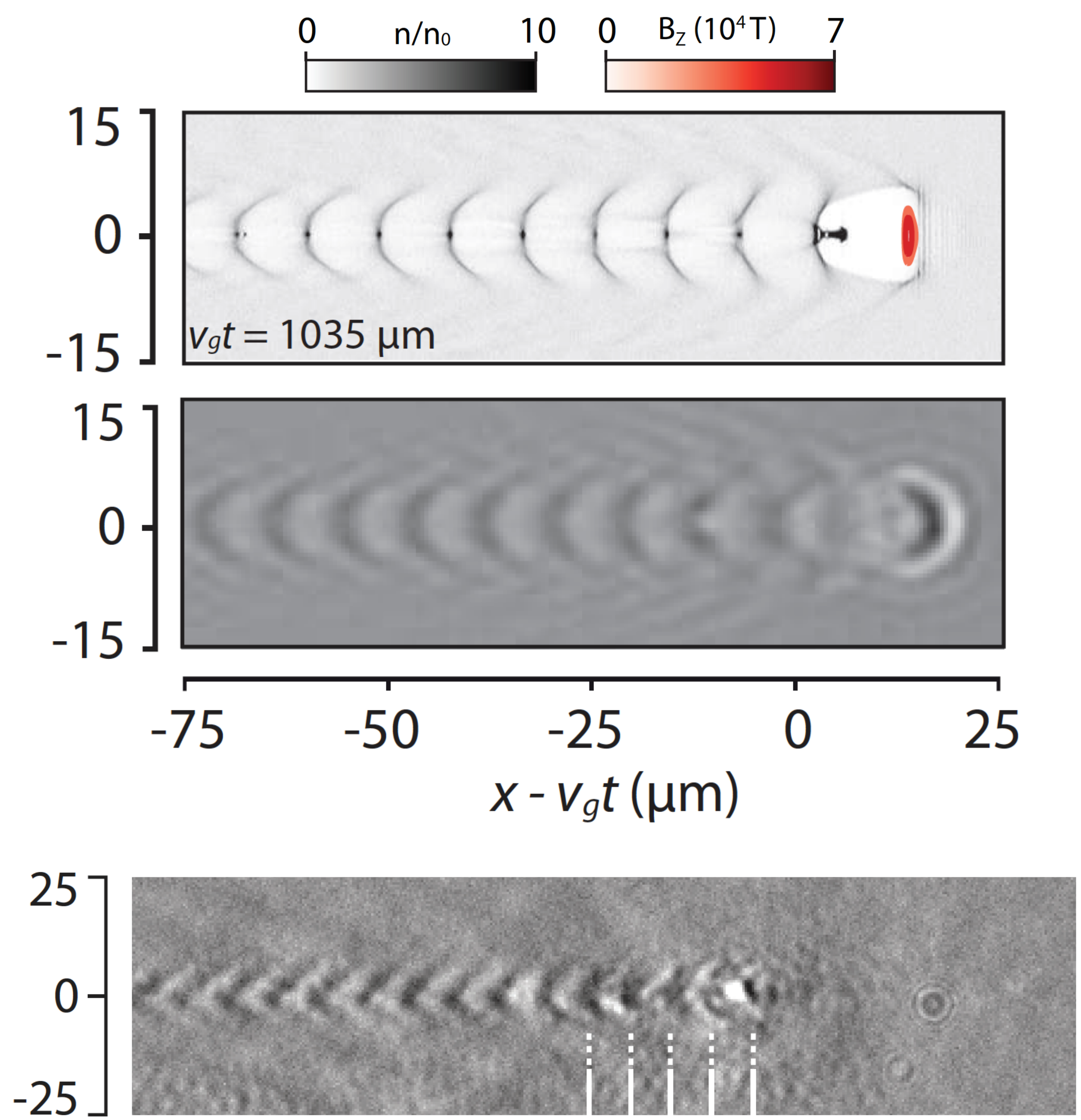
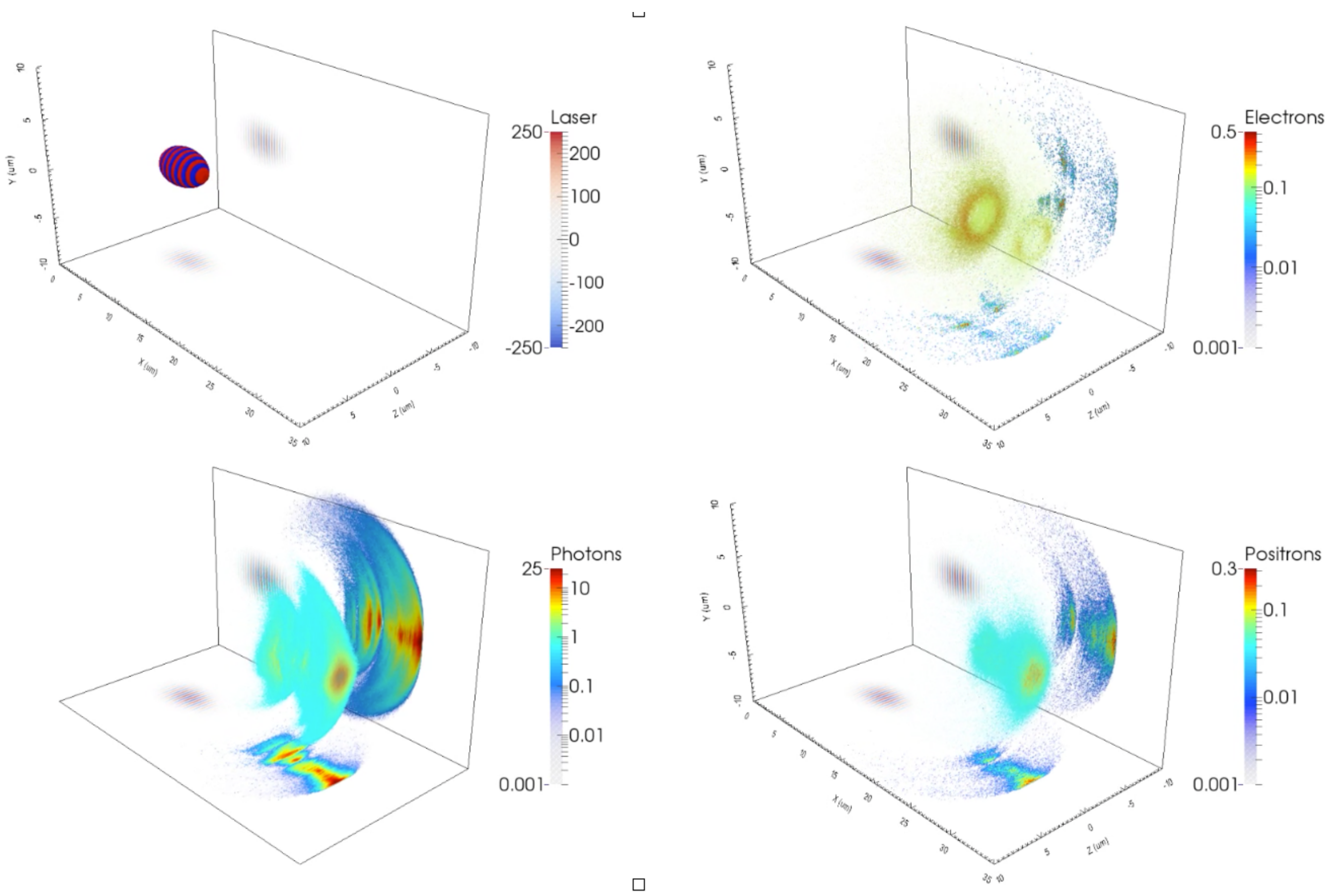
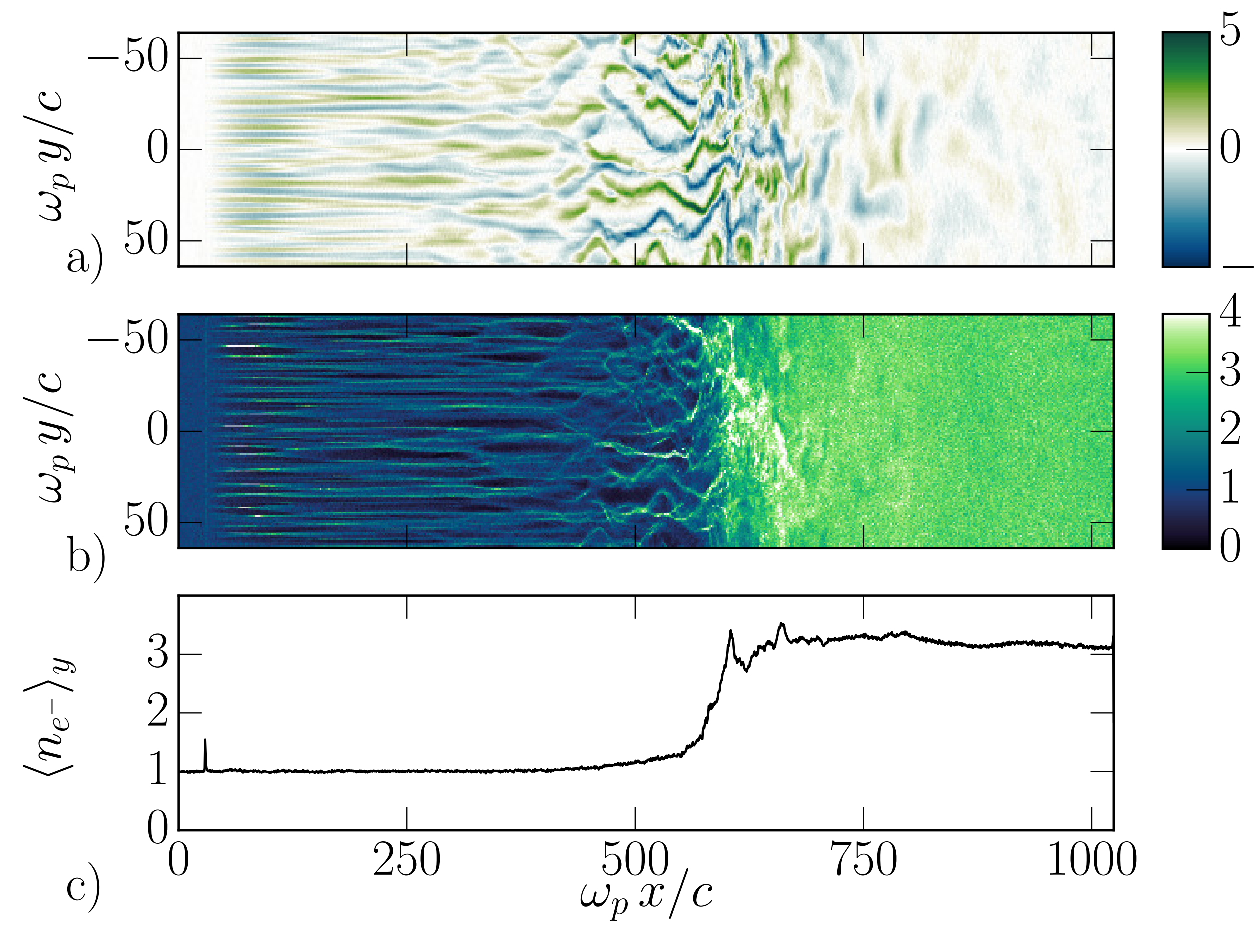


A. Sävert *et al.*, Phys. Rev. Lett. **115**, 055002 (2015)

PIC codes are very versatile: they can be applied to a wide range of physical scenarii, from laser-plasma interaction to astrophysics





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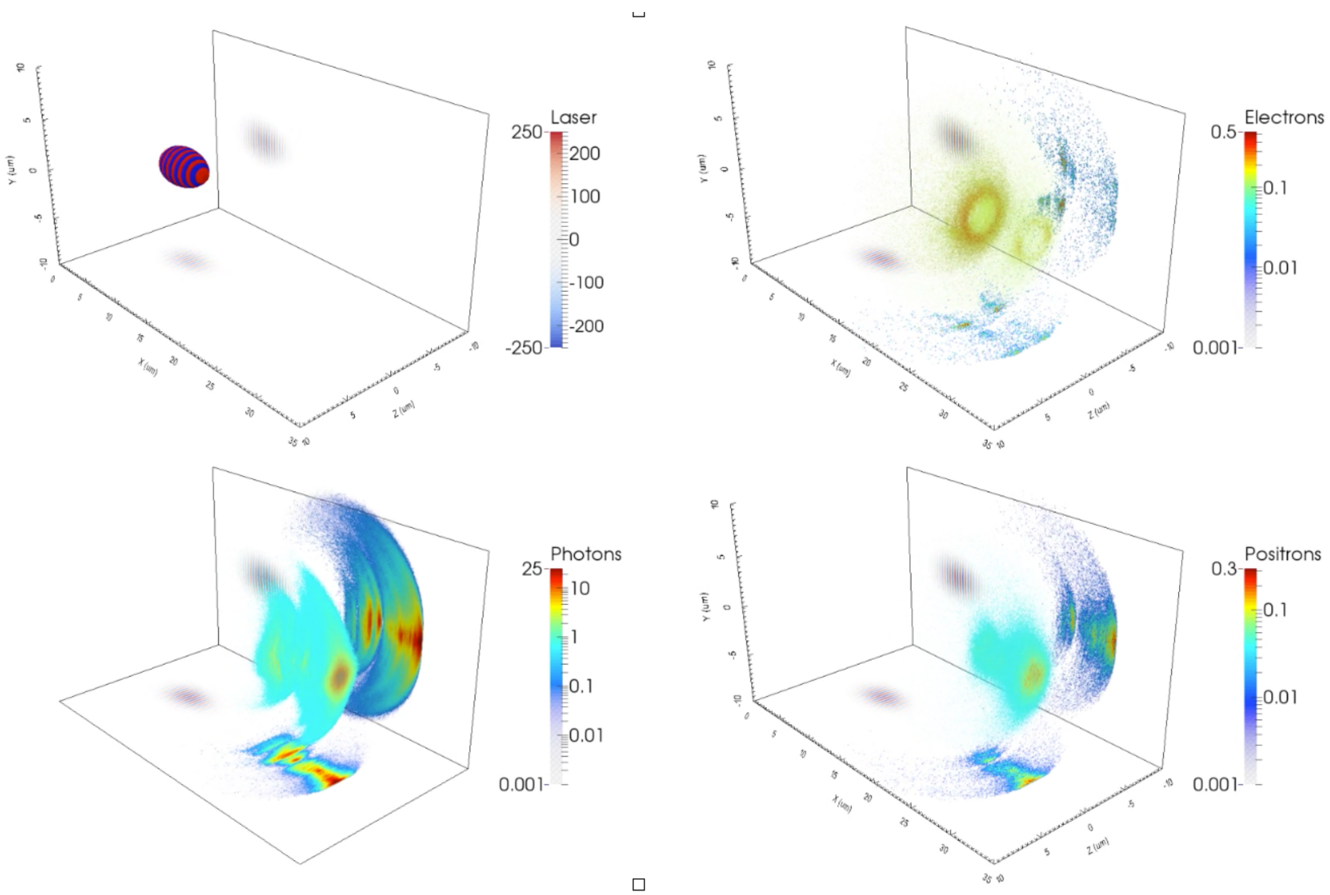
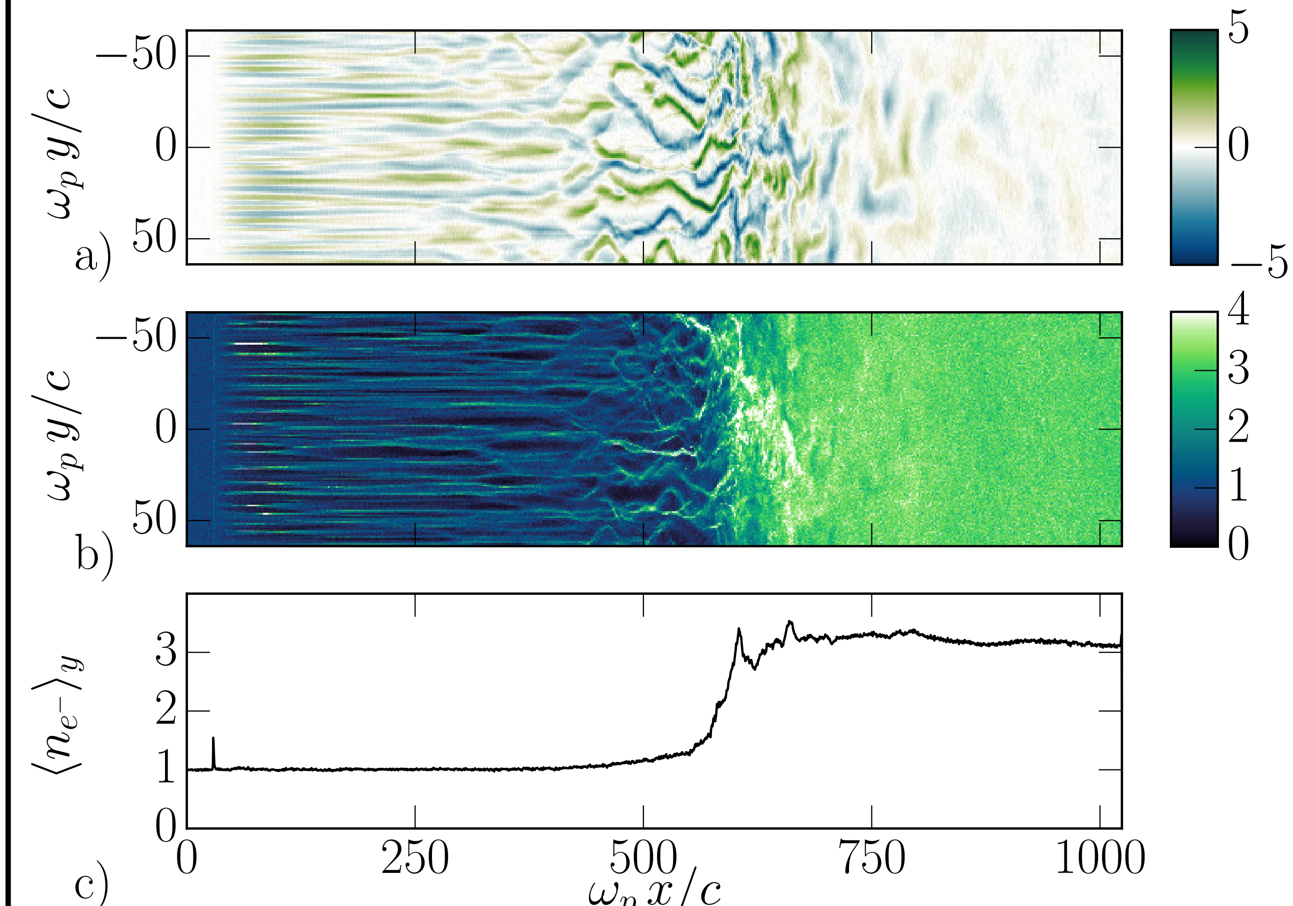
Pair production on multi-petawatt laser facilities



M. Lobet *et al.*, arXiv:1510.02301v2 (2015)

Relativistic shocks in electron-positron plasmas



Plotnikov, Grassi & Grech, MNRAS

Conclusions

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- Implementation on new & future architectures requires a strong input (co-development) from HPC specialists

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