Extreme Plasma Physics

Thomas Grismayer
GoLP / Instituto de Plasmas e Fusão Nuclear
Instituto Superior Técnico, Lisbon, Portugal

epp.tecnico.ulisboa.pt || golp.tecnico.ulisboa.pt
What do we mean by extreme?
Coherent and incoherent phenomena / scales

Radiation Reaction
Classical, relativistic and QED

Can we create abundant pair-plasma in the lab?
Pair production and QED cascades

Strong field QED in astro environments
Relativistic reconnection, Neutron stars
# Typical scales of CED and QED

<table>
<thead>
<tr>
<th>Scale</th>
<th>CED</th>
<th>QED</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Energy scale</strong></td>
<td>Electron rest energy</td>
<td>$\mathcal{E} = mc^2 = 0.5$ MeV</td>
</tr>
<tr>
<td><strong>Momentum scale</strong></td>
<td>$p = \mathcal{E}/c = 0.5$ MeV/c</td>
<td></td>
</tr>
<tr>
<td><strong>Length scale</strong></td>
<td>Classical electron radius</td>
<td>Compton length</td>
</tr>
<tr>
<td></td>
<td>$r_0 = e^2/mc^2 = 2.8 \times 10^{-13}$ cm</td>
<td>$\lambda_C = \hbar/p = 3.9 \times 10^{-11}$ cm</td>
</tr>
<tr>
<td></td>
<td>(Thomson cross section)</td>
<td>(Heisenberg uncertainty principle)</td>
</tr>
<tr>
<td><strong>Time scale</strong></td>
<td>$r_0/c = 10^{-23}$ s</td>
<td>$\lambda_C/c = 1.3 \times 10^{-21}$ s</td>
</tr>
<tr>
<td><strong>Field scale</strong></td>
<td>Critical field CED</td>
<td>Critical field QED</td>
</tr>
<tr>
<td></td>
<td>$E_0 = \mathcal{E}/er_0 = 1.8 \times 10^{18}$ V/cm</td>
<td>$E_S = \mathcal{E}/e\lambda_C = 1.3 \times 10^{16}$ V/cm</td>
</tr>
<tr>
<td><strong>Intensity scale</strong></td>
<td>$I_0 = cE_0^2/4\pi = 8.6 \times 10^{33}$ W/cm$^2$</td>
<td>$I_S = cE_S^2/4\pi = 4.6 \times 10^{29}$ W/cm$^2$</td>
</tr>
</tbody>
</table>
Incoherent and coherent QED processes

Incoherent QED processes

Pair production  
\[ \gamma\gamma \rightarrow e^+e^- \]
\[ \gamma e \rightarrow ee^+e^- \]
\[ ee \rightarrow eee^+e^- \]

Photon production  
\[ ee \rightarrow e\gamma \]
\[ e\gamma \rightarrow e\gamma\gamma \]

Annihilation  
\[ e^+e^- \rightarrow \gamma\gamma \]

Comptonization  
\[ e\gamma \rightarrow e\gamma \]

Cross section  
\[ \sigma \sim \alpha^n \sigma_T \times f(\mathcal{E}) \]

Coherent QED processes

Non-linear Breit Wheeler, trident, non linear Compton

Schwinger mechanism, vacuum polarisation

Photon splitting  

Probability rate  
\[ \frac{dP}{dt} \sim \left( \frac{\alpha c}{\lambda_C} \right) f\left( \frac{E}{E_S}, \mathcal{E} \right) \]
Orders of magnitude...

**QED Photons interaction**

**Near-future facilities**
- Pulse duration: 30-150 fs
- Focal width: $\sim \mu m$
- Intensity: $\sim 10^{21} - 10^{25} W/cm^2$
- Extreme acceleration regime

**Normalised vector potential $a_0$**
- **Non relativistic**
  - $a_0 \ll 1$, $I \ll 10^{18} W/cm^2$
- **Weakly nonlinear, relativistic**
  - $a_0 \sim 1$, $I \sim 10^{18} W/cm^2$
- **Relativistic, nonlinear**
  - $a_0 \sim 10$, $I \sim 10^{20} W/cm^2$
- **QED**
  - $a_0 \sim 1000$, $I \sim 10^{24} W/cm^2$

New facilities open possibilities to explore exotic physics.

\[
a_0 = \frac{eA}{mc^2} = 0.8 \times 10^{-9} \sqrt{I} \left[ \frac{W}{cm^2} \right] \lambda[\mu m]
\]
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**Radiation Reaction in CED**

**Larmor’s formula:** instantaneous emitted power for non relativistic particle

\[ P = \frac{2}{3} \frac{e^2}{c^3} a^2 \]

**Thomson scattering:** dipole approximation

\[
E = E_0 \sin \omega t \quad \langle P \rangle = \frac{1}{3} \frac{e^4}{m c^3} E_0^2 \quad \langle S \rangle = \frac{c}{8\pi} \frac{E_0^2}{r_e^2}
\]

- *incident wave*
- *averaged power*
- *averaged Poynting flux*

\[ \sigma_T = \frac{\langle P \rangle}{\langle S \rangle} = \frac{8\pi}{3} r_e^2 \]

**Radiation reaction:** the force acting on a particle by virtue of the radiation it produces?

\[
\int F_{rad} \cdot v dt = - \frac{2}{3} \frac{e^2}{c^3} \int a^2 dt = - \frac{2}{3} \frac{e^2}{c^3} \left( [a \cdot u] - \int \dot{a} \cdot v dt \right)
\]

\[ F_{rad} = \frac{2}{3} \frac{e^2}{c^3} \dot{a} \]
Radiation Reaction in CED

Dipole radiation pattern
Radiation reaction models

Different approaches of calculating the damping force

\[ \frac{dp}{dt} = F_L + F_{rad} \]

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[ \frac{dp}{dt} = F_L - \frac{2 e^4 \gamma}{3 m^3 c^5} p \left( E_\perp + \frac{p}{\gamma mc} \times B \right)^2 ] [Bell 2008]</td>
</tr>
<tr>
<td>2</td>
<td>[ \frac{dp}{dt} = F_L + \frac{2 e^3}{3 mc^3} \left{ \gamma \left( \frac{\partial}{\partial t} + \frac{p}{\gamma m} \cdot \nabla \right) E + \frac{p}{\gamma mc} \times \left( \frac{\partial}{\partial t} + \frac{p}{\gamma m} \cdot \nabla \right) B \right} + \frac{e}{mc} \left( E \times B + \frac{1}{\gamma mc} B \times (B \times p) + \frac{1}{\gamma mc} E(p \cdot E) \right) - \frac{e \gamma}{m^2 c^2} p \left( \left( E + \frac{p}{\gamma mc} \times B \right)^2 - \frac{1}{\gamma^2 m^2 c^2} (E \cdot p)^2 \right) ] [Landau &amp; Lifshitz 1951]</td>
</tr>
<tr>
<td>3</td>
<td>[ \frac{dp}{dt} = F_L + \frac{2 e^3}{3 m^2 c^4} \left{ \frac{1}{\gamma^2 m^2 c^2} p \cdot (p \cdot F_L) \right} \times B - \frac{2 e^2 p}{3 m^3 c^5} \left( \frac{F_L - \frac{1}{\gamma^2 m^2 c^2} p \cdot (p \cdot F_L)}{1 + \frac{2 e^2}{3 \gamma^2 m^3 c^5} (p \cdot F_L)} \cdot F_L \right) ] [Sokolov 2009]</td>
</tr>
<tr>
<td>4</td>
<td>[ \frac{dp}{dt} = F_L - \frac{2 e^4 \gamma^5}{3 mc^2} \left( \left( E + \frac{p}{\gamma mc} \times B \right)^2 - \frac{1}{\gamma^2 m^2 c^2}</td>
</tr>
<tr>
<td>5</td>
<td>[ \frac{dp}{dt} = F_L + \frac{2 e^2}{3 mc^3} \left{ \gamma \frac{dF_L}{dt} - \frac{\gamma}{m^2 c^2} \frac{dp}{dt} \times (p \times F_L) + \frac{1}{\gamma m^4 c^4} \left( p \cdot \frac{dp}{dt} \right) (p \times (p \times F_L)) \right} ] [Ford 1993]</td>
</tr>
<tr>
<td>6</td>
<td>[ \frac{dp}{dt} = F_L + \frac{2 e^3}{3 mc^3} \left{ \frac{e}{mc} \left( E \times B + \frac{1}{\gamma mc} B \times (B \times p) + \frac{1}{\gamma mc} E(p \cdot E) \right) - \frac{e \gamma}{m^2 c^2} p \left( \left( E + \frac{p}{\gamma mc} \times B \right)^2 - \frac{1}{\gamma^2 m^2 c^2} (E \cdot p)^2 \right) \right} ]</td>
</tr>
</tbody>
</table>
Reduced L&L is best for PIC

L&L captures physically relevant solutions of LAD equation

Without radiation reaction

\[
\frac{dp}{dt} = e \left( \mathbf{E} + \frac{p}{\gamma mc} \times \mathbf{B} \right)
\]

With radiation reaction

\[
\frac{dp}{dt} = e \left( \mathbf{E} + \frac{p}{\gamma mc} \times \mathbf{B} \right) + \frac{2e^3}{3mc^3} \left\{ \gamma \left( \frac{\partial}{\partial t} + \frac{p}{\gamma m} \cdot \nabla \right) \mathbf{E} + \frac{p}{\gamma mc} \times \left( \frac{\partial}{\partial t} + \frac{p}{\gamma m} \cdot \nabla \right) \mathbf{B} \right\} + \frac{e}{mc} \left( \mathbf{E} \times \mathbf{B} + \frac{1}{\gamma mc} \mathbf{B} \times (\mathbf{B} \times p) + \frac{1}{\gamma mc} \mathbf{E} (p \cdot \mathbf{E}) \right) - \frac{e\gamma}{m^2c^2} p \left( \left( \mathbf{E} + \frac{p}{\gamma mc} \times \mathbf{B} \right)^2 - \frac{1}{\gamma^2 m^2 c^2} (\mathbf{E} \cdot p)^2 \right) \]

L&L reduced

\[
\frac{A}{B} \sim \frac{1}{2\pi\gamma a_0}
\]
OSIRIS 4.0

osiris framework

- Massively Parallel, Fully Relativistic Particle-in-Cell (PIC) Code
- Visualization and Data Analysis Infrastructure
- Developed by the osiris.consortium ⇒ UCLA + IST

code features

- Scalability to ~ 1.6 M cores
- SIMD hardware optimized
- Parallel I/O
- Dynamic Load Balancing
- Classical radiation reaction
- Particle merging
- GPGPU support
- Xeon Phi support
- QED Module

Ricardo Fonseca
ricardo.fonseca@tecnico.ulisboa.pt
Frank Tsung
tsung@physics.ucla.edu

http://epp.tecnico.ulisboa.pt/
http://plasmasim.physics.ucla.edu/
PIC loop with classical radiation reaction

\[
\frac{dp}{dt} = F_L + F_{rad}
\]

Integration of equations of motion:
- moving particles
  \( F_p \rightarrow u_p \rightarrow x_p \)

Interpolation:
- evaluating force on particles
  \( (E, B)_i \rightarrow F_p \)

Deposition:
- calculating current on grid
  \( (x, u)_p \rightarrow j_i \)

Integration of field equations:
- updating fields
  \( (E, B)_i \leftarrow J_i \)
Energy loss in simple setup

\[ -\frac{d\gamma}{dt} = \frac{2e^2\omega_0}{3mc^3} 2a_0^2 (\gamma^2 - 1) \]
\[ -\frac{d\gamma}{dt} = \frac{2e^2\omega_0}{3mc^3} 4a_0^2 (\gamma^2 - 1) \]
\[ -\frac{d\gamma}{dt} = \frac{2e^2\omega_0}{3mc^3} B^2 (\gamma^2 - 1) \]

Linear polarization \( a_0 = 25 \)
Circular polarization \( a_0 = 25 / \sqrt{2} \)
Synchrotron \( B = 2 \times 25 / \sqrt{2} \)
Full-scale 3D classical radiation reaction

~ 40% energy loss for a 1 GeV beam at $10^{21}$ W/cm$^2$

Laser wakefield accelerator in bubble regime

Second laser $I \sim 10^{21}$ W/cm$^2$

Accelerated electrons

X-ray detector

M. Vranic et al., PRL 113, 134801 (2014)
Classical RR shrinks beam energy spectrum*

In quantum interaction we expect energy spread and divergence to grow**

Energy loss versus intensity

M. Vranic et al., PRL 113, 1348001 (2014)
Anomalous radiative trapping

A. Gonoskov et al., PRL 113, 014801 (2014)
The interest in physics at high intensities rises in the very beginning of quantum electrodynamics when Klein showed there is probability of passage of a Dirac electron through an arbitrarily high potential barrier, which Sauter showed to be satisfied for relativistic particles. The electron rest energy, which is called the Schwinger field, performs a work equal to electron rest mass energy in a strong electromagnetic field of a plane wave to a certain approximation. This motivated a lot of research, and many different QED processes can occur in the presence of strong field. However, the strong field contributions of invariants can be neglected. The first condition in (2.6) is trivially satisfied because all the fields we can achieve are orders of magnitude smaller than the strong fields and is mainly concerned with the transition regime between the classical and the linear Maxwell electrodynamics in the presence of strong electromagnetic field.

To identify if the interaction is classical or not, we can use the characteristic value of such a process, which is photon emission by an electron which has a classical limit. For electrons, we express the parameter of electromagnetic field in quantum electrodynamics:

\[ E_s = \frac{m^2 c^3}{e \hbar} \quad \chi = \frac{\sqrt{(p_\mu F^{\mu \nu})^2}}{E_s mc} \]

\[ \chi_R = \frac{\sqrt{(k_\mu F^{\mu \nu})^2}}{E_s mc} \]

**Classical:** \( \chi \ll 1 \)

**QED:** \( \chi \simeq 1 \)

Photon emission has a probabilistic character. Radiation reaction is discrete. Energy spread and divergence are expected to grow.

Implementation of QED effects

**Radiation Reaction**

**Different types of Radiation reaction models**

\[ \frac{dp}{dt} = F_L + \begin{cases} \frac{F_{rad}}{d^2 P} & \text{Continuous damping rate}^* \\ \frac{d^2 P}{dt d\chi_\gamma} & \text{QED probabilistic approach}^{**} \end{cases} \]

**Implementation in PIC codes**

- Continuous damping rate: particle pusher with \( F_{rad} \gamma < 10 \)
- QED probabilistic approach: particle pusher + Monte Carlo module
  - every \( \Delta t \) : probability of photon emission
  - Select a photon in QED synchrotron spectrum
  - Update particle momentum due to quantum recoil

- The QED approach can be generalized to any external EM fields under the conditions:
  - quasi-static fields \( \chi_e^2 \gg \text{Max}(f, g) \quad (f, g) \ll 1 \)
  - weak fields \( \chi_e^2 \gg \text{Max}(f, g) \quad (f, g) \ll 1 \)

\[ f = F_{\mu \nu}^2 / E_{\mu \nu}^2 \quad g = F_{\mu \nu}^* F_{\mu \nu} / E_{\mu \nu}^2 \quad E_{\text{crit}} = m^2 c^3 / e \hbar \quad \chi_{e, \gamma} = \frac{|F_{\mu \nu} p_{e, \gamma}^\nu|}{E_{\text{crit}} \gamma m c} \]

* Landau & Lifshitz (Theory of Fields)
Quantum effects are strongest for the case of counter-propagation

But, the interaction at 90 degrees has only a factor of two lower electron chi

\[ a_0 = 0.8 \sqrt{I[10^{18} \text{ W/cm}^2] \lambda[\mu\text{m}]} \]

**Counter-propagation**

\[ \chi \approx 2 \gamma_0 a_0 \times 2 \times 10^{-6} \]

**Co-propagation**

\[ \chi \approx \frac{a_0}{2 \gamma_0} \times 2 \times 10^{-6} \]

**Interaction at 90 deg.**

\[ \chi \approx \gamma_0 a_0 \times 2 \times 10^{-6} \]
Energy spread increases due to stochasticity

M.Vranic et al, NJP 18, 073035 (2016)
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Integration of equations of motion:
-moving particles
\[ \frac{dp}{dt} = F_L + \frac{dP_\gamma}{dt \, d\chi} \text{ Probabilistic} \]

Integration of field equations:
-updating fields
\[ \frac{\partial E}{\partial t} = c \nabla \times B - 4\pi j \]
\[ \frac{\partial B}{\partial t} = -c \nabla \times E \]

Deposition:
-calculating current on grid
\[ (x, u)_p \rightarrow j_i \]

Emission of photons
-Probability of pair creation
➡ new particles

Interpolation:
-evaluating force on particles
\[ (E, B)_i \rightarrow F_p \]

Particle Merging

T. Grismayer et al., POP (2016), F. Niel et al., PRE (2018)
Standing wave configurations for QED cascades

Laser 1

Laser 2

Laser parameters
- $a_0 = 1000$, $\lambda = 1$ um,
- $\tau = 30$ fs, $W_0 = 3.2$ um

Linear | Double clockwise | Clockwise-anti clockwise

$E, B$
3D OSIRIS QED - colliding laser cascades at $\chi \gg 1$

**Laser parameters**

- $a_0 = 1000$, $\lambda = 1$ um,
- $\tau = 30$ fs, $W_0 = 3.2$ um

**Particles remain in the x-y plane**

**Particles explore the whole space**

**Particles rotate in the y-z plane**
Cascade’s growth rate in 2 lasers setup

ELI range intensity

T. Grismayer et al. PRE 95, 023210 (2017)
M.A Fedotov et al. PRL 105, 080402 (2010)
V. F. Bashmakov et al. PoP 21, 013105 (2014)
Cascade’s growth rate in 2 lasers setup

ELI range intensity

Zone for cascades linear polarization

$P_{\text{laser}} > 100 \text{ PW for } W_0 \sim 3\lambda$  
(focal spot for these runs)

T. Grismayer et al. PRE 95, 023210 (2017)
M.A Fedotov et al. PRL 105, 080402 (2010)
V.F. Bashmakov et al. PoP 21, 013105 (2014)
Cascade’s growth rate in 2 lasers setup

ELI range intensity

Laser absorption
linear polarization

$P_{\text{laser}} > 200 \text{ PW, } W_0 \sim 3\lambda$
(focal spot for these runs)

T. Grismayer et al. PRE 95, 023210 (2017)
M.A Fedotov et al. PRL 105, 080402 (2010)
V. F. Bashmakov et al. PoP 21, 013105 (2014)
Laser absorption in QED cascades

The pulses start to overlap

Early development of the cascade

Zone of relativistically critical density

Creation of a large-scale overdense relativistic hot plasma

The pulses are reflected on the critical interface

G.N. Nerush et al., PRL 106, 035001 (2011)

A simple model for laser absorption

When does the laser absorption become important?

a) $t_a > \tau_0 / 2$

- The absorption time is bigger than half of the pulse duration → fraction of the pulse can escape

b) $t_a < \tau_0 / 2$

- The absorption time is smaller than half of the pulse duration → the reflected wave can reform a standing wave

$$
\eta = \frac{\mathcal{E}_a}{\mathcal{E}_p} = 1 - \frac{t_a}{\tau}
$$

E.N. Nerush et al., PRL 106, 035001 (2011)
When does the laser absorption become important?

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- the absorption time is smaller than half of the pulse duration → the reflected wave can reform a standing wave

\[ \eta = \frac{\mathcal{E}_a}{\mathcal{E}_p} = 1 - \frac{t_a}{\tau} \]

Model 1D

\[ t_a = \frac{\ln(a_0 n_c / n_0)}{\Gamma} \]

Model 2D-3D

\[ t_a = \frac{\ln(a_0 n_c \sigma_0^2 \tau_0 c / n_0 \lambda_0^3)}{\Gamma} \]


*E.N Nerush et al., PRL 106, 035001 (2011)
4-laser cascades - effect of polarisation

Growth rates, absorption and output radiation

M. Vranic et al., PPCF 59, 014040 (2017)
“In plane” - vortex electric field structure

Time = 84.50 [1/\omega_p]
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Magnetic reconnection: energy conversion

Classical magnetic Islands reach pressure balance

$$\frac{y}{\rho_L}$$

$$\frac{B}{B_0}$$

$$\frac{ct}{L_y} = 1.2$$

Classical relativistic reconnection

**Magnetic tension**

$$\frac{B \cdot \nabla B}{4\pi}$$

**Plasma pressure**

$$\nabla \left( P + \frac{B^2}{8\pi} \right)$$

Pinch equilibrium (Bennett)
Radiation induced compression of magnetic field

Classical relativistic reconnection

Radiative reconnection

Classical

QED
Radiation induced compression of density

Classical relativistic reconnection

Radiative QED reconnection

electron density $n_e/n_b$  ct/$L_y = 1.2$

$y/\rho_L$

$y/\rho_L$

Classical

QED


**Gamma rays/ Pairs produced inside islands**

- **Early t = 1.2 \( L_y/c \)**
  - 5 6 7 8 9 10 20 30
  - \( \varepsilon/n_b T_b \)
  - 350 300 250 200 150
  - \( y/\rho_L \)
  - \( x/\rho_L \)

- **Late t = 2.2 \( L_y/c \)**
  - 5 6 7 8 9 10 20 30
  - \( \varepsilon/n_b T_b \)
  - 350 300 250 200 150
  - \( y/\rho_L \)
  - \( x/\rho_L \)

**Pair creation rate density**

- **Pairs**
  - 0 1 2 3 4 5 6 7 8 9 10
  - \( N_p c \)
  - \( L_y \rho_L \)
  - \( x/\rho_L \)
  - \( y/\rho_L \)