New Insights into Plasma Turbulence

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Note about these slides

These slides contain additional material to my lecture on plasma (fluid) turbulence at the Les Houches school on "*The multiple approaches to plasma physics: from laboratory to astrophysics*" (May 2019).

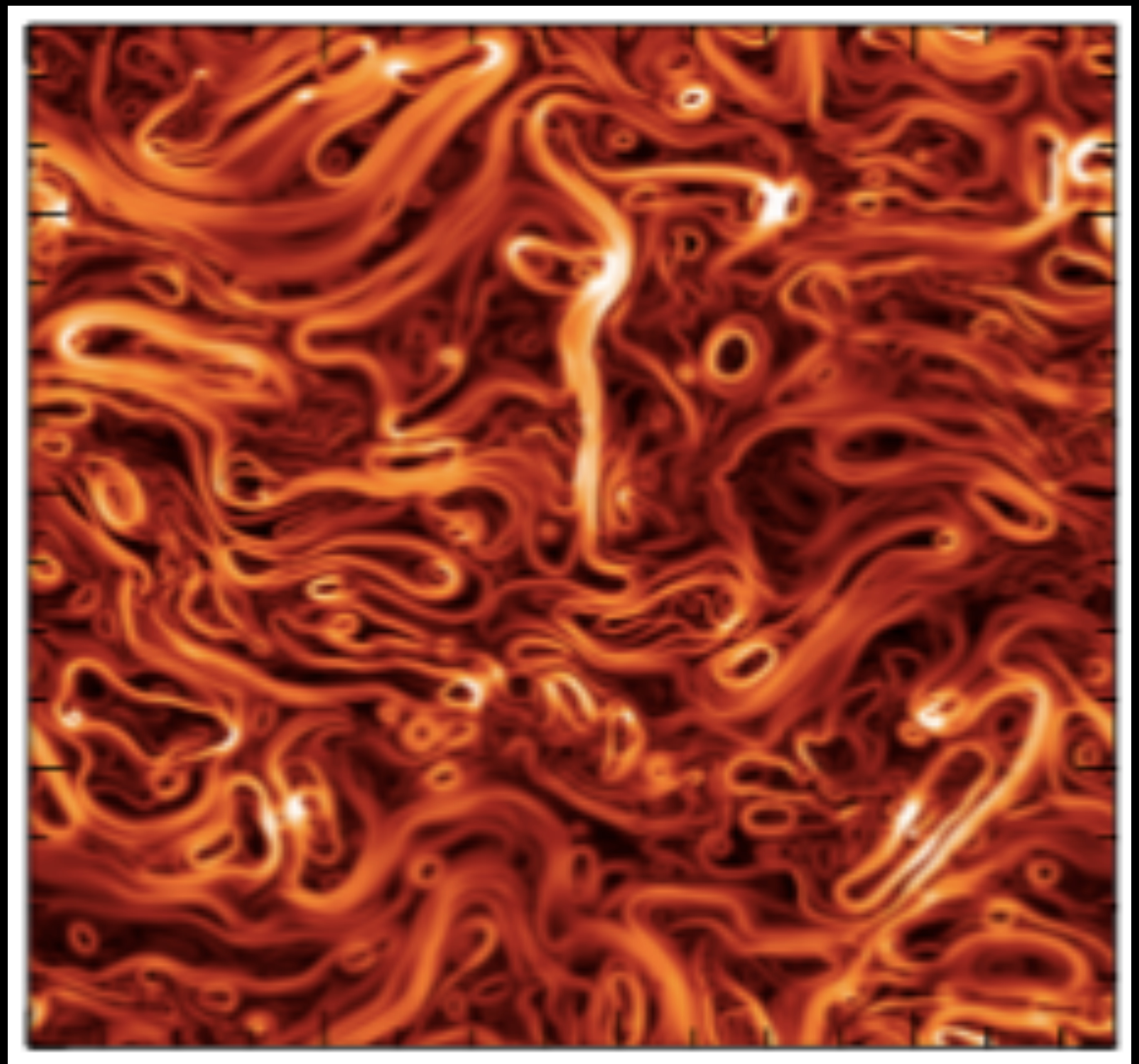
I did most of the lecture on the blackboard and ended up not showing most of these slides. They contain some things that we covered in my lecture, and also more advanced material that we did not. I hope they are useful to you in illustrating some of the questions that are currently being researched. There are also some references for further reading provided at the end.

Introduction

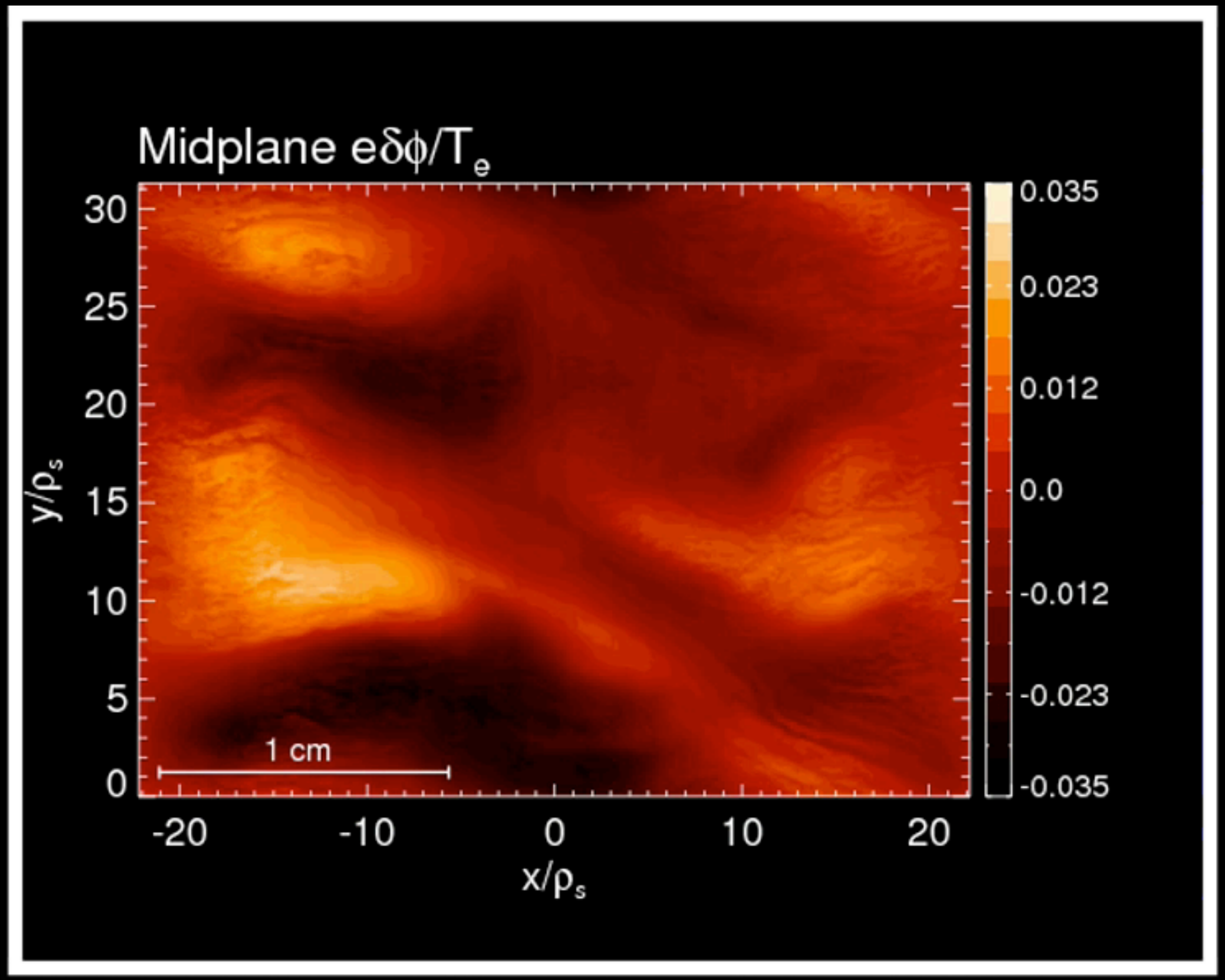
Turbulence is ubiquitous

Magnetized plasmas are found everywhere in the universe.

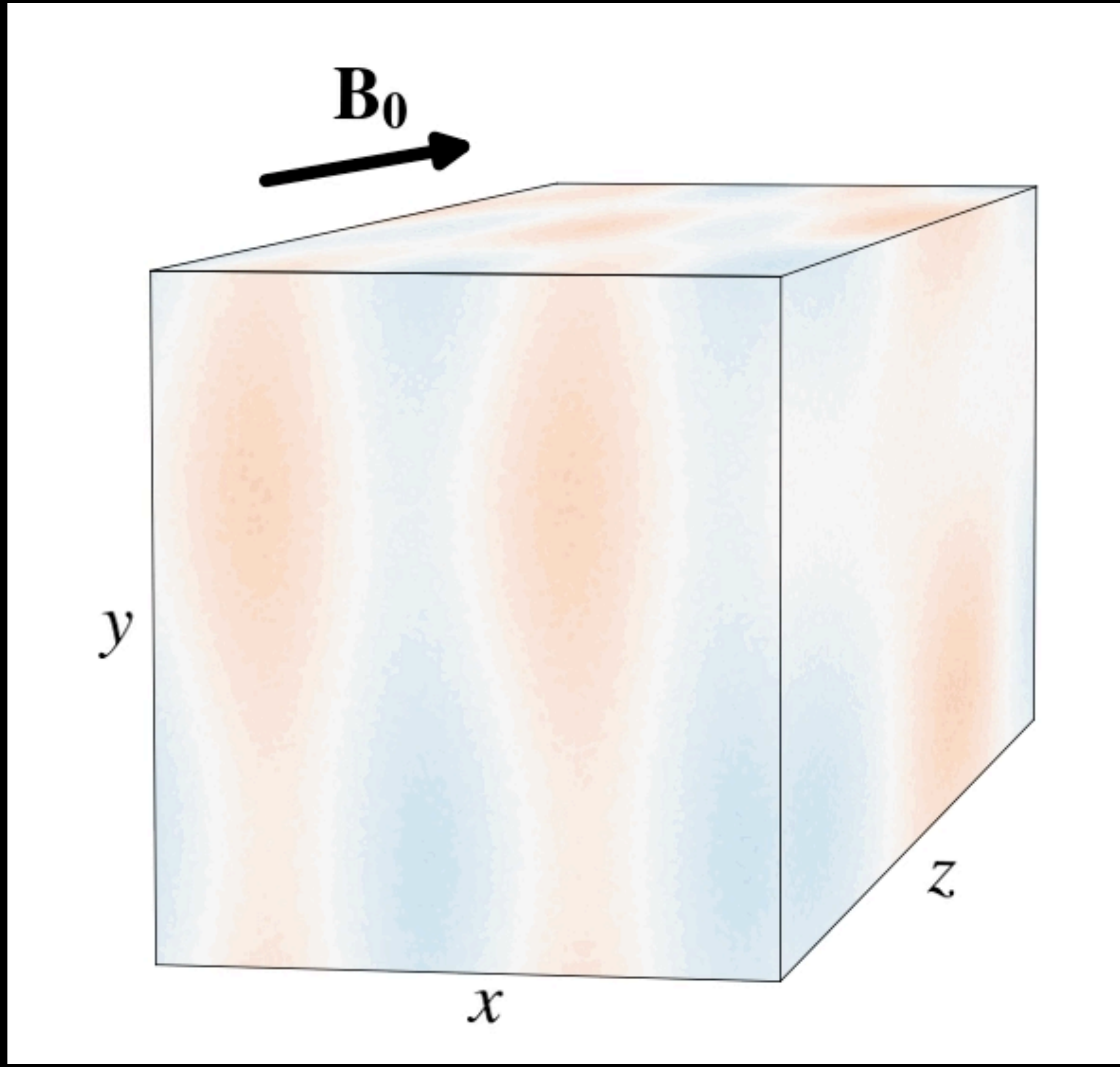
They tend to exist in a turbulent state.



G. Inchingolo *et al.*, ApJ 2018



N. Howard *et al.*, NF 2015



D. Groselj *et al.*, PRL 2018

Introduction

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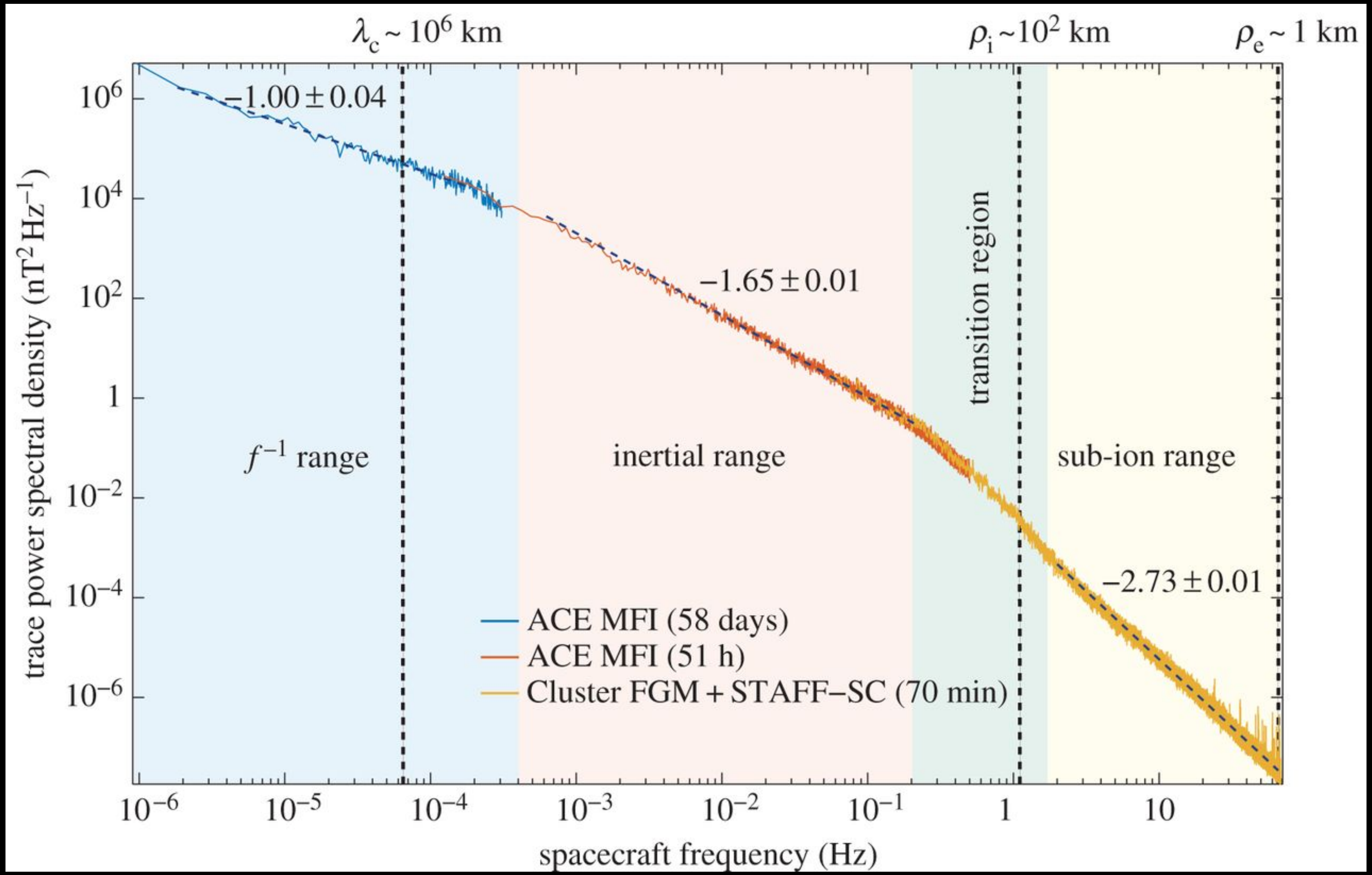
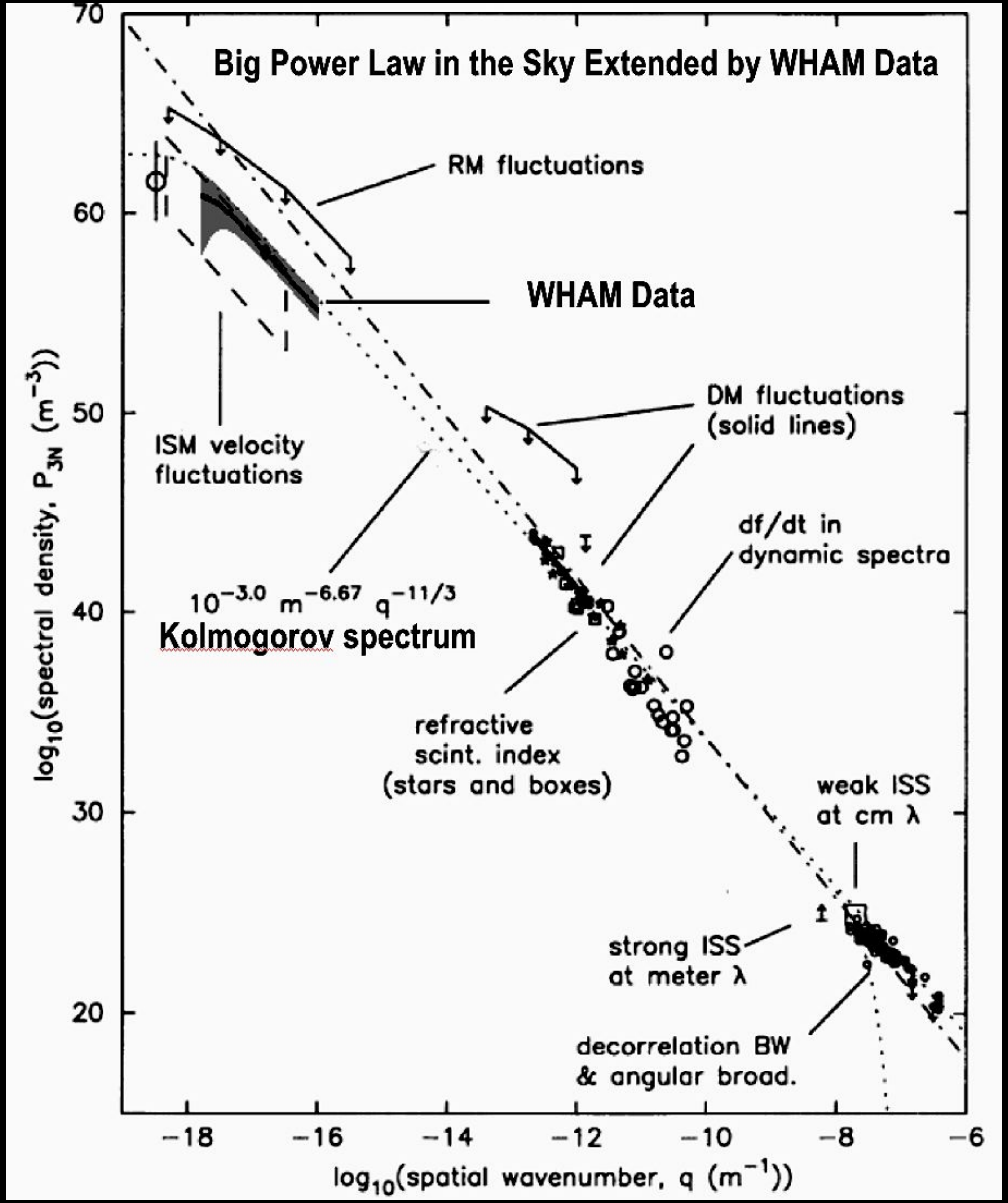




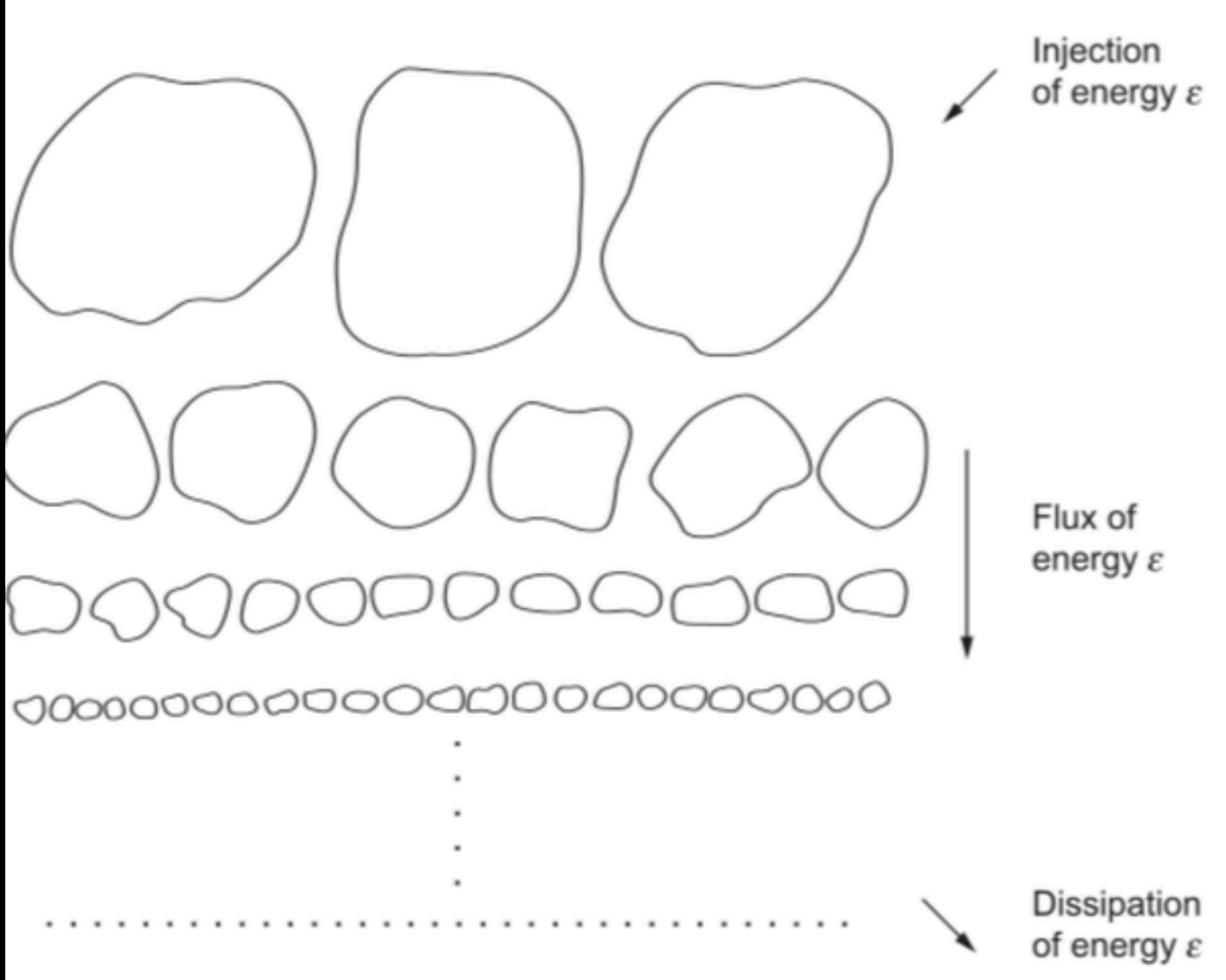
A (very rough) overview of how we think of turbulence

Energy cascades from large to small scales locally in k-space

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \nu\frac{\partial^2 V}{\partial x^2}$$

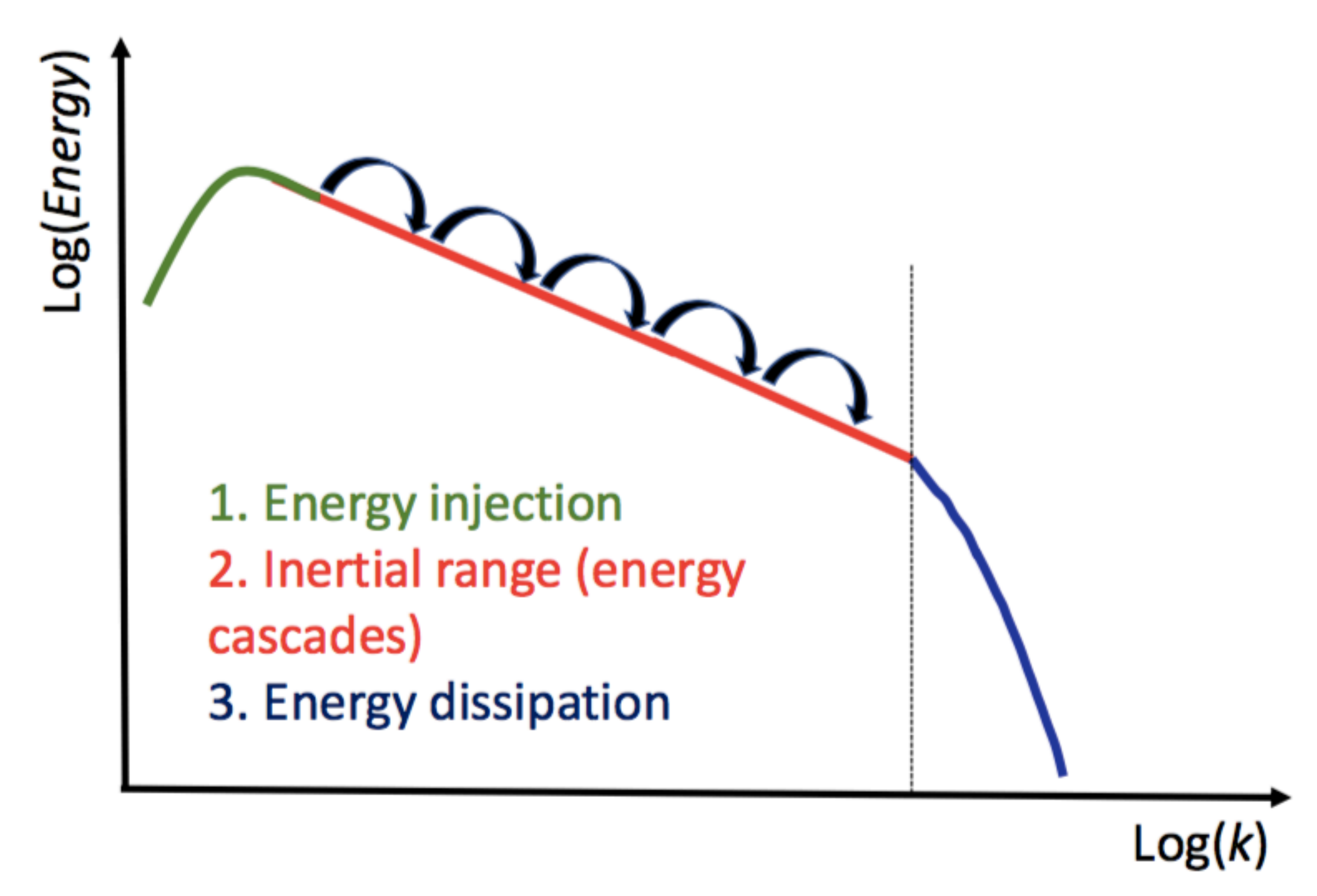
$$V(x, t=0) = V_0 \sin(k_0 x)$$

$$V(x,\Delta t) = V(0) - \Delta t V\frac{\partial V}{\partial x}$$
$$= V_0\sin(k_0 x) - \frac{1}{2}k_0\Delta t V_0^2 \sin(2k_0 x)$$



A (very rough) overview of how we think of turbulence

Energy cascades from large to small scales locally in k-space



Kolmogorov-like view of plasma turbulence:

1. Energy is externally injected into the system

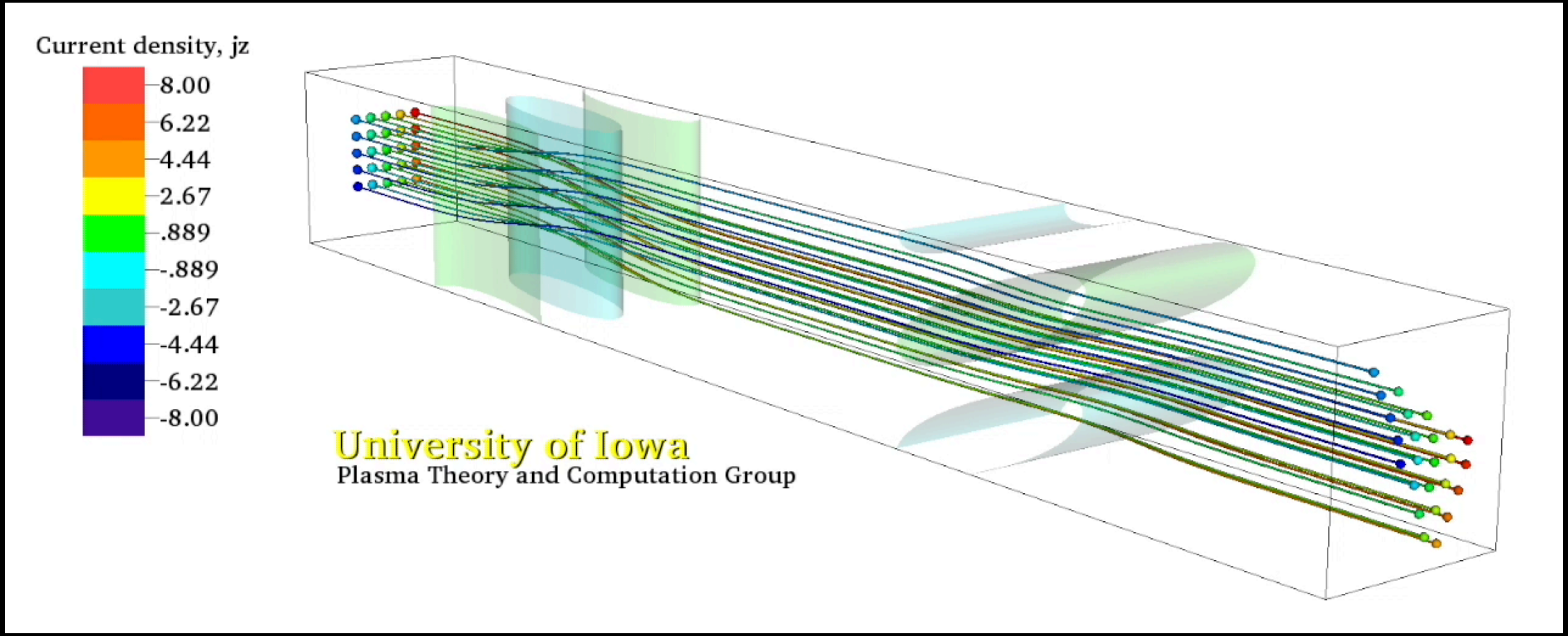
2. Energy cascades, no dissipation

3. Dissipation (resistivity, viscosity) becomes important, damps the energy.

Elsasser formulation of MHD

Only counter-propagating Alfvén wave packets can lead to energy cascade

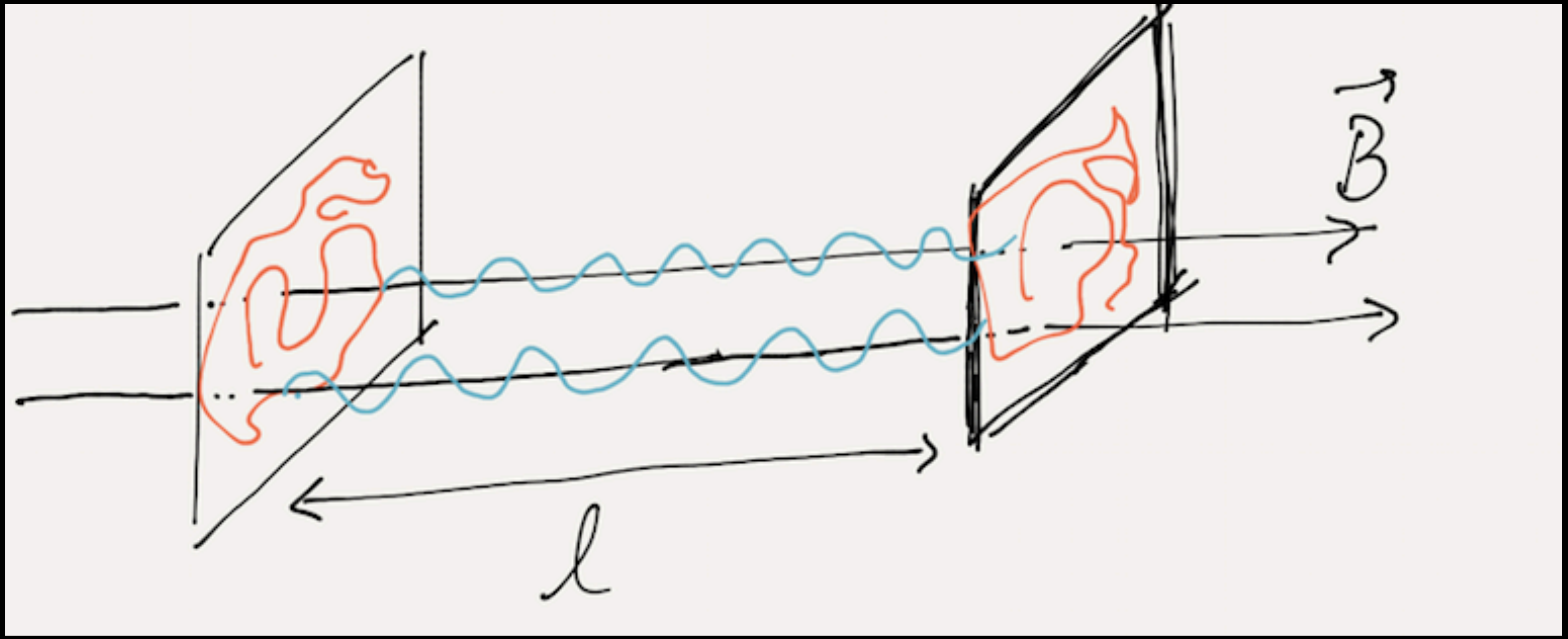
$$\partial_t \mathbf{z}^\pm \mp v_A \nabla_\parallel \mathbf{z}^\pm + \mathbf{z}^\mp \cdot \nabla \mathbf{z}^\pm = -\nabla p$$

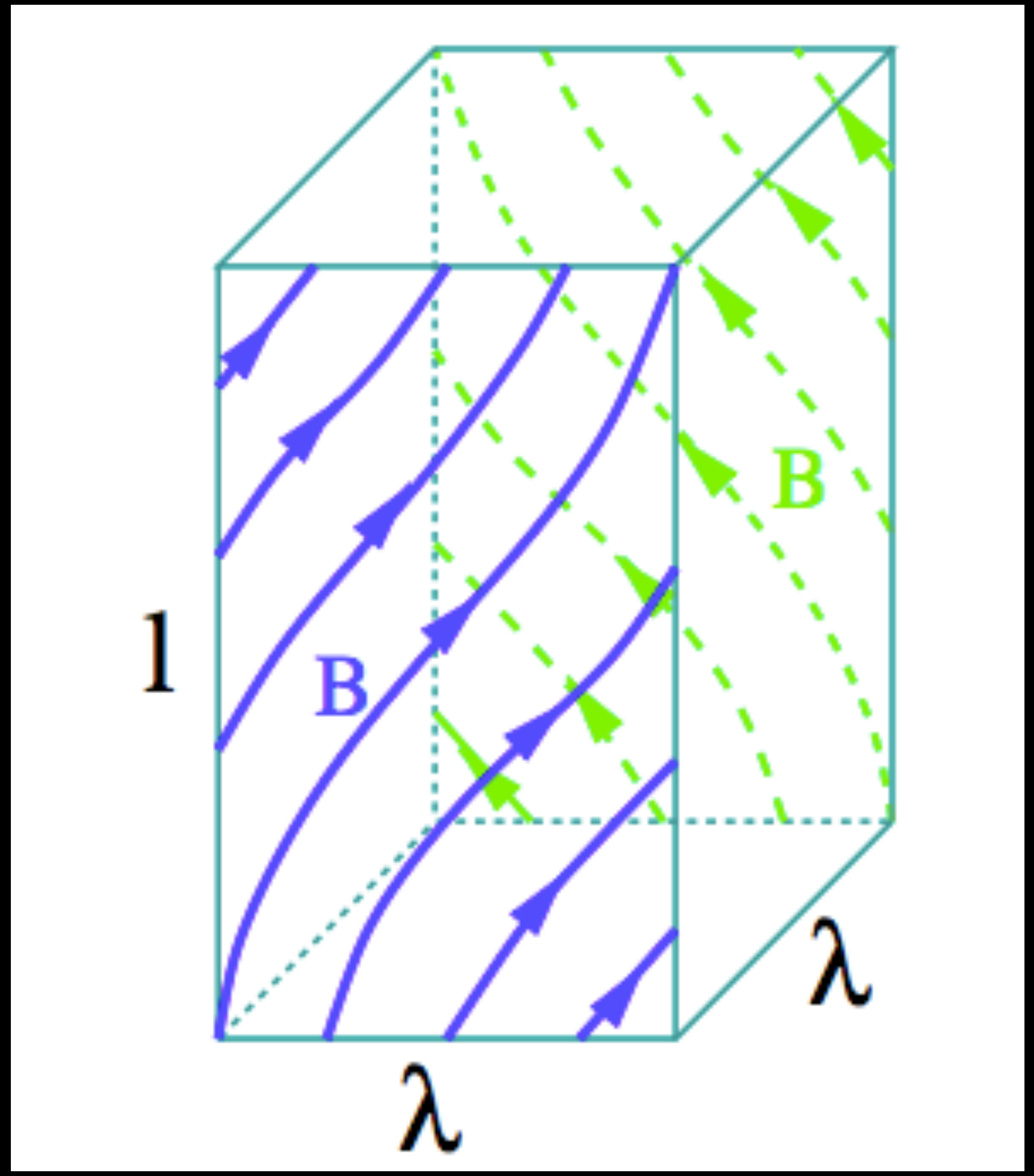


Critical balance (GS95)

Balance between the linear and nonlinear timescales

Two perpendicular planes can only be causally connected (i.e., correlated) if their distance can be covered by an Alfvén wave faster than the characteristic timescale of the perpendicular dynamics:





Critical Balance: $l/v_{A_\lambda} \sim \lambda/\delta u_\lambda$

Goldreich-Sridhar (GS95): eddies' dimension perpendicular to the background field are comparable; become filaments at small scales. (key idea: critical balance).

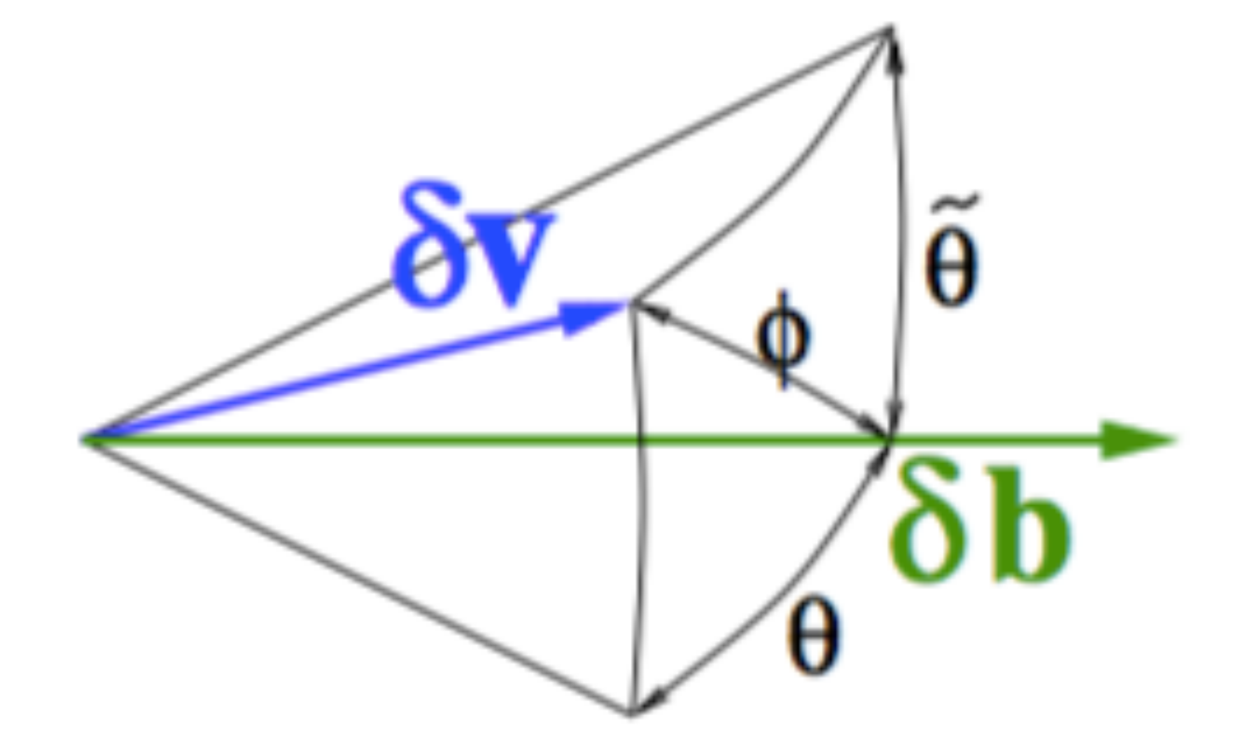
$E(k) \sim k^{-5/3}$

Boldyrev '06 dynamic alignment

Velocity and magnetic field fluctuations tend to align

$$E = \frac{1}{2}\int (b^2 + v^2) d^3x$$

$$H^C = \int (\mathbf{v} \cdot \mathbf{b}) d^3x$$



Dissipation of cross helicity is *not positive definite*:

integral will decay more slowly than that of the energy.

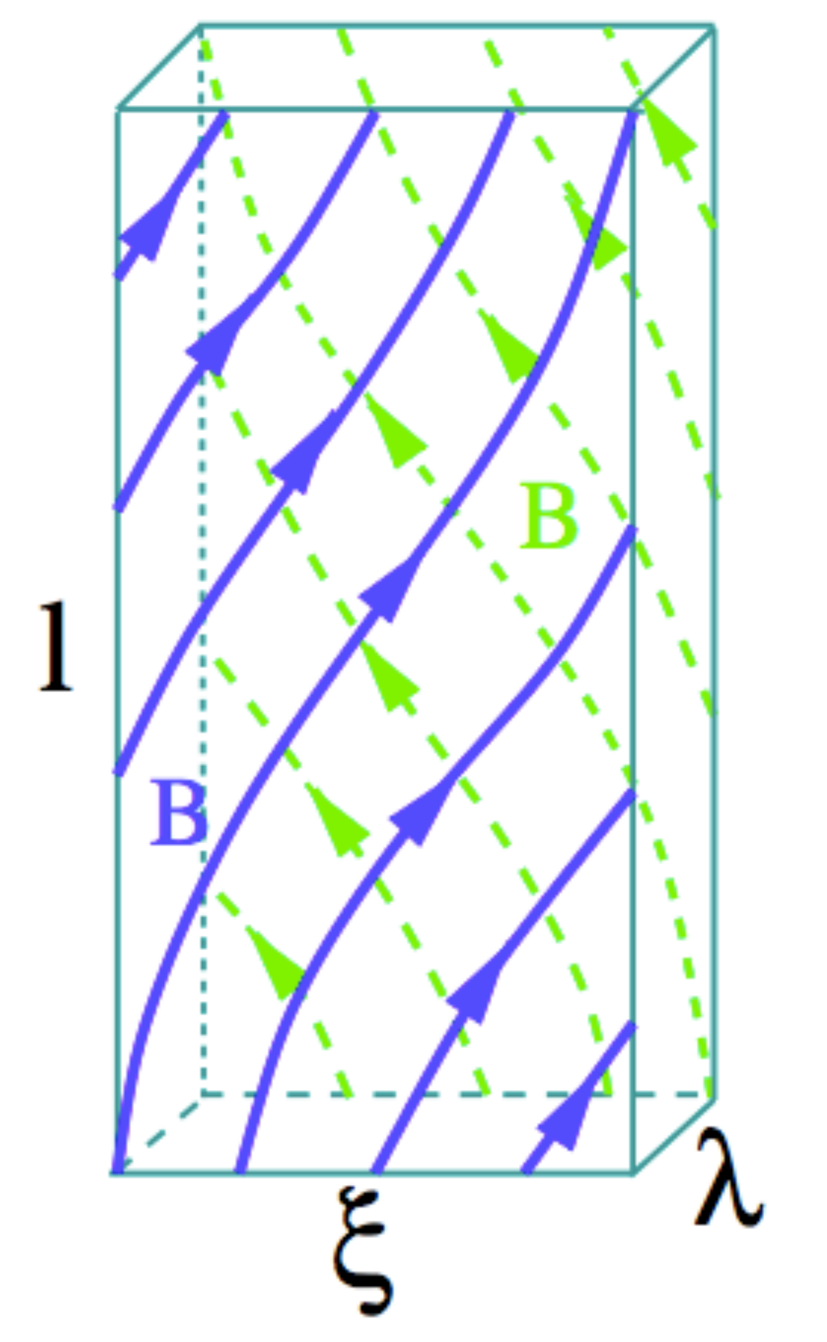
This selective decay means turbulence approaches state were

b=**v** or **b**=-**v** —> *dynamic alignment* (or Alfvenization effect) of turbulence.

Perfect alignment cannot be reached because that is inconsistent with constant energy flux over scales.

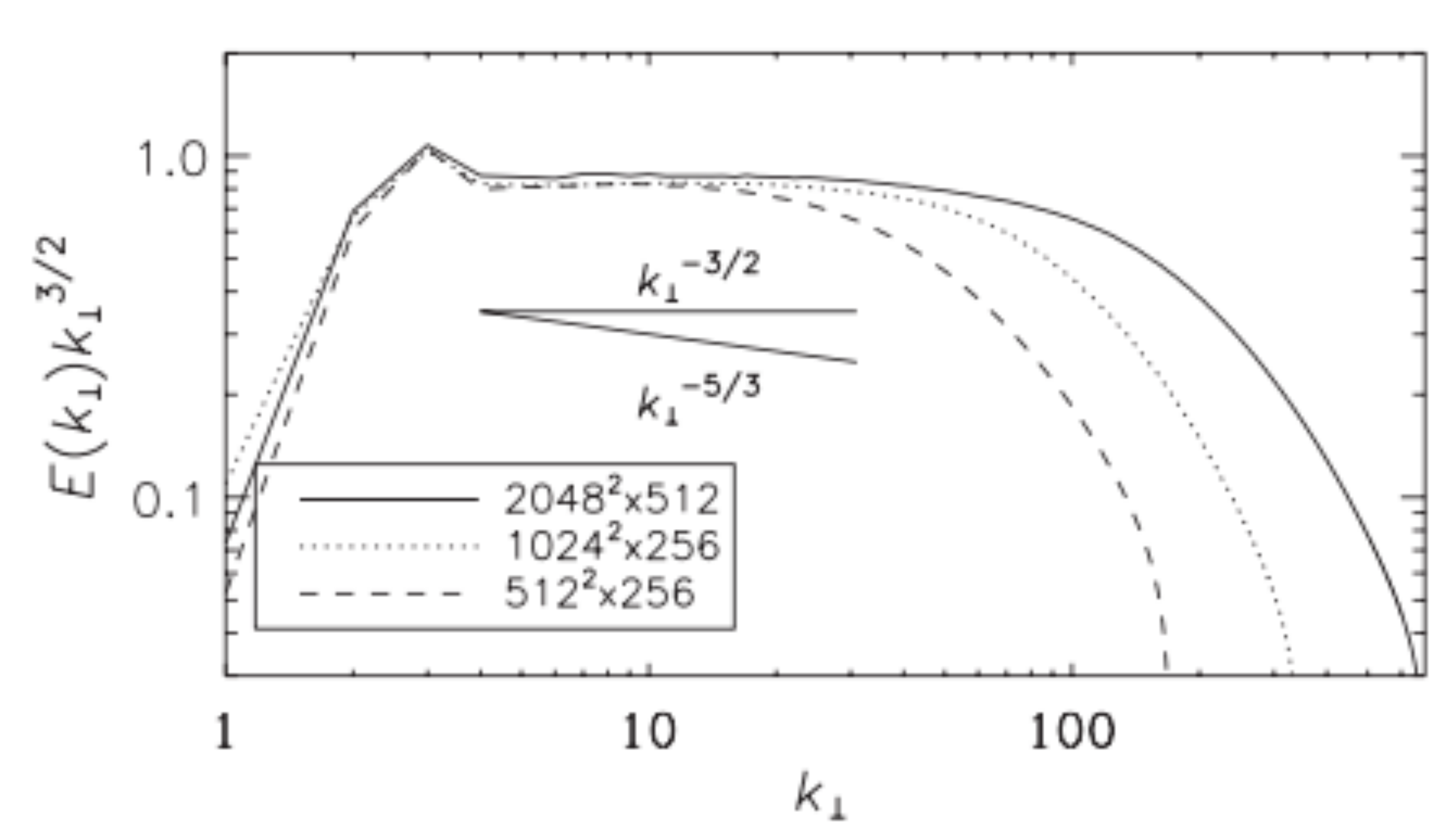
Boldyrev '06 3D anisotropic eddies

MHD turbulent eddies have progressively elongated cross-sections in the perp direction



Boldyrev '06: eddies fully anisotropic; ξ/λ >>1; become high-aspect ratio current sheets at small scales. (key idea: critical balance + dynamic alignment).

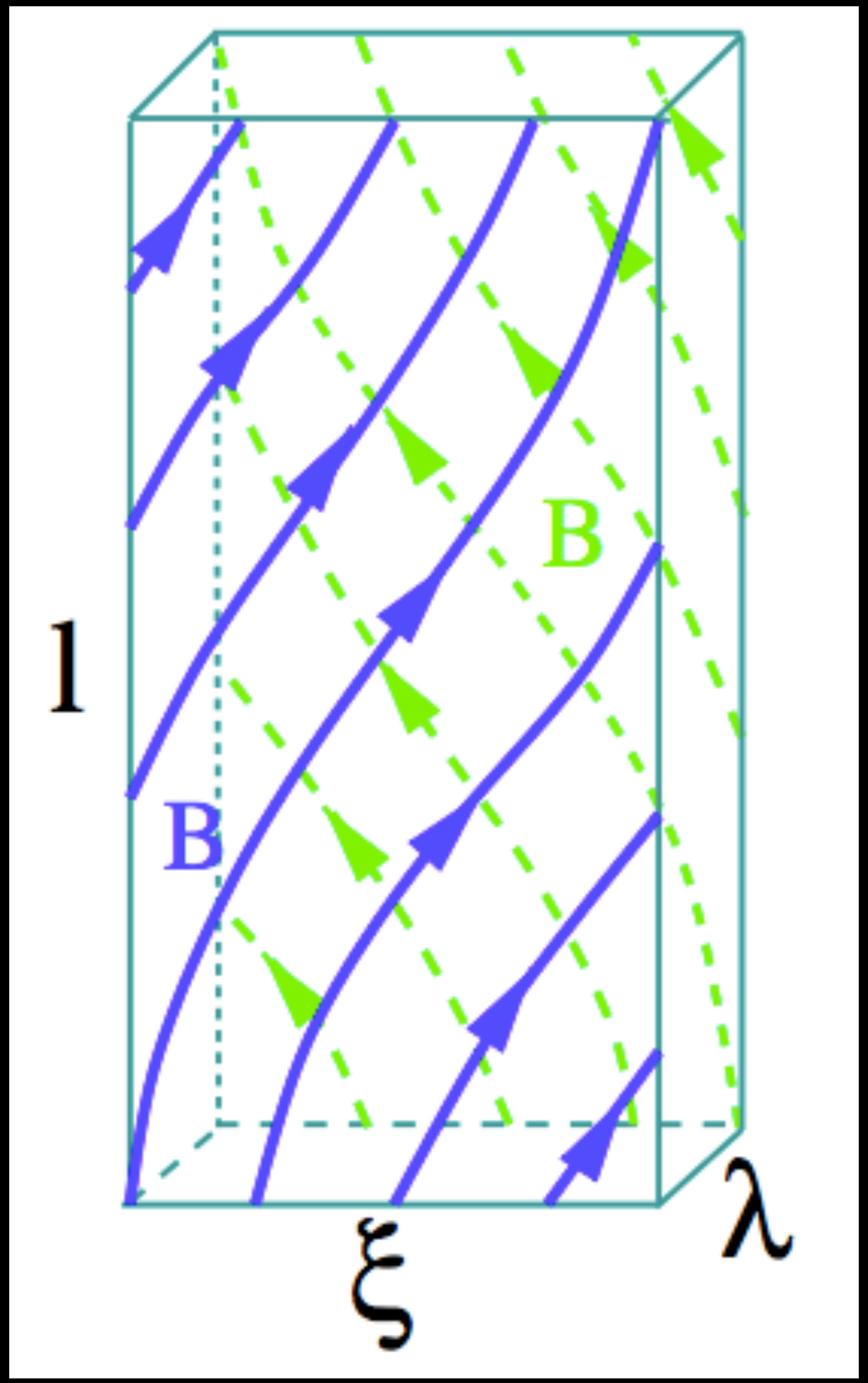
$E(k) \sim k^{-3/2}$



Magnetic energy (compensated) spectrum. Perez *et al.*, PRX '12

Eddies in Boldyrev '06

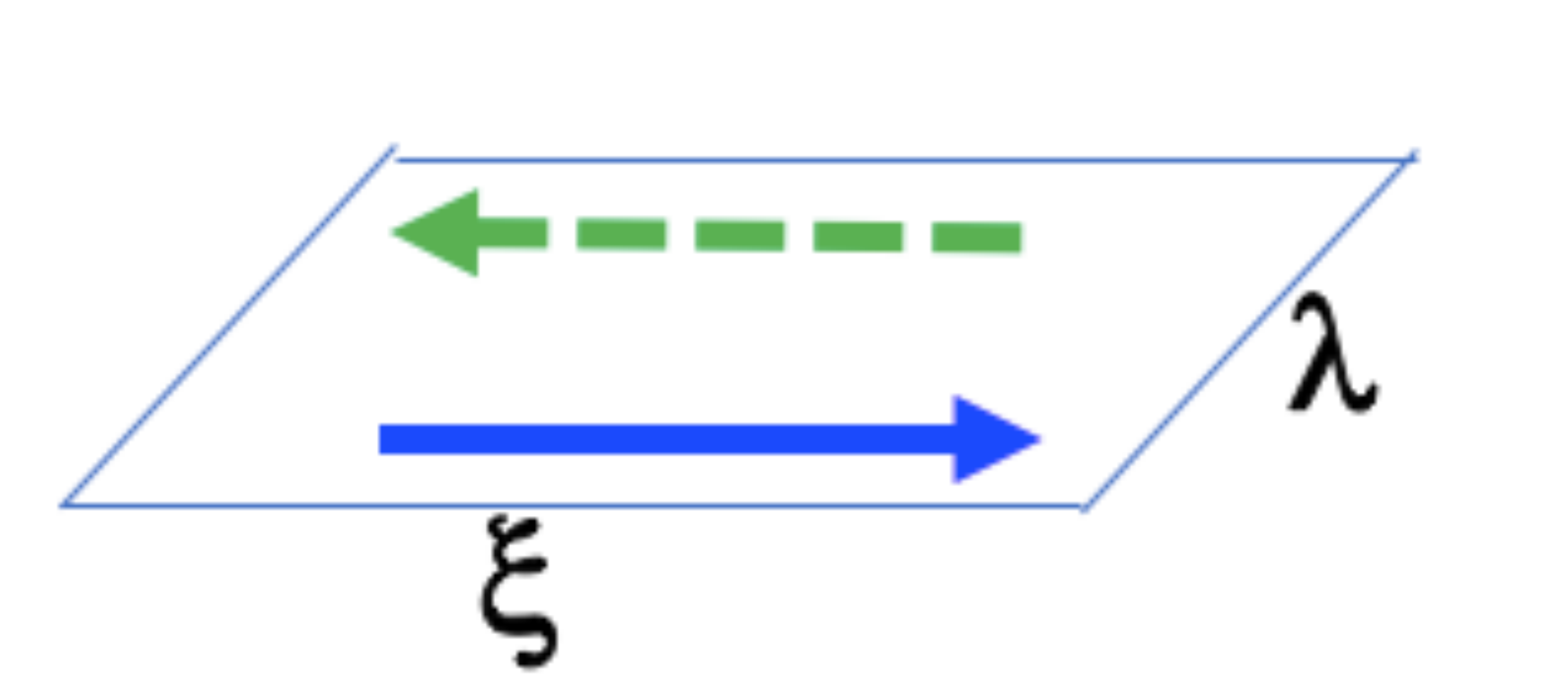
3D anisotropy as a function of scale



Boldyrev '06 predicts 3D structure of the turbulent eddies as a function of scale (λ)

$$\xi \sim L(\lambda/L)^{3/4},$$
$$\ell \sim L(\lambda/L)^{1/2},$$
$$b \sim B_0(\lambda/L)^{1/4},$$
$$\tau \sim \ell/V_{A,0} \sim \lambda^{1/2}L^{1/2}/V_{A,0}$$

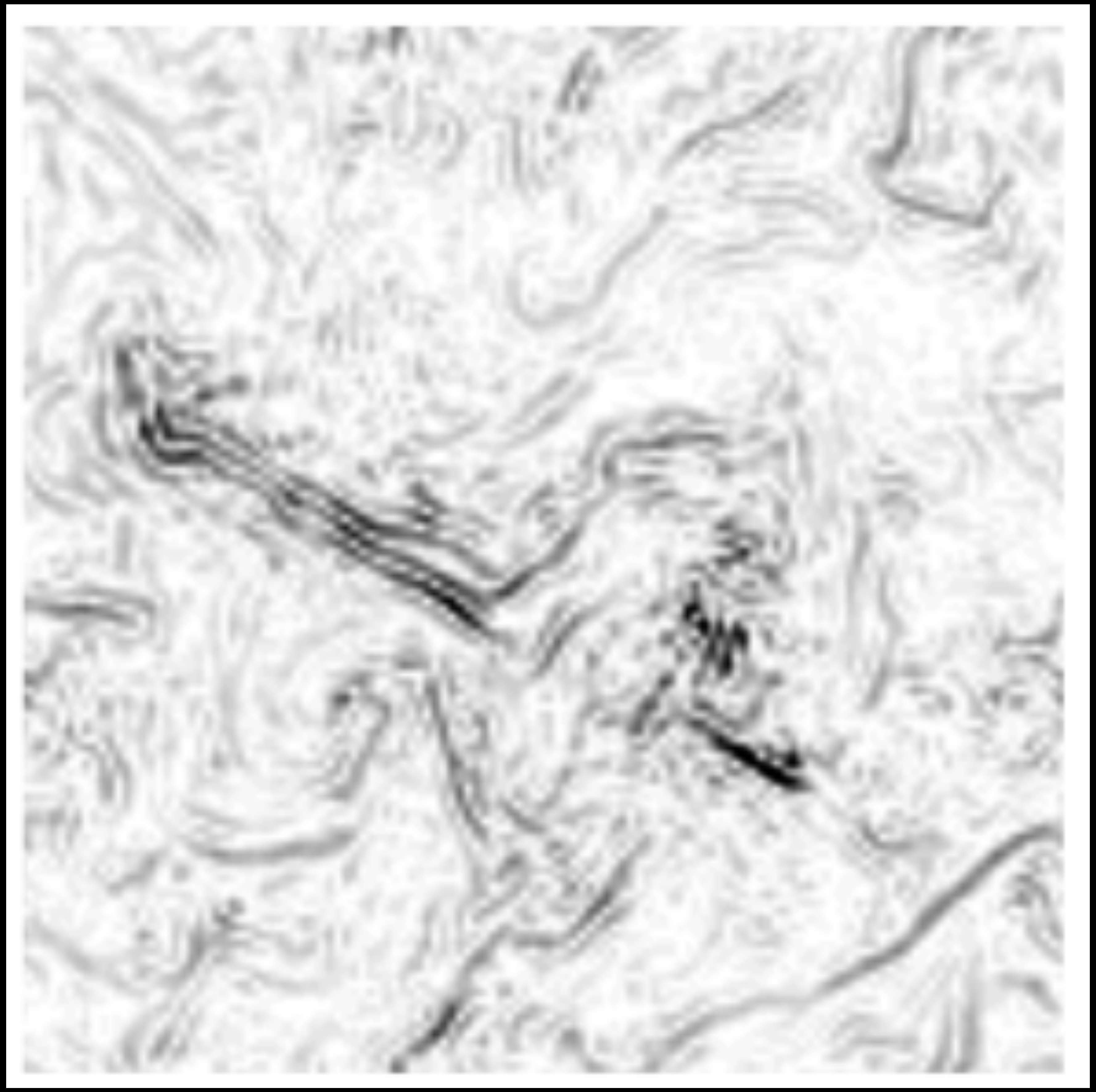
In the perpendicular plane, think of current sheets of length ξ, thickness λ, and upstream field *b*.



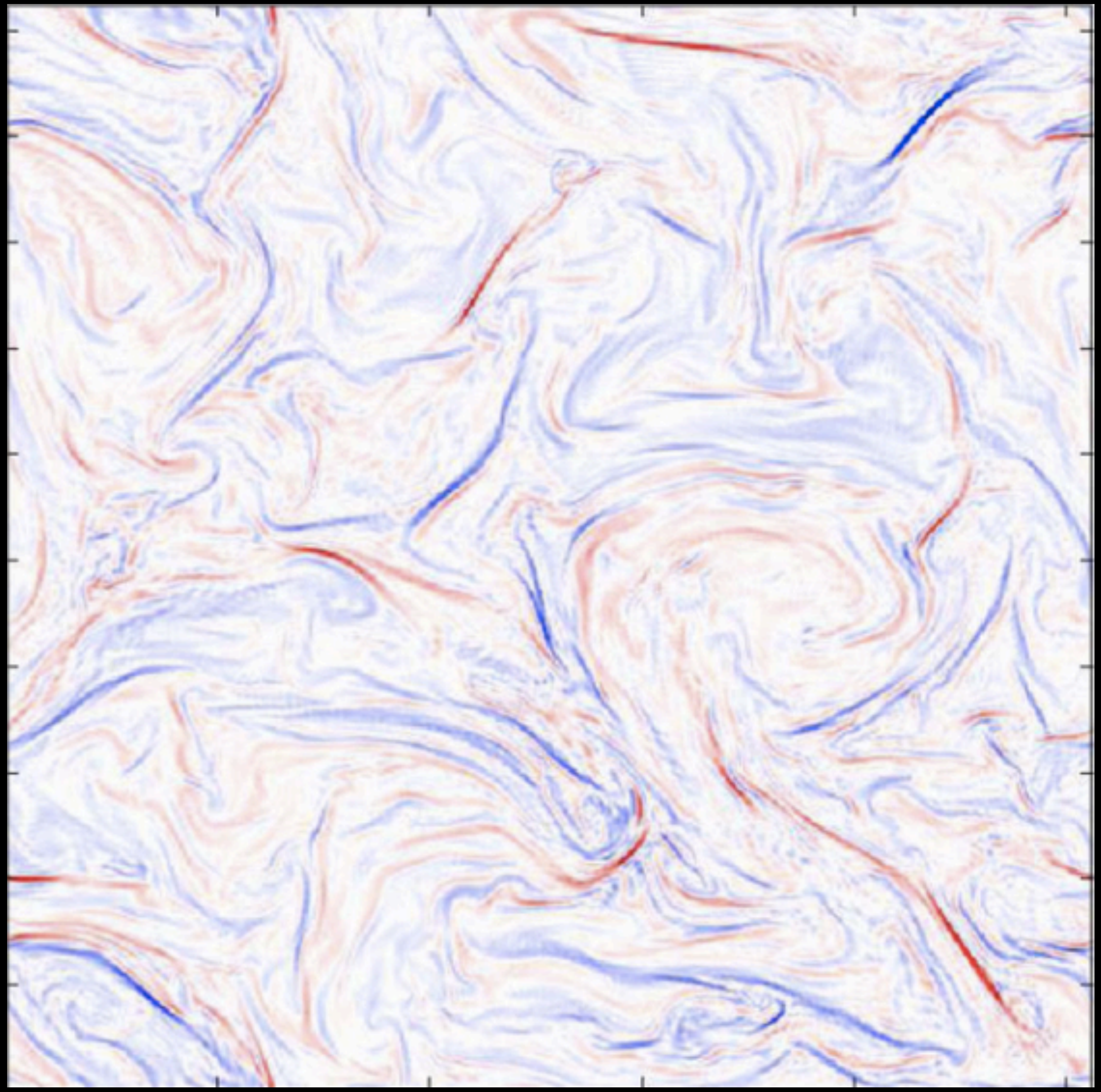
The eddies last for a time interval τ, unless they are first disrupted by reconnection.

Current sheet formation is predicted and observed

MHD turbulence simulations show abundant evidence for current sheet formation



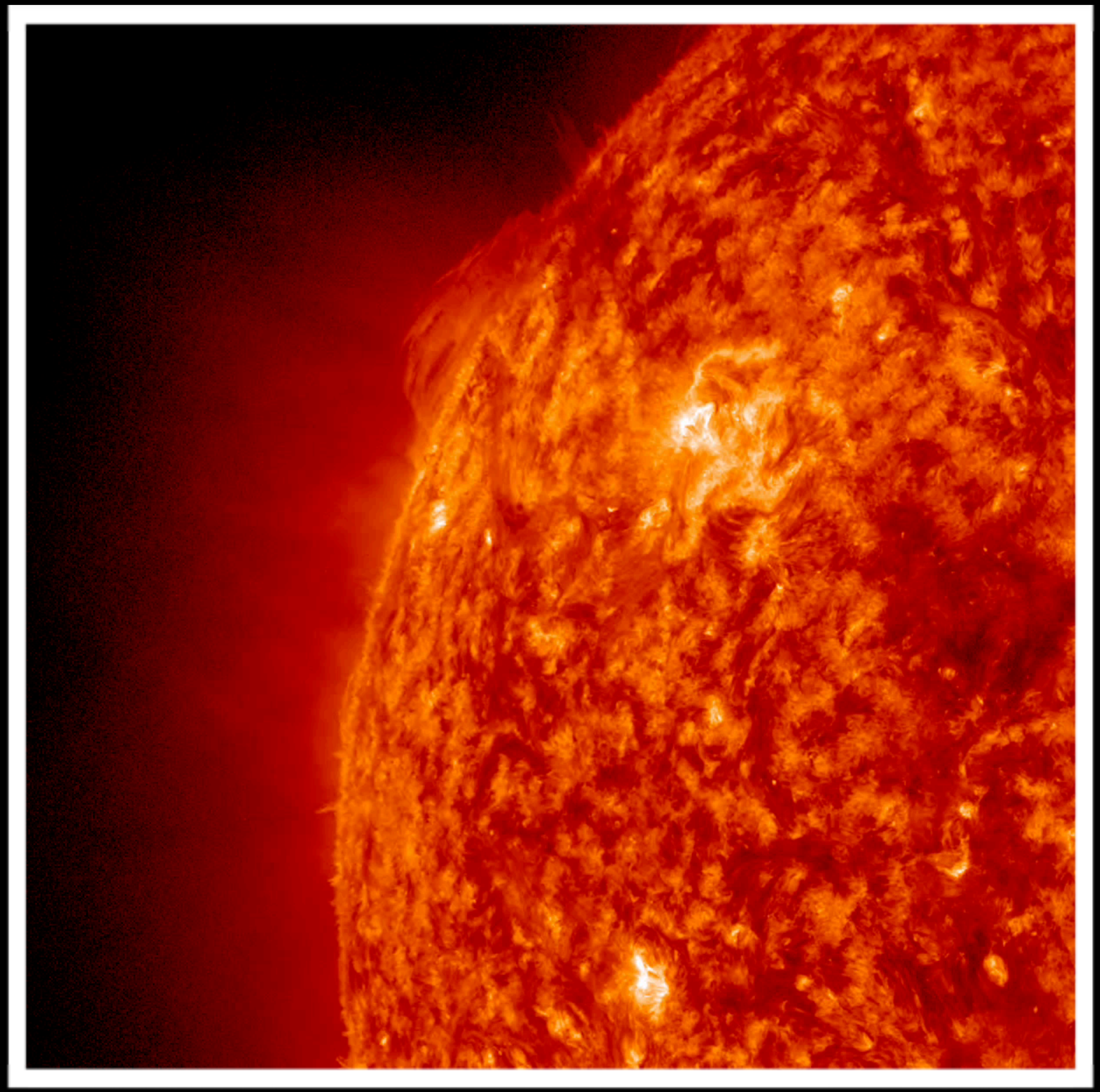
Maron & Goldreich, ApJ '01

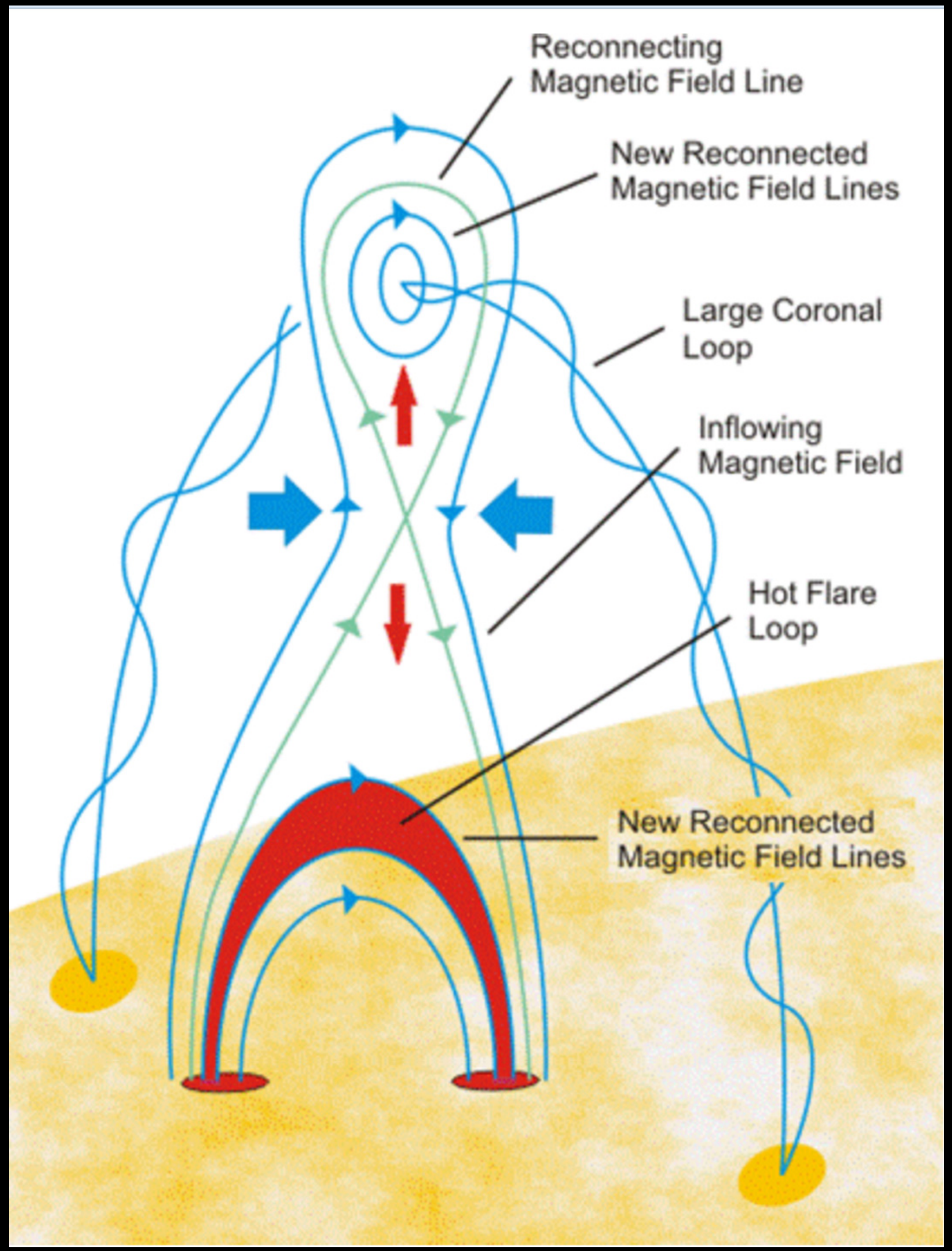


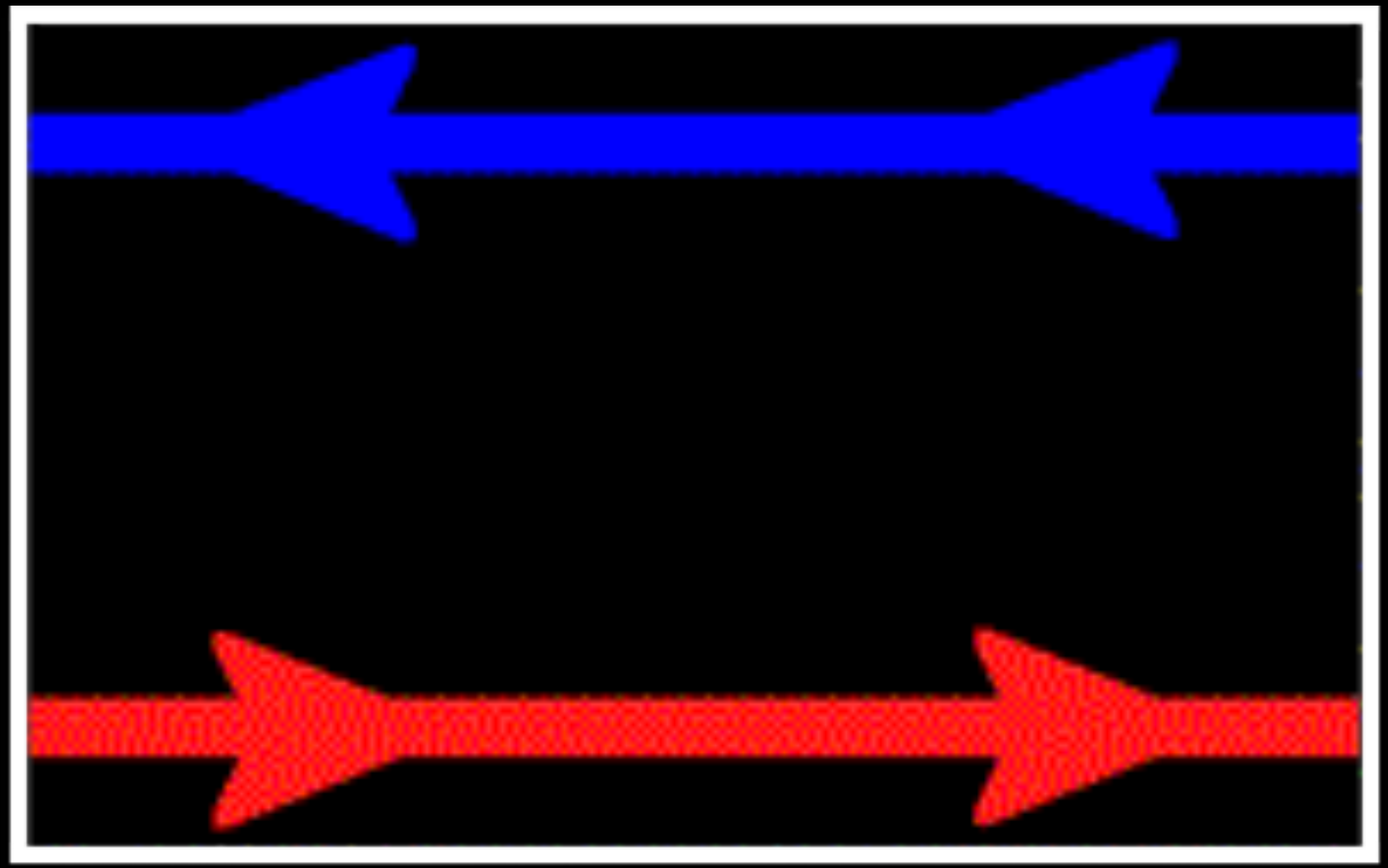
Zhdankin *et al.*, ApJ '13

Quick primer on magnetic reconnection

Topological change of the magnetic field enabled by non-ideal effects

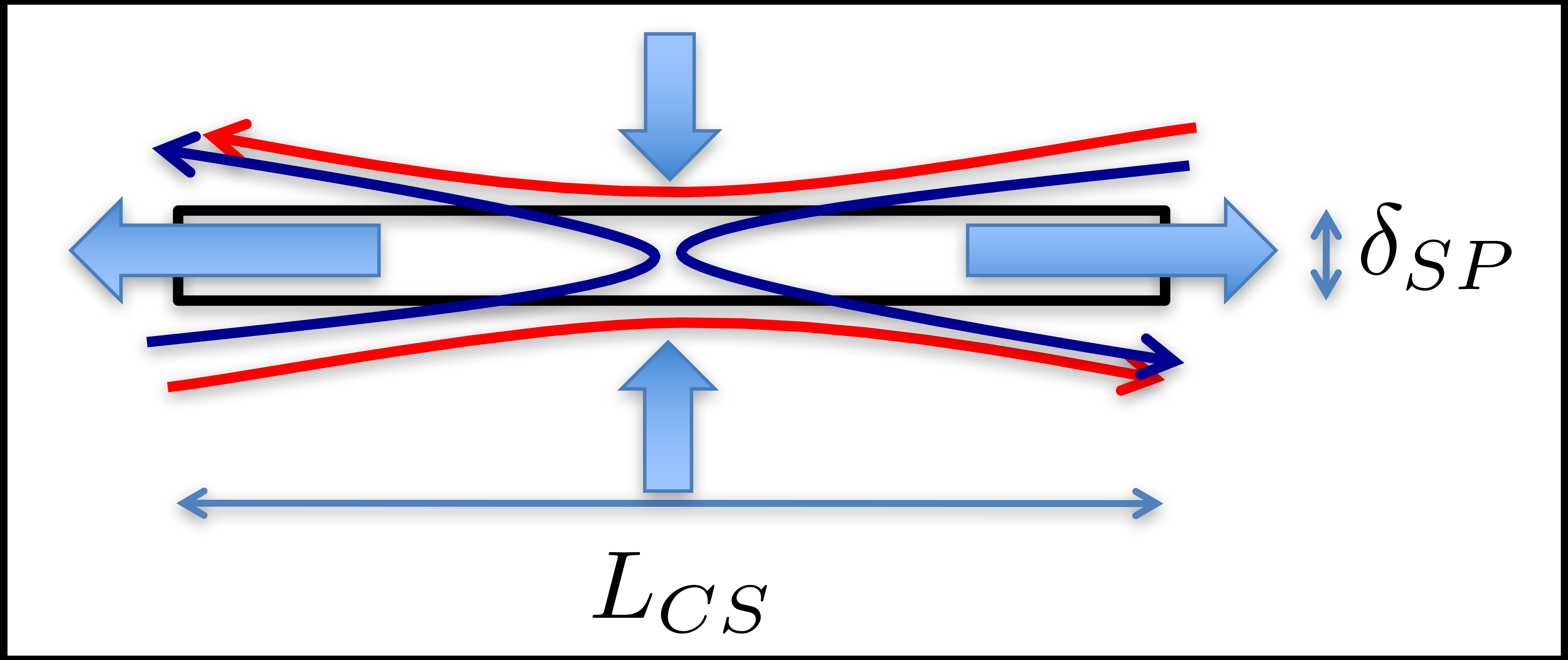






Quick primer on magnetic reconnection

Sweet-Parker model of reconnection



$$S = L_{CS} V_A / \chi_m;$$

$$\delta_{SP}/L_{CS} \sim S^{-1/2},$$

$$u_{in}/V_A \sim S^{-1/2},$$

$$cE \sim V_A B_0 S^{-1/2}$$

Onset of Dissipation in Turbulence

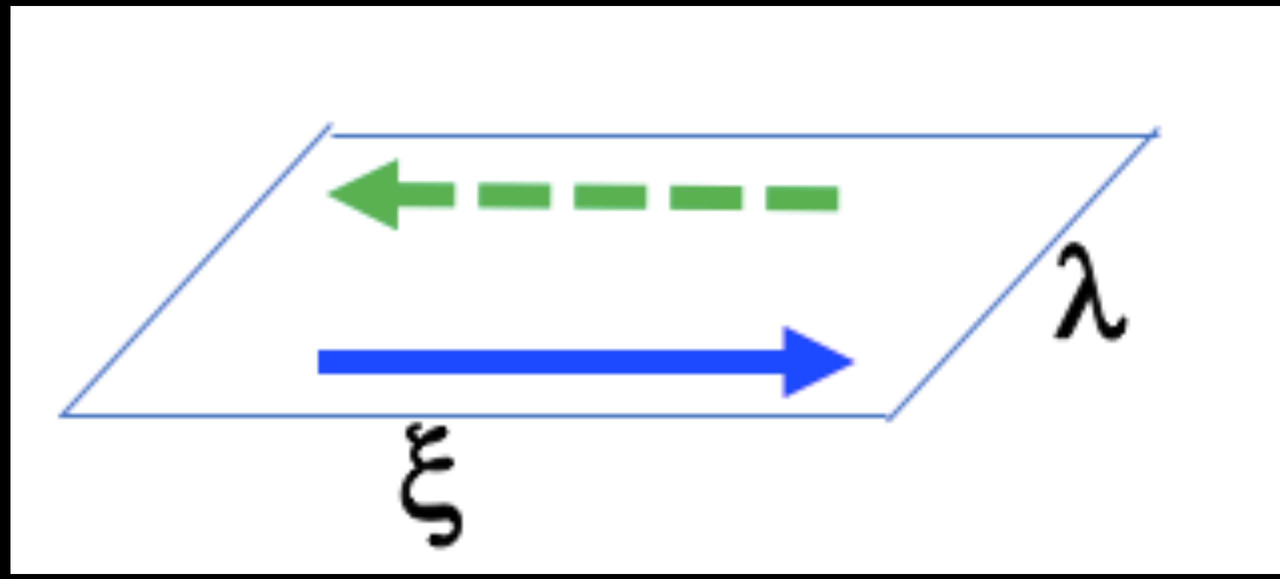
Usual way to estimate dissipation scale leads to paradoxical result

Usual way to estimate dissipation scale is to compare the eddy turnover time to the dissipation time:

$$\lambda^{1/2} L^{1/2} / V_{A,0} \sim \lambda^2 / \eta$$

This leads to:

$$\lambda / L \sim S_L^{-2/3} \sim R_m^{-2/3}$$



This dissipation-scale eddy has the *exact same aspect ratio of a Sweet-Parker current sheet* (Zhdankin *et al.* '13, Loureiro & Boldyrev '17):

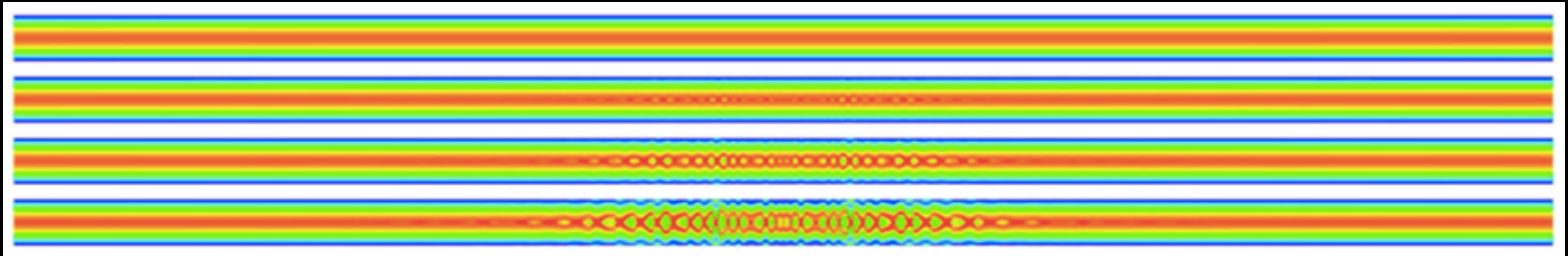
$$\lambda / \xi \sim (\xi V_{A,\lambda} / \eta)^{-1/2} \sim S_L^{-2/3}$$

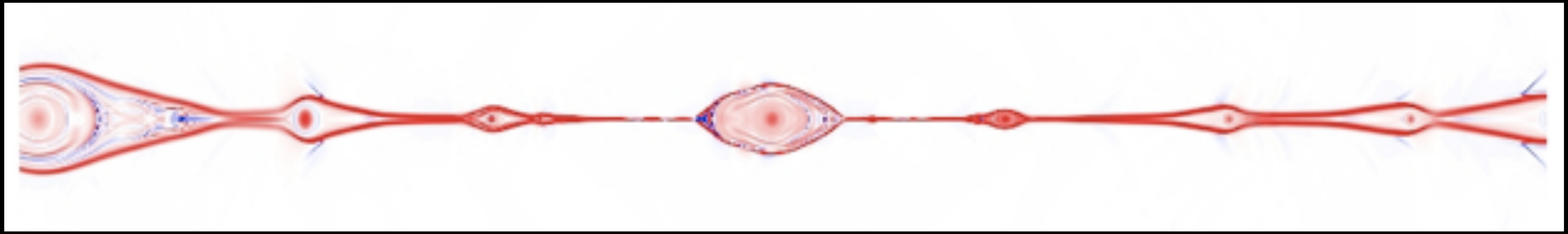
But this cannot be right because Sweet-Parker current sheets are strongly unstable.

Current Sheet Instability

Fully developed Sweet-Parker-like current sheets are unstable

Last decade of research on magnetic reconnection has demonstrated that Sweet-Parker current sheets are unstable to a super-Alfvenic instability (Loureiro *et al.* 2007; Samtaney *et al.* 2009; Bhattacharjee *et al.* 2009; Huang *et al.* 2010; many others: see Loureiro & Uzdensky PPCF 2016 for a review).

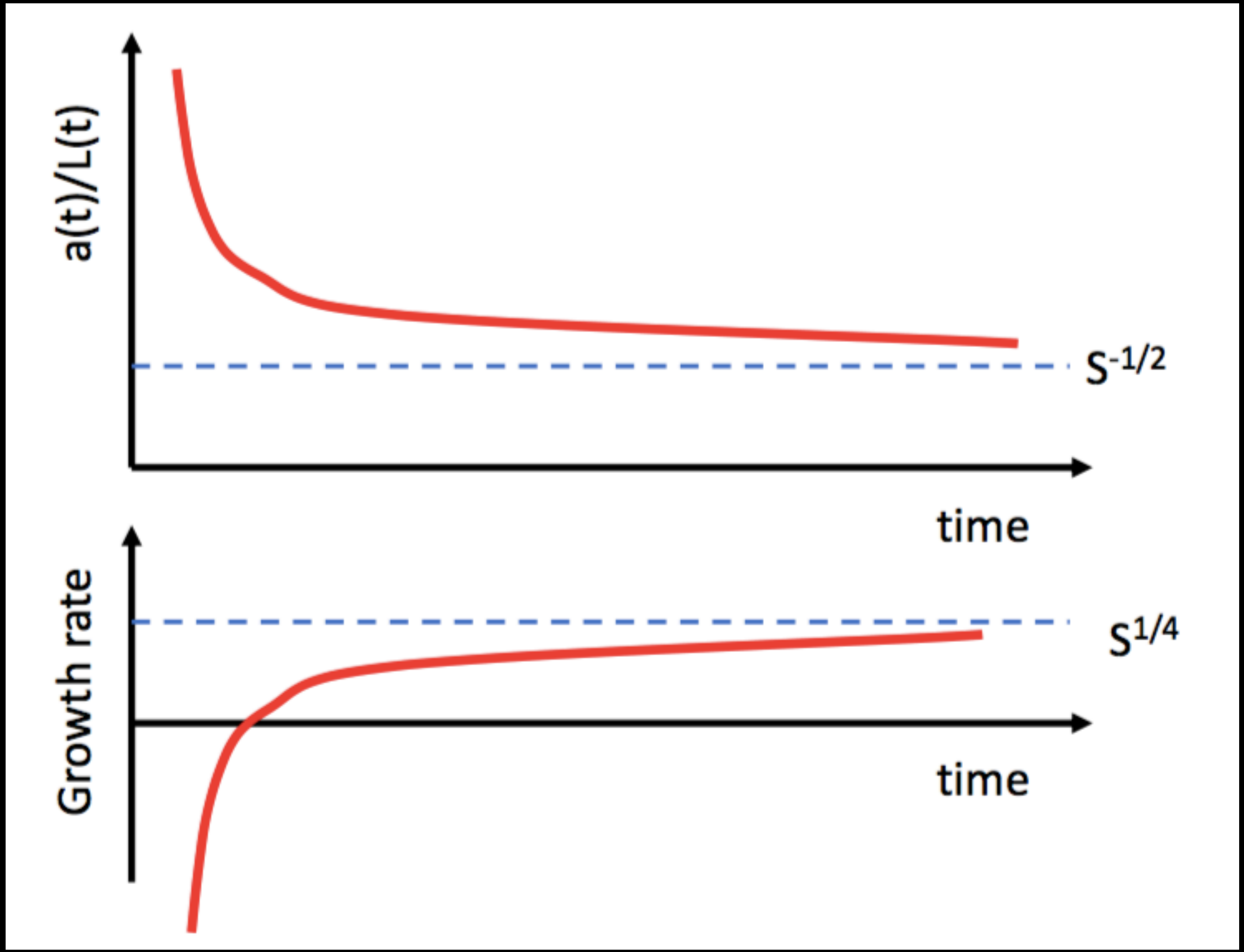




Reconnection Onset in a Forming Sheet

Instability must arise as the current sheet is forming

In fact, this instability means that Sweet-Parker current sheets can never really form: as one is trying to form, it is disrupted by its own instability along the way (Pucci & Velli '14, Uzdensky & Loureiro, '16, Comisso *et al.* '16, Tolman *et al.* '18)



A forming current sheet must become unstable before attaining the Sweet-Parker aspect ratio ~ $S^{-1/2}$

The important moment of time is when

$$\gamma[a(t), L(t)]\tau_{CS} \sim 1$$

Uzdensky & Loureiro, PRL '16

Dynamic Reconnection Onset

Matching the turbulence and reconnection timescales

At what scale does the *eddy turnover time* become comparable to the *tearing mode growth time* in the eddy?

$$\gamma_{\rm tear}\tau \sim 1$$

This leads to the prediction of a *critical scale* below which reconnection is faster than turbulence:

$$\lambda_{cr}/L \sim S_L^{-4/7}$$

Result is easily extended to high Pm plasmas: $\lambda_{cr}/L \sim S_L^{-4/7} Pm^{-2/7}$

Loureiro & Boldyrev, PRL 2017

Boldyrev & Loureiro, ApJ 2017

Mallet *et al.*, MNRAS 2017

Spectrum

Prediction of the existence of a new, sub-inertial range

Spectrum can be computed from enforcing constant energy flux: $$\gamma_{nl} V_{A,\lambda}^2 = \epsilon$$

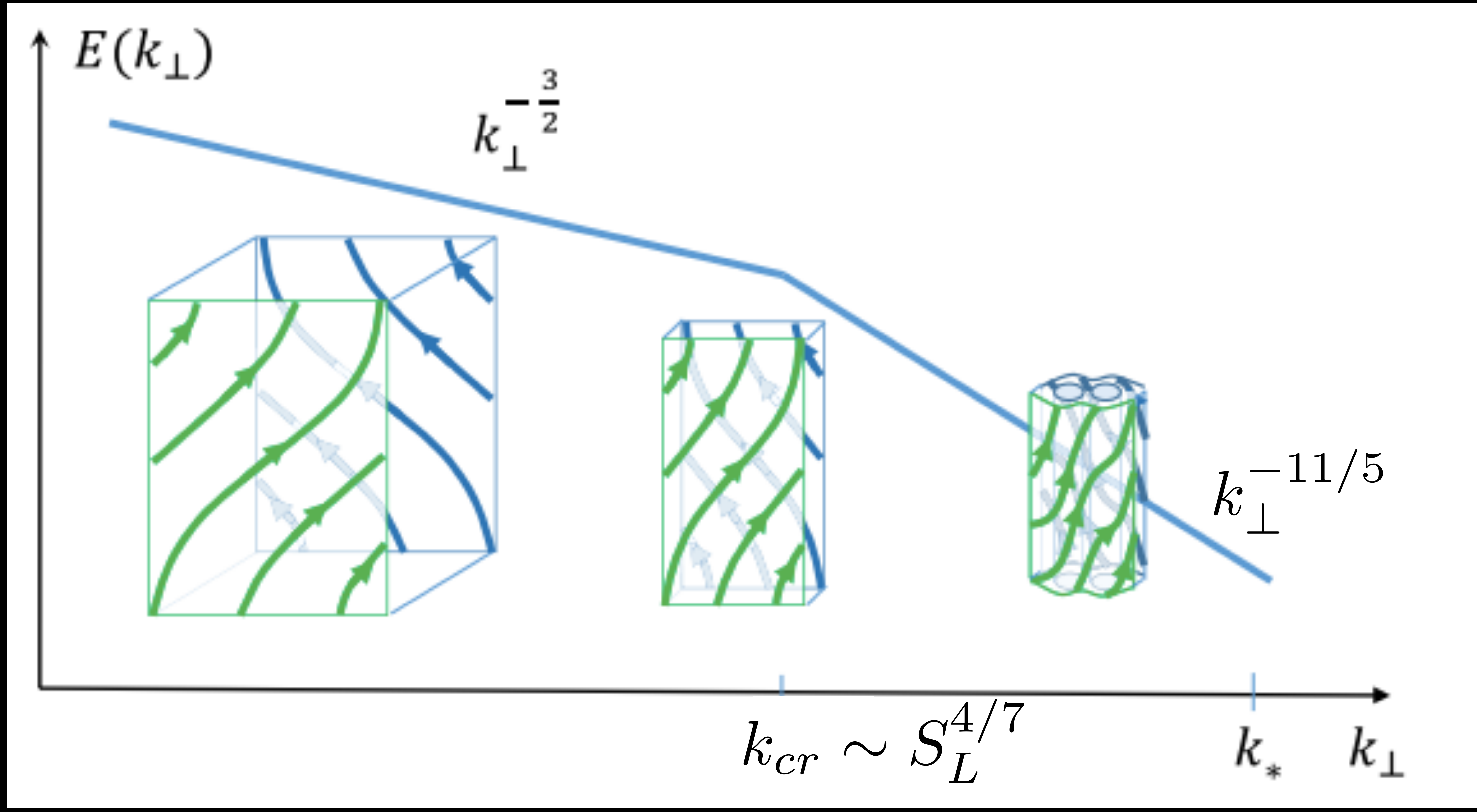
where $\epsilon \sim V_{A,0}^3 / L_0$ is the constant rate of energy cascade over scales.

We assume that when the tearing mode becomes nonlinear, it sets the timescale of the eddy:

$$\gamma_{nl} \sim \gamma_{\text{tear}}$$

Obtain:

$$E(k_\perp) dk_\perp \sim \epsilon^{4/5} \eta^{-2/5} k_\perp^{-11/5} dk_\perp$$



Boldyrev & Loureiro, ApJ 2017

Spectrum

Dissipation scale

The energy dissipation per unit time is $\eta \int^{k_*} k_\perp^2 E(k_\perp) dk_\perp \sim \epsilon$

which leads to the estimate of the *dissipation scale* $k_* L \sim S_L^{3/4}$

It's easy to check that the Lundquist number at the dissipation scale is $S_{\lambda_*} = \lambda_* v_{A_{\lambda_*}} / \eta \sim 1$

Similarly, can check that at the dissipation scale the eddy becomes isotropic in the perpendicular plane (which explains why the dissipation scale that we obtain is the same as in GS95).

Extension to the kinetic reconnection regime

Collisionless reconnection in MHD-scale eddies

In many realistic plasmas, collisions are so infrequent that reconnection in a MHD-scale eddy will trigger kinetic effects:

$$\lambda \gg \rho_i \gg \delta \sim d_e$$

This can be handled with kinetic tearing mode theory (reconnection is caused by electron inertia, instead of collisions).

Different cases can be analyzed, depending on electron beta.

Invariably, obtain spectra that scale as

$$E(k_\perp) \propto k_\perp^{-3} \quad \text{or} \quad E(k_\perp) \propto k_\perp^{-8/3}$$

depending on what shape is assumed for the reconnecting magnetic field.

Loureiro & Boldyrev, ApJ '17. See also Mallet *et al.*, JPP '17.

Extension to the kinetic reconnection regime

Collisionless reconnection in MHD-scale eddies

Consider a low beta plasma: $m_e/m_i \ll \beta \ll 1$

Find the critical scale for reconnection onset as: $\lambda_{cr}^{(1)}/L \sim (d_e/L)^{4/9}(\rho_s/L)^{4/9}$

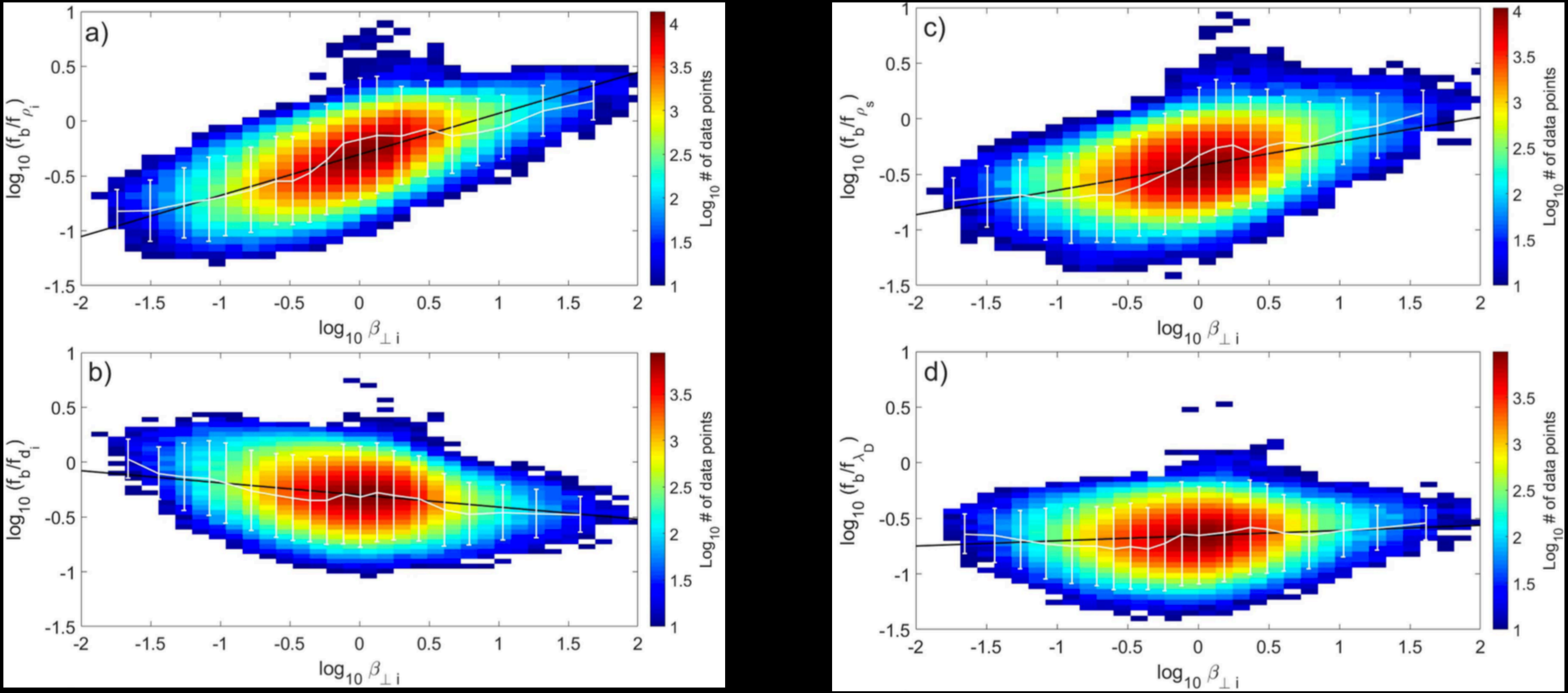
Valid if $\lambda_{cr} \gg \rho_s$ i.e., $\rho_s/L \ll (m_e/m_i)^2\beta_e^{-2}$

Similar estimates can be made for ultra low beta, or beta~1.

Critical scales then differ, but spectral index remains unchanged.

Evidence for reconnection in SW turbulence

Analysis of solar wind data shows evidence of a reconnection-induced spectral break



Vech *et al.*, ApJ 2018

Reconnection in the kinetic turbulence range

Collisionless reconnection at sub-Larmor radius scales

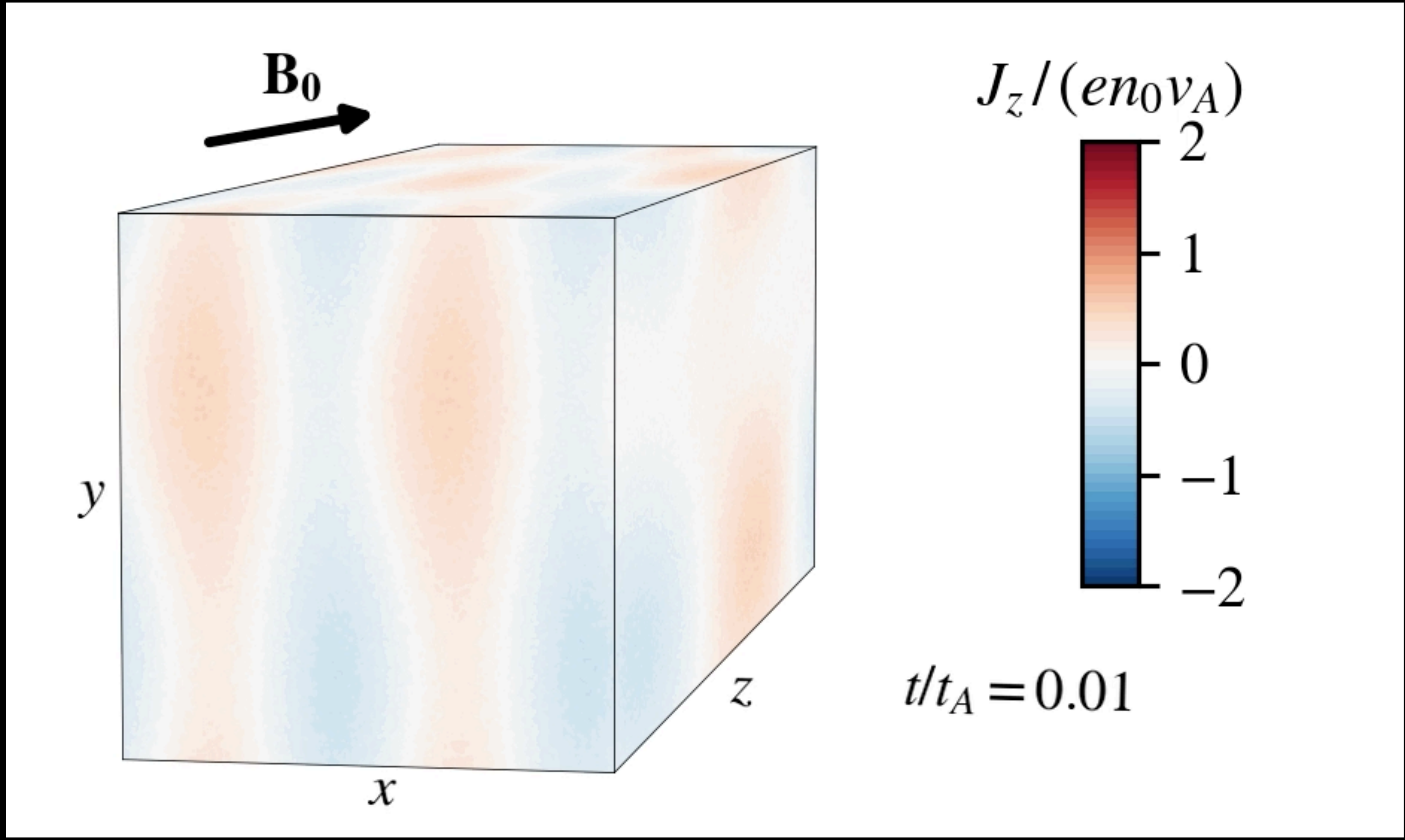
Can we extend these ideas to the kinetic turbulent range, i.e.,

$$\lambda \ll \max(\rho_i, \rho_s)$$

Uncertain: no theory to describe the eddy aspect ratio, etc.

Numerical simulations do suggest current sheet presence at these scales.

Cannot estimate the critical scale for the transition to the reconnection range – this requires knowing what the eddies look like.



D. Groselj *et al.*, PRL '18

But can estimate the spectrum given expression for the tearing mode growth rate at those scales. Again, we obtain:

$$E(k_\perp) \propto k_\perp^{-3} \quad \text{or} \quad E(k_\perp) \propto k_\perp^{-8/3}$$

Boldyrev & Loureiro, ApJ 2017

Magnetized pair plasmas

Fluid equations for a low beta, non-relativistic pair plasma

Start from two-fluid equations for electrons and positrons:

$$n^{\pm}m\left(\frac{\partial \mathbf{v}^{\pm}}{\partial t}+\mathbf{v}^{\pm}\cdot\nabla\mathbf{v}^{\pm}-\mu^{\pm}\nabla^2\mathbf{v}^{\pm}\right)=\pm n^{\pm}e\left(\mathbf{E}+\frac{\mathbf{v}^{\pm}\times\mathbf{B}}{c}\right)-\nabla p^{\pm}\mp\mathcal{R}$$

Sum and subtract:

$$nm\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}+\frac{1}{n^2e^2}\mathbf{j}\cdot\nabla\mathbf{j}-\mu\nabla^2\mathbf{v}\right)=\frac{\mathbf{j}\times\mathbf{B}}{c}-\nabla p$$

v is center-of-mass velocity

$$\frac{m}{ne^2}\left(\frac{\partial \mathbf{j}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{j}+\mathbf{j}\cdot\nabla\mathbf{v}-\mu\nabla^2\mathbf{j}\right)=\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}}{c}-\eta\mathbf{j}.$$

Magnetized pair plasmas

Fluid equations for a low beta, non-relativistic pair plasma

Impose reduced-MHD-like ordering: $\mathbf{B} = B_0\hat{z} + \mathbf{B}_\perp, \text{with } B_\perp/B_0 \sim \epsilon$

Introduce potentials: $\mathbf{v}_\perp = \hat{z} \times \nabla_\perp \phi;$ $\quad \dfrac{\mathbf{B}_\perp}{\sqrt{4\pi\rho}} = \hat{z} \times \nabla_\perp \psi,$

$$\frac{\partial}{\partial t}\nabla_\perp^2 \phi + \{\phi, \nabla_\perp^2 \phi\} = \{\psi, \nabla_\perp^2 \psi\} + V_A \frac{\partial}{\partial z}\nabla_\perp^2 \psi + \mu \nabla_\perp^2 \phi,$$

momentum equation

$$\frac{\partial}{\partial t}\left(1 - d_e^2 \nabla_\perp^2\right)\psi + \left\{\phi, (1 - d_e^2 \nabla_\perp^2)\psi\right\} = V_A \frac{\partial \phi}{\partial z} + \eta \nabla_\perp^2 \psi - \mu d_e^2 \nabla_\perp^4 \psi$$

Ohm's law

Magnetized pair plasmas

Invariants and waves

These equations have two *exact* invariants at *all* scales:

$$\mathcal{E} = \frac{1}{2}\int dV \left\{(\nabla_\perp \psi)^2 + d_e^2 \left(\nabla_\perp^2 \psi\right)^2 + (\nabla_\perp \phi)^2\right\}$$

energy

$$\mathcal{H}^C = \int dV \left\{\nabla_\perp^2 \phi \left(1 - d_e^2 \nabla_\perp^2\right) \psi\right\}$$

cross helicity

Only one wave is supported:

$$\omega_l = \pm \frac{k_z V_A}{\sqrt{1 + k_\perp^2 d_e^2}}$$

Alfvén wave modified at kinetic scales by electron inertia

Turbulence in magnetized pair plasmas

Turbulence at MHD and kinetic scales

At **MHD scales** ($kd_e \ll 1$), the equations are the same as for "normal" (ion-electron) plasmas.

So turbulence will be the same:

expect $k^{-3/2}$ up until the reconnection scale,

followed by a transition to a k^{-3} (or -8/3) due to reconnection

At **kinetic scales** ($kd_e \gg 1$), *no reconnection is possible*. So we expect to have just a normal energy cascade.

$$\mathcal{E} = \frac{1}{2}\int dV \left\{(\nabla_\perp\psi)^2 + d_e^2\left(\nabla_\perp^2\psi\right)^2 + (\nabla_\perp\phi)^2\right\}$$

$$\Rightarrow \frac{1}{2}\int dV \left\{d_e^2(\nabla_\perp^2\psi)^2 + (\nabla_\perp\phi)^2\right\}$$

expect equipartition between parallel and perpendicular kinetic energies at these scales.

Turbulence in magnetized pair plasmas

Turbulence at kinetic scales (cont'd)

$$k_\perp^2 \phi_\lambda^2 / \tau_\lambda \sim \varepsilon$$

$$\tau_\lambda = 1/\omega_{nl} \sim 1/(k_\perp^2 \phi_\lambda)$$

This results in:

$$\phi_\lambda \sim \varepsilon^{1/3} k_\perp^{-4/3} \quad \Rightarrow \quad E_\phi(k_\perp) dk_\perp \sim \varepsilon^{2/3} k_\perp^{-11/3} dk_\perp$$

and same scaling for magnetic energy, by equipartition

Finally, *declare* that the fluctuations are *critically balanced* at these scales: $\omega_l \sim \omega_{nl}$

$$k_{||} \sim \varepsilon^{1/3} d_e V_A^{-1} k_\perp^{5/3}$$

Loureiro & Boldyrev, ApJL 2018

Turbulence in magnetized pair plasmas

Dealing with the cross helicity

Recall cross-helicity: $$\mathcal{H}^C = \int dV \left\{\nabla_\perp^2 \phi \left(1 - d_e^2 \nabla_\perp^2\right) \psi\right\}$$ It is *not positive definite*

Estimate the flux of cross helicity as $$\left(k_\perp^2 \phi_\lambda\right) \left(d_e^2 k_\perp^2 \psi_\lambda\right) R_\lambda / \tau_\lambda \sim \varepsilon^c$$

where R is a dimensionless cancelation factor at scale lambda. From the energy invariant:

$$k_\perp^2 d_e^2 \psi_\lambda^2 \sim \phi_\lambda^2$$ so the cross helicity flux becomes $$k_\perp d_e (k_\perp^2 \phi_\lambda^2) R_\lambda / \tau_\lambda \sim \varepsilon^c$$

But energy flux (constant) is $$k_\perp^2 \phi_\lambda^2 / \tau_\lambda \sim \varepsilon$$ Thus: $$R_\lambda \propto 1/(k_\perp d_e).$$

Conclusions

If current understanding of MHD turbulence is correct (GS95+Boldyrev '06), reconnection has to become important:

- Eddies become current sheets of progressively larger aspect ratios at small scales
- Therefore, they are progressively more unstable to the tearing mode

Can compute the scale at which reconnection becomes important. This marks the onset of a new, sub-inertial range whose spectrum can also be calculated.

These ideas can be extended to the kinetic regime. In all cases, we obtain spectra that scale as $k_\perp^{-8/3}$ or $k_\perp^{-3}$, in good agreement with observations and simulations.

Acknowledgements

NSF-DOE Partnership in Basic Plasma Science and Engineering, Award No. DE-SC0016215 and NSF CAREER award no. 1654168.

Further reading

Some (somewhat biased) suggestions for further reading:

Fluid turbulence:

– P. Davidson, *“Turbulence: an introduction for scientists and engineers”*

– U. Frisch, *“Turbulence: the legacy of A. N. Kolmogorov”*

MHD turbulence:

– D. Biskamp, *“Magnetohydrodynamic turbulence”*

– A. Schekochihin, http://www-thphys.physics.ox.ac.uk/research/plasma/JPP/papers17/schekochihin2a.pdf

Kinetic turbulence:

– A. Schekochihin, *“Astrophysical Gyrokinetics: Kinetic and Fluid Turbulent Cascades in Magnetized Weakly Collisional Plasmas”*, Astrophys. J. Supp., 2009