Tokamaks and stellarators

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PERON ANNOUNCES NEW WAY TO MAKE ATOM YIELD POWER

- Reports Argentina has devised thermonuclear reaction that does not use uranium

- News conference by Argentinian dictator Juan Peron:
  - Argentina had built a fusion pilot plant
  - The reaction required “enormously high temperatures of millions of degrees”

- Met with skepticism from Fermi and other leading physicists
Lyman Spitzer Jr, 1914-1997

- Head of Princeton Astrophysics (36)
- Pioneered the study of the interstellar medium

\[ ^6\text{Li} + n \rightarrow ^4\text{He} \ (2.05\text{MeV}) + T \ (2.73\text{MeV}) \]

\[ ^7\text{Li} + n \rightarrow ^4\text{He} + T + n - 2.47 \text{ MeV} \]
Magnetic confinement

without magnetic field

with magnetic field

ions, electrons
Motion in an inhomogenous magnetic field

• If \( B \) varies slowly on the length scale of the gyroradius \( \rho \)

\[
\rho_* = \frac{\rho}{R} \ll 1, \quad (R^{-1} = |\nabla B|/B)
\]

the particles drift slowly across the field

\[
\begin{align*}
\mathbf{r} &= \mathbf{R} + \vec{\rho}, \\
\dot{\mathbf{R}} &= v_\parallel \mathbf{b} + \frac{v_\perp^2/2 + v_\parallel^2}{\Omega} \mathbf{b} \times \nabla \ln B = v_\parallel \mathbf{b} + \mathbf{v}_d \\
\mathbf{b} &= \mathbf{B}/B
\end{align*}
\]

• Lagrangian (Taylor 1964)

\[
L = \frac{mv_\parallel^2}{2} + e \mathbf{A} \cdot \dot{\mathbf{R}} - \mu B
\]

\[
\mu = \frac{mv_\perp^2}{2B} = \text{adiabatic invariant}
\]
The tokamak

Usually credited to Tamm and Sakharov (1951), but also conceived by others (Spitzer, Schlüter, ...).
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Mirroring

\[ E = \frac{mv^2}{2} + \mu B = \text{constant}, \quad \mu = \frac{mv^2}{2B} \]

\[ \Rightarrow \quad B < \frac{E}{\mu} \]
Spitzer’s first idea
The classical stellarator
The classical stellarator
"Unwrap" the surface, and plot consider the field lines:

\[
\theta = \text{poloidal angle}
\]

\[
\varphi = \text{toroidal angle}
\]

The rotational transform is defined as the winding number

\[
\iota = \lim_{l \to \infty} \frac{\theta(l)}{\varphi(l)}
\]
Ampère's law

\[ \oint_C \mathbf{B} \cdot d\mathbf{r} = 0 \]
Analogy to hydrodynamics

MHD equation of motion:

\[ \rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p \]

Equilibrium without flow:

\[ \mathbf{J} \times \mathbf{B} = \nabla p \quad \Rightarrow \quad \mathbf{B} \cdot \nabla p = 0 \]

\[ (\nabla \times \mathbf{B}) \times \mathbf{B} = \frac{\nabla p}{\mu_0} \]

Compare with hydrodynamics:

\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla P, \quad \nabla \cdot \mathbf{u} = 0 \]

Stationary flows satisfy:

\[ \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \left( \frac{P}{\rho} \right) \quad \Rightarrow \quad (\nabla \times \mathbf{u}) \times \mathbf{u} = -\nabla \left( \frac{P}{\rho} + \frac{u^2}{2} \right) \]
Analogy to hydrodynamics

Isomorphism:

\[ u \Leftrightarrow B \]
\[ \nabla \times u \Leftrightarrow \nabla \times B \]

Magnetic field lines may twist in the absence of current.

Fluid streamlines may twist in the absence of vorticity.
First stellarator

- Achieved $T = 0.5$ MK
Tokamak breakthrough: 1968

IAEA Fusion Energy Conference, Novosibirsk 1968:

- Soviet tokamak programme
- $T_e$ up to 1 keV!

First dismissed, but later confirmed by UK scientists.
Tokamak genealogy
Tokamaks

ASDEX Upgrade
Garching (D)

EAST
Chengdu (C)

JET
Culham (GB)

DIII-D
San Diego (USA)

ITER
Cadarache (F)

KSTAR
Daejon (KR)

JT-60SA
Naka (JA)

Person
Stellarators

- LHD
  Toki (JA)

- Wendelstein 7-X
  Greifswald (D)

- TJ-II Madrid (E)

- Wendelstein 7-AS
  Garching (D)

- HSX
  Madison (USA)
ITER

- Built in CEA Cadarache
  - 1st Plasma: 2025
  - EU, USA, China, India, Japan, South Korea, Russia

Parameters

R \( [m] \) \quad 6,2
a \( [m] \) \quad 2,0
\( T_{\text{Puls}} \) \( [s] \) \quad 300
\( P_{\text{Fusion}} \) \( [MW] \) \quad 500
Energy multiplication (Q) \quad > 10
Cost \( [\text{Mrd} \, \€] \) \quad 15
The fusion triple product

Let $n = \text{density}$, $T = \text{temperature}$, $\tau = \text{energy confinement time}$

$$\tau = \frac{\overline{\text{thermal plasma energy}}}{\overline{\text{heating power}}}$$

Energy density $\sim nT$

Rate of collisions $\sim n^2$

Fusion yield per collision $\sim T^2$

Energy production $> \text{energy loss}$ if $n^2 T^2 > nT/\tau$

$nT\tau > 5 \text{ atmosphere seconds}$

Thus we need
- high density
- high temperature ($\sim 100 \text{ million degrees}$)
- long confinement time
Confinement time

\[ \tau_E \propto A^{0.40} I^{0.90} P^{-0.65} R^{1.90} a^{0.20} \kappa^{0.80} B^{0.05} n^{0.30} \]

Measured \( \tau_E \) in s

Großer Radius (m)

ITER

DB2P8=1

\( \tau_E \) from scaling law in s

ITER

JET

ASDEX-U
European Fusion Experiment JET (Joint European Torus)

- Largest and most successful fusion experiment in the world
- Collaborative EU project in Culham near Oxford
- Start of construction in 1978; in use since 1983
- 16 MW fusion power with efficiency of 0.64 for about 1 sec
- 4 MW fusion power with efficiency of 0.2, for about 5 sec.
- Temperatures up to 400 Million °C
- Discharges up to 1 Minute
- Plasma current up to 7 Million Amperes
The Tokamak JET from the outside
JET: the world’s largest tokamak (still)

- a high performance DT shot on JET
- achieved the following performance for a pulse lasting several seconds (at $B_\phi = 3.6$ T and $P_h = 25$ MW)

<table>
<thead>
<tr>
<th>density</th>
<th>ion temperature</th>
<th>electron temperature</th>
<th>average $\beta$</th>
<th>confinement time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n(0)$</td>
<td>$T_i(0)$</td>
<td>$T_e(0)$</td>
<td>$\langle\beta\rangle \simeq \beta(0)/3$</td>
<td>$\tau_E$</td>
</tr>
<tr>
<td>$0.4 \times 10^{20}$ m$^{-3}$</td>
<td>28 keV</td>
<td>14 keV</td>
<td>0.018</td>
<td>0.9 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\langle p \rangle \tau_E$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.84 atm s</td>
</tr>
</tbody>
</table>
Some tokamak theory

Consider a tokamak with inverse aspect ratio

\[ \epsilon = \frac{r}{R} \ll 1 \quad \text{and} \quad q = \frac{r B}{R B_\theta} \geq 1 \]

The magnetic field is then approximately toroidal

\[ B \approx F \nabla \varphi \quad \Rightarrow \quad B \approx \frac{F}{R} = B_0 (1 - \epsilon \cos \theta) \]

Particles at \( \theta = \pi/2 \) are trapped if

\[ \mu B_0 (1 + \epsilon) > \frac{mv^2}{2} \]

\[ \left( \frac{v_{||}}{v} \right)^2 < \epsilon \]

trapped-particle fraction \( \sim \sqrt{\epsilon} \)
Random walk and diffusion

If random steps are taken with

\[ \text{step length } = \Delta x \]

\[ \text{time between steps } = \Delta t \]

the result after a large number of steps is diffusion.

The distribution function satisfies

\[ \frac{\partial f}{\partial t} = D \frac{\partial^2 f}{\partial x^2} \]

where

\[ D = \frac{\Delta x^2}{2\Delta t} \]
Neoclassical transport

Banana orbits in a tokamak

Effective collision frequency for trapped particles:

- enhanced because velocity vector needs only be scattered slightly

\[ \nu_{\text{eff}} \sim \frac{\nu}{\epsilon} \]
Neoclassical transport in tokamaks

Random-walk estimate

\[ D_{\text{banana}} \sim \epsilon^{1/2} \nu_{\text{eff}} \delta r^2 \sim \frac{q^2}{\epsilon^{3/2}} \nu \rho^2 \sim 10^2 \nu \rho^2 \]

fraction of trapped particles

Proper theory obtained by solving the drift kinetic equation by expansion in \( \rho_* \ll 1 \)

\[ \frac{\partial f}{\partial t} + (v_\parallel + v_d) \cdot \nabla f = C(f) \]

\[ v_\parallel \nabla_\parallel f_0 = C(f_0) \Rightarrow f_0 = n(\psi) \left( \frac{m}{2\pi T(\psi)} \right)^{3/2} e^{-mv^2/2T(\psi)} - e^{\phi_0/T(\psi)} \]

\[ v_\parallel \nabla_\parallel f_1 - C(f_1) = -v_d \cdot \nabla f_0 \]

Particle flux

\[ \langle \Gamma \cdot \nabla \psi \rangle = \left\langle \int f_1 v_d \cdot \nabla \psi d^3v \right\rangle \]
Neoclassical transport (of electrons) in stellarators

Random walk due to bad orbits:

- Step length
  \[ \Delta r \sim v_d \Delta t \]

- Time between steps
  \[ \Delta t \sim \frac{\epsilon_h}{\nu} \]

- Diffusion coefficient
  \[ D_{1/\nu} \sim \epsilon_h^{1/2} \frac{\Delta r^2}{\Delta t} \sim \frac{\epsilon_h^{3/2} v_d^2}{\nu} \]

- Large at high temperatures, since
  \[ D_{1/\nu} \propto \frac{m^{1/2} T^{7/2}}{n B^2 R^2} \]
Other collisionality regimes

\[ \nu_\ast = \frac{\nu R}{v_T} \sim \frac{R}{\lambda} \]

\[ \lambda = \text{mean free path} \]
Stability

- Economy of magnetic confinement measured by

\[ \beta = \frac{\langle p \rangle}{\langle B^2 \rangle / 2\mu_0} \]

- Usually limited in tokamaks to a few percent.

- Toroidal plasma current limited by kink modes

\[ q = \left. \frac{rB_\varphi}{RB_\theta} \right|_{\text{edge}} > 2 \]

- Disruptions when the density or current becomes too large.
Tokamak disruptions

- A stability limit is reached.
- Plasma interacts with wall and cools quickly.
- Resistivity increases, $\eta \sim T_e^{-3/2}$
- Large electric field is induced, trying to maintain the plasma current.
- Runaway electrons are generated, which
  - are accelerated to ~20 MeV.
  - carry much of the original current.
  - usually hit the wall => hard X-rays.
  - can cause serious damage.
  - occasionally remain in the cool plasma (~10 eV) for several s.

(JET: Gill et al, Nucl. Fusion, 2000)
The pressure is limited by equilibrium, not stability

- Shafranov shift
- Ergodisation

No disruptions

No Greenwald density limit

$n_e(0)$ up to $10^{21}\text{m}^{-3}$ in LHD
Challenges

- **Stability**: when the pressure or current is high enough, the plasma tends to become unstable
  - require strong magnetic fields

- **Turbulence**: limits the energy confinement for a device of given size
  - need strong magnetic field or large device

- **Power loading**: high volume to area ratio leads to high wall heat flux.

- **Neutron activation**: materials must withstand high neutron fluxes
Optimisation

The (vacuum) magnetic field is defined by the shape of the boundary

\[ \mathbf{B} = \nabla V \]
\[ \nabla^2 V = 0 \]
\[ \mathbf{B} \cdot \mathbf{n} = 0 \]

Find current distribution on a surface that produces the desired interior field

Magnetic field coils

Plasma

Magnetic field line
Wendelstein 7-X

Optimisation criteria:

• good nested magnetic surfaces
• good finite-pressure equilibrium
  • minimized plasma current
• good magnetohydrodynamic stability properties
• modular coils
• small transport
Particle orbits in Wendelstein 7-X
Wendelstein 7-X

Plasma volume ~ 30 m³
- major radius 5.5 m
- effective minor radius 0.55 m

Magnetic field 3 T
- magnetic energy 900 MJ

Superconducting coils
- 50 non-planar coils
- 20 planar coils

Pulse duration 30 minutes

Heating systems
- 8 MW ECRH (140 GHz)
- 5-10 MW NBI
- 2 MW ICRH (35 MHz)

Actively cooled plasma facing components
- up to 10 MW/m²
Wendelstein 7-X from inside out

Shape of the magnetic surface from optimisation
Heat-load target elements

Actively cooled target elements: 10 MW/m²
Heat-load target elements

Actively cooled target elements: 10 MW/m²
Plasma vacuum vessel

Vacuum ~ $10^{-6}$ mbar

Development of welding technologies for 3D-shaped vessel
Magnetic field coils

Magnetic field 3T (7T)
Superconducting non-planar coils with cable-in-conduit conductor (NbTi)
Coil support structure

High precision ~ mm
Large forces ~ 100 t
Cryostat vessel

Cryogenic temperatures (LHe)
Assembly 2011
Visualising magnetic field lines

- Evacuated plasma vessel \(<10^{-5}\) mbar and magnetic field of \(>50\) mT

- Cathode inserted to emit electron beam along the magnetic field

- Optical detection of the e-beam by interaction with
  - background gas
  - fluorescent rod intersecting the beam: Poincaré-plot

- Sweeping electron emitter and fluorescent rod detector
Magnetic field lines: July 2015

Electron beam propagates ~ 1.5 km around the torus

- Emission from N$_2$ and H$_2$O
- Field components measured to an accuracy of 10$^{-5}$
Fusion triple product

- Figure of merit for fusion plasmas:

![Graph showing the figure of merit for fusion plasmas with data points labeled for late 2017 and late 2018. The graph includes various symbols representing different experiments such as JT-60, JET, TFTR, DIII-D, EAST, KSTAR, Tore Supra, LHD, W7-X, and NSTX. The x-axis represents the duration (s) ranging from 0.1 to 10^8, and the y-axis shows the product of n_i(0)T_i(0)/n_e(1keV-m^-3-s). The graph also includes time references such as hour, day, month, and year.]
Summary

• Quest for fusion energy dominated by two concepts
  • Tokamak (axisymmetric, leading contender)
  • Stellarator (3D, trying to catch up)

• ITER should demonstrate net energy gain (Q>10) in the tokamak

• Other superconducting tokamaks underway
  • Japan, China, Korea
  • MIT

• Wendelstein 7-X will test stellarator optimisation on a smaller scale.
  • First results promising