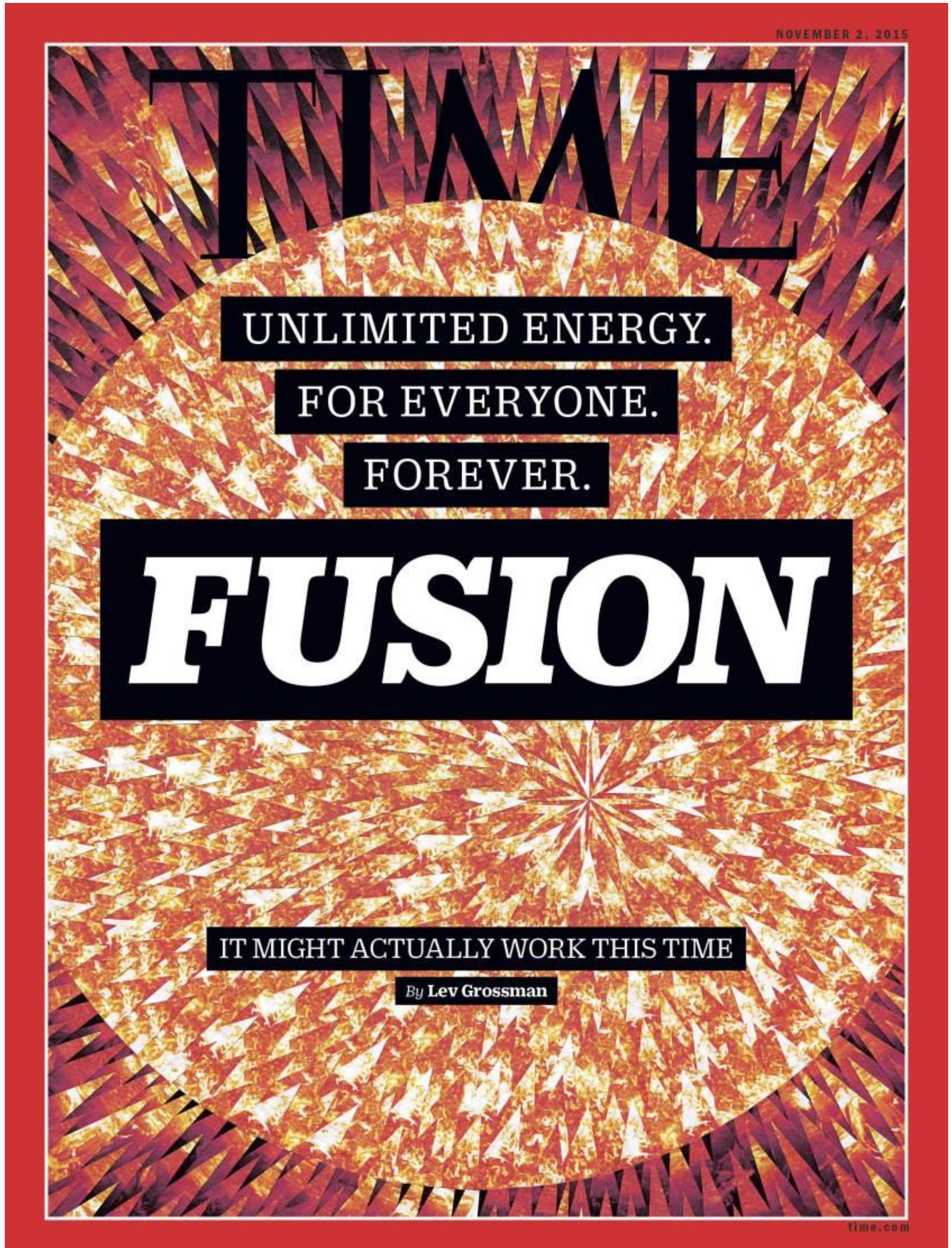
Tokamaks and stellarators

Per Helander
Max Planck Institute for Plasma Physics, Greifswald


NOVEMBER 2, 2015
TIME
UNLIMITED ENERGY.
FOR EVERYONE.
FOREVER.
FUSION
IT MIGHT ACTUALLY WORK THIS TIME
By Lev Grossman
time.com

New York Times, 25 March 1951

PERON ANNOUNCES NEW WAY TO MAKE ATOM YIELD POWER

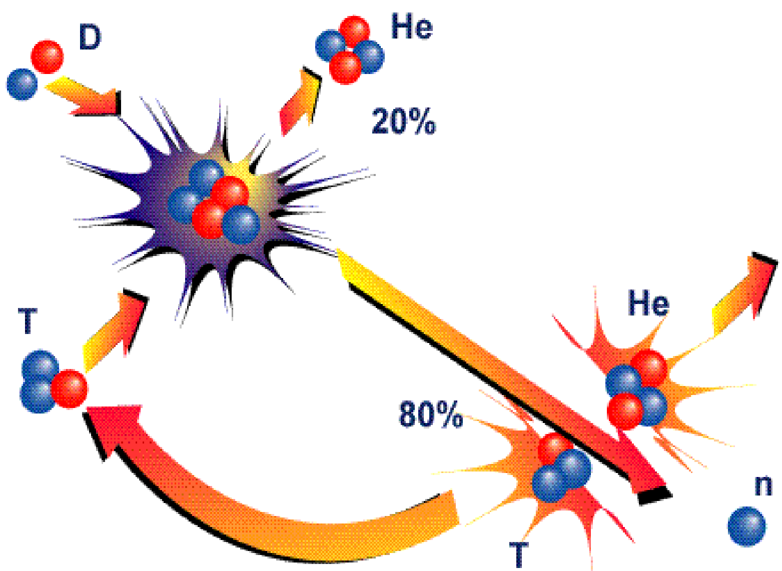
- **Reports Argentina has devised thermonuclear reaction that does not use uranium**

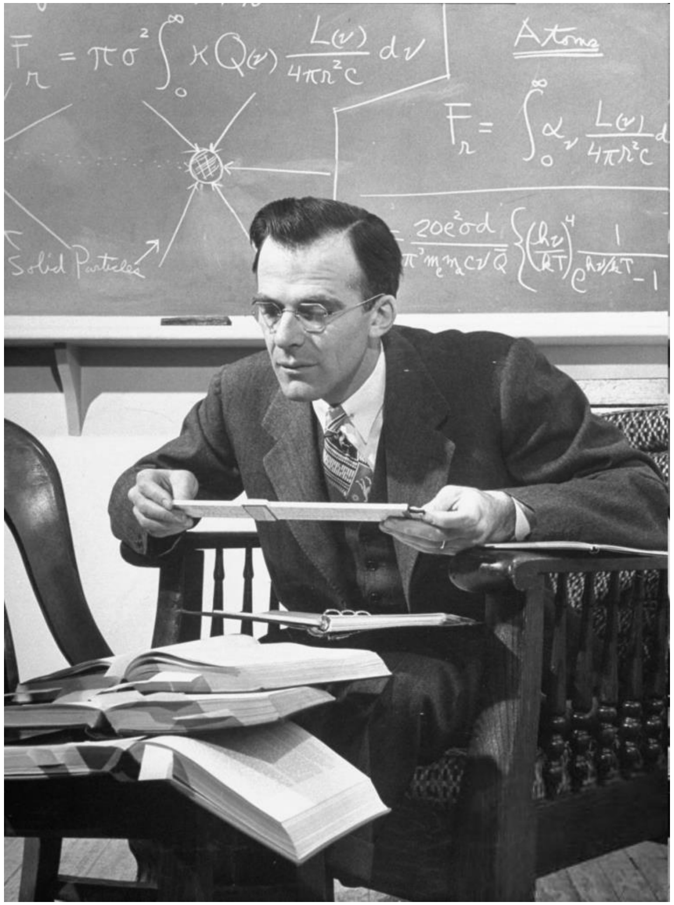
- News conference by Argentinian dictator Juan Peron:
  - Argentina had built a fusion pilot plant
  - The reaction required “enormously high temperatures of millions of degrees“
- Met with skepticism from Fermi and other leading physicists

Lyman Spitzer Jr, 1914-1997

- Head of Princeton Astrophysics (36)

- Pioneered the study of the interstellar medium



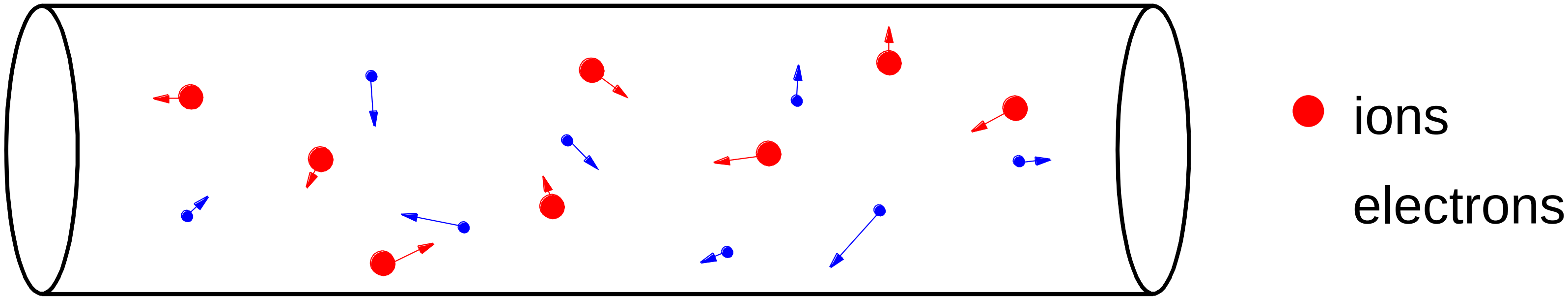


$^{6}Li + n \rightarrow {}^{4}He\ (2.05 MeV) + T\ (2.73 MeV)$

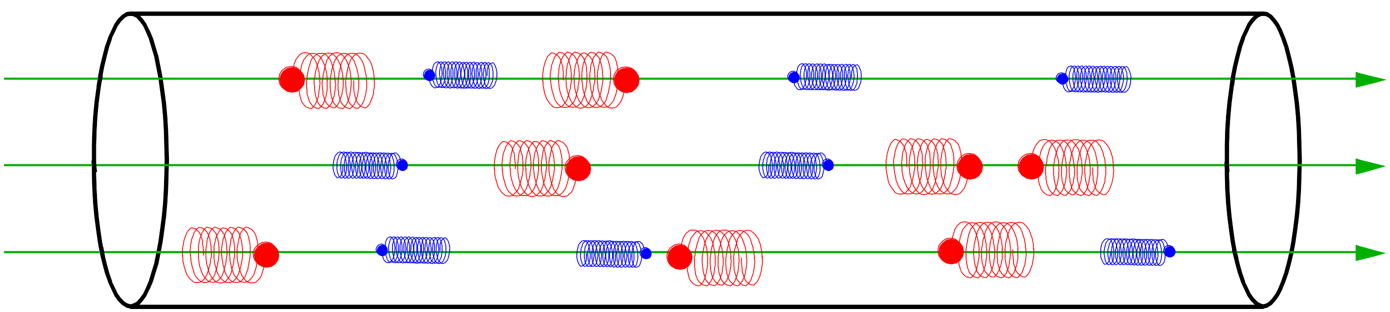
$^{7}Li + n \rightarrow {}^{4}He + T + n - 2.47\ MeV$

Magnetic confinement

without magnetic field



with magnetic field



Motion in an inhomogenous magnetic field

- If **B** varies slowly on the length scale of the gyroradius ρ

$$\rho_* = \rho/R \ll 1, \qquad (R^{-1} = |\nabla \mathbf{B}|/B)$$

the particles drift slowly across the field

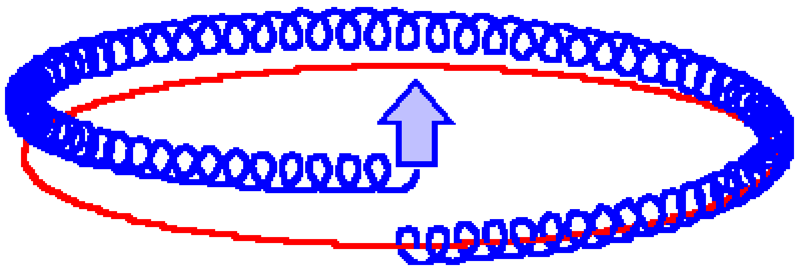
$$\mathbf{r} = \mathbf{R} + \vec{\rho},$$

$$\dot{\mathbf{R}} = v_\parallel \mathbf{b} + \frac{v_\perp^2/2 + v_\parallel^2}{\Omega} \mathbf{b} \times \nabla \ln B = v_\parallel \mathbf{b} + \mathbf{v}_d \qquad \mathbf{b} = \mathbf{B}/B$$

- Lagrangian (Taylor 1964)

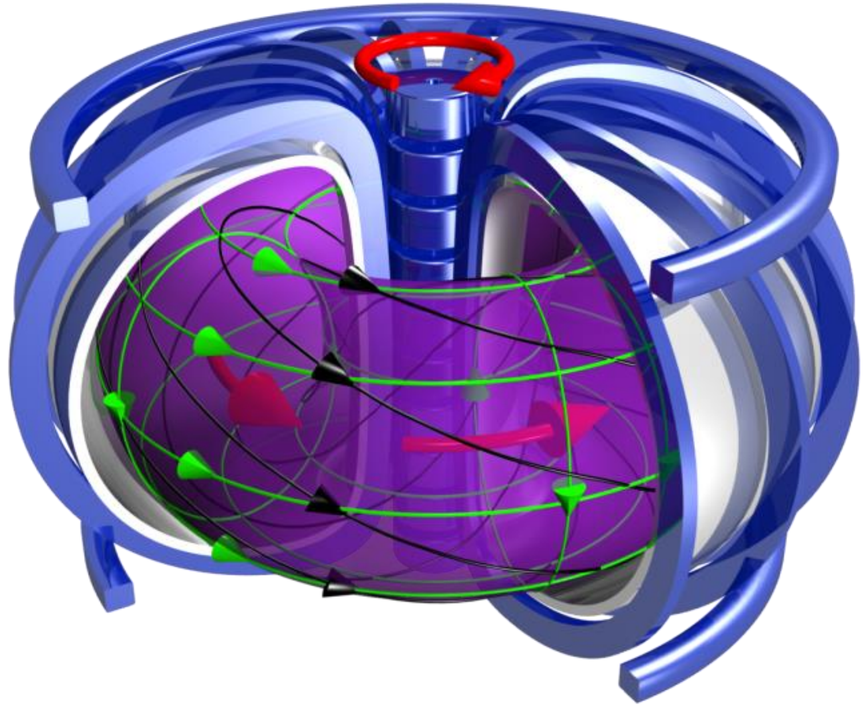
$$L = \frac{m v_\parallel^2}{2} + e\mathbf{A}\cdot\dot{\mathbf{R}} - \mu B$$

$$\mu = \frac{m v_\perp^2}{2B} = \text{adiabatic invariant}$$



The tokamak

Usually credited to Tamm and Sakharov (1951), but also conceived by others (Spitzer, Schlüter, …).



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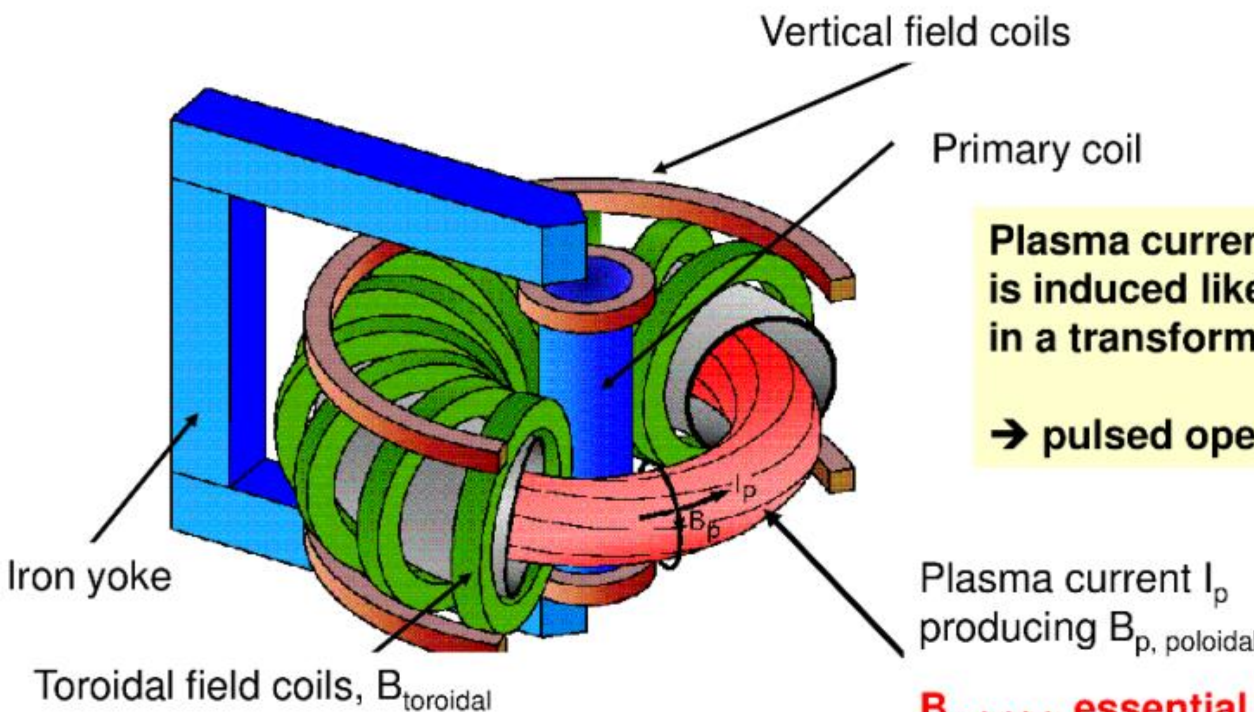
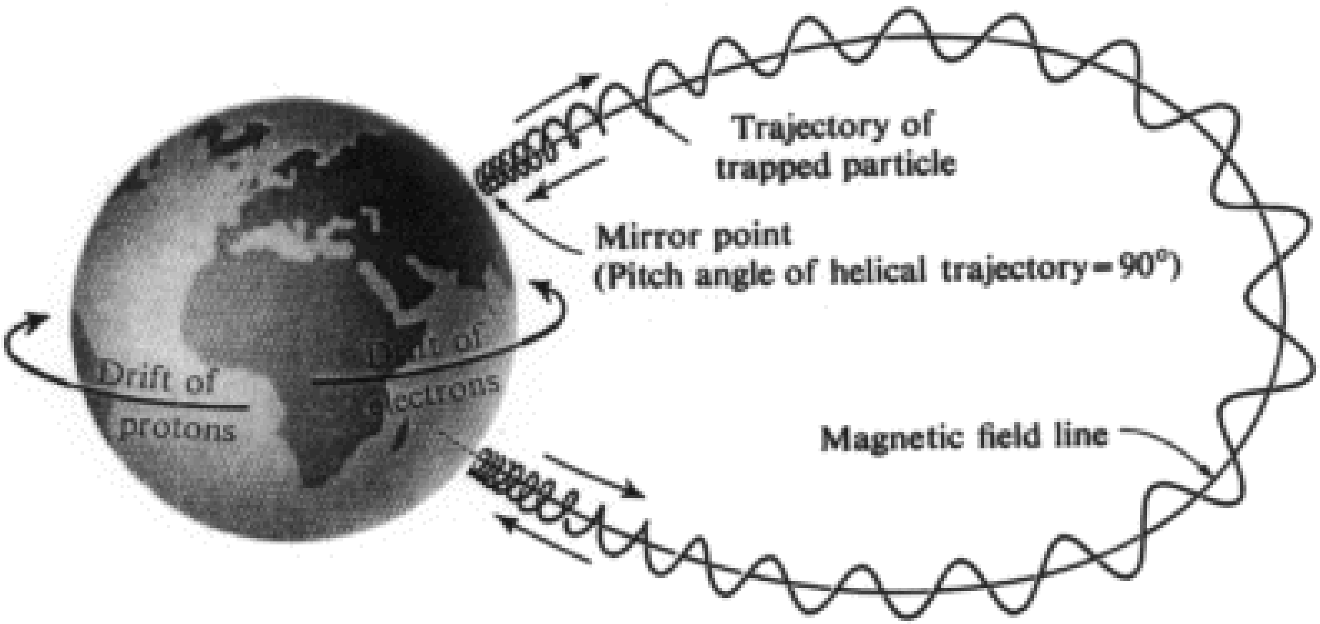


Plasma current is induced like in a transformer

→ pulsed operation

$B_{poloidal}$, essential for confinement

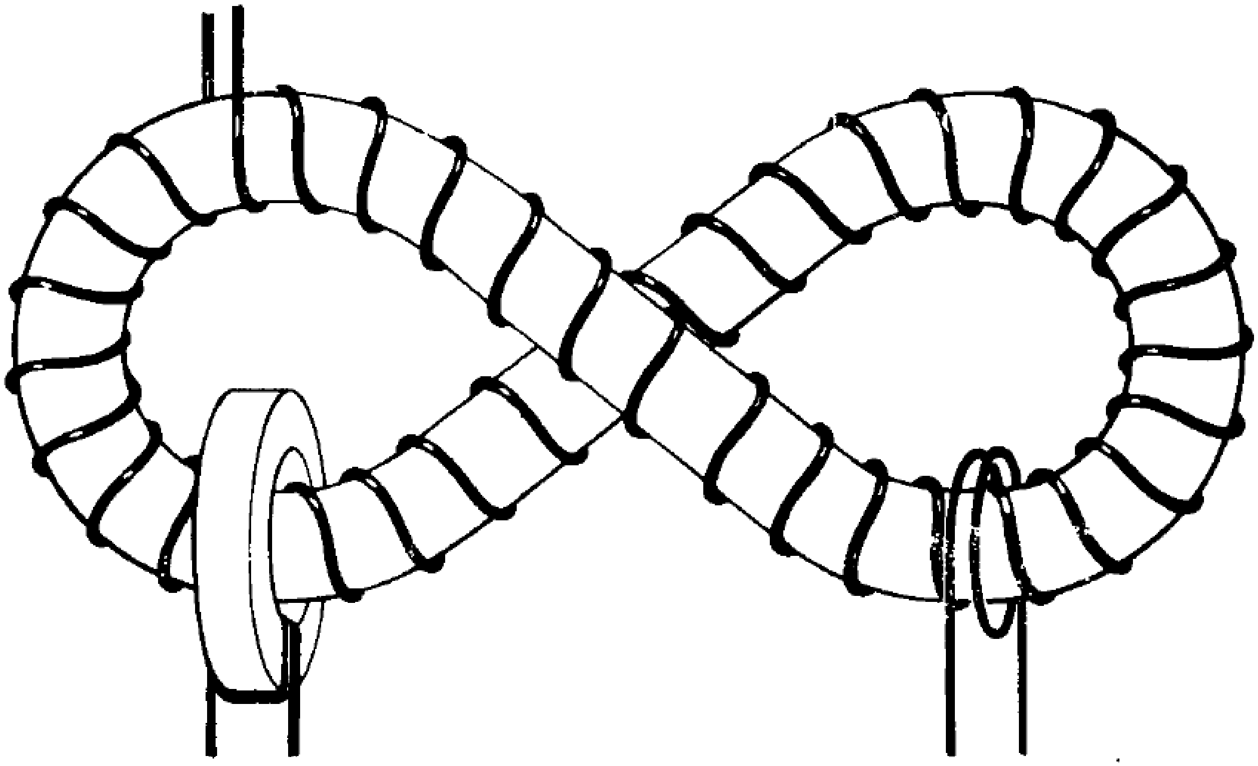
Mirroring



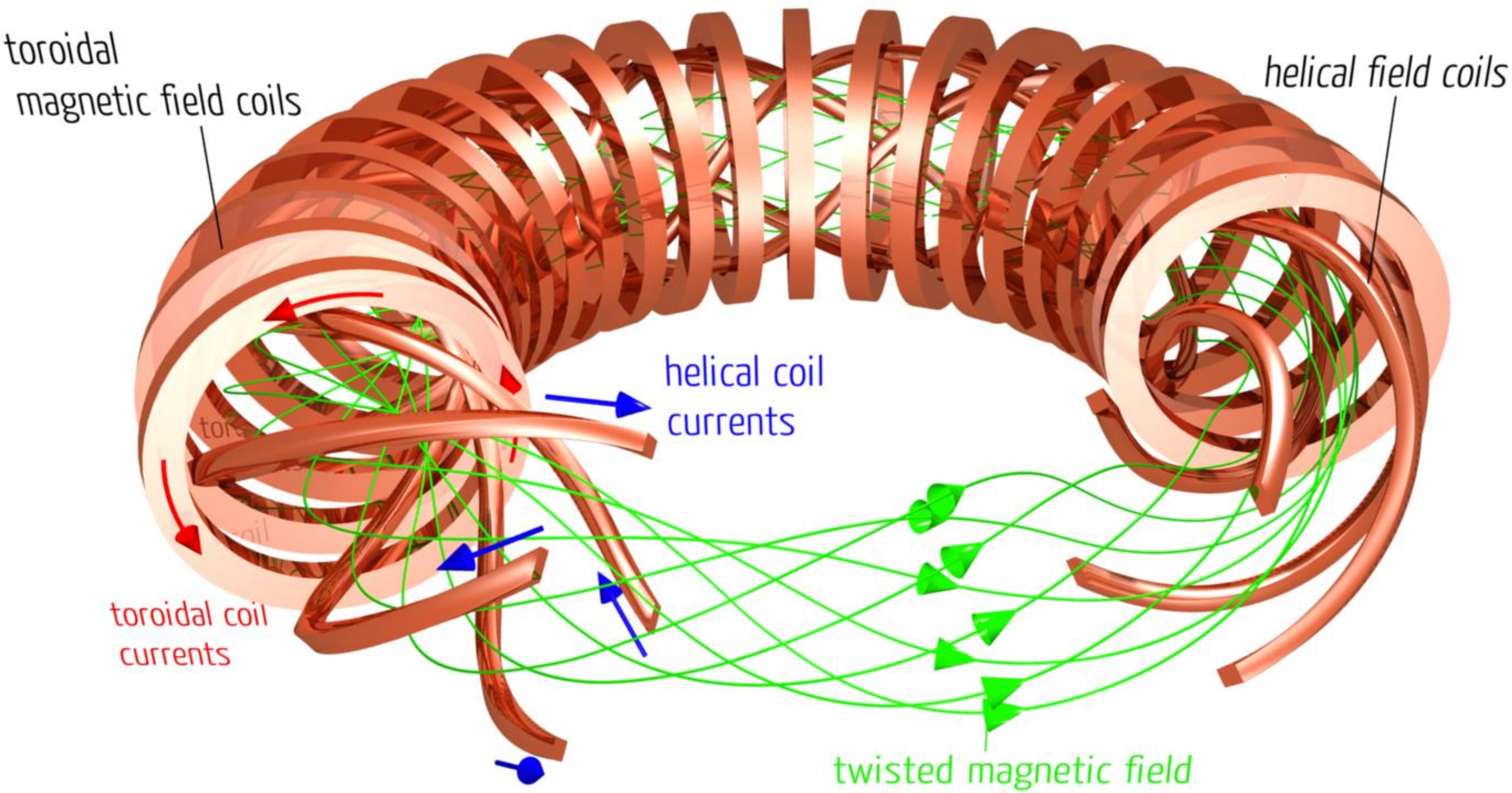
$$E = \frac{mv_\parallel^2}{2} + \mu B = \text{constant}, \qquad \mu = \frac{mv_\perp^2}{2B}$$

$$\Rightarrow \quad B < \frac{E}{\mu}$$

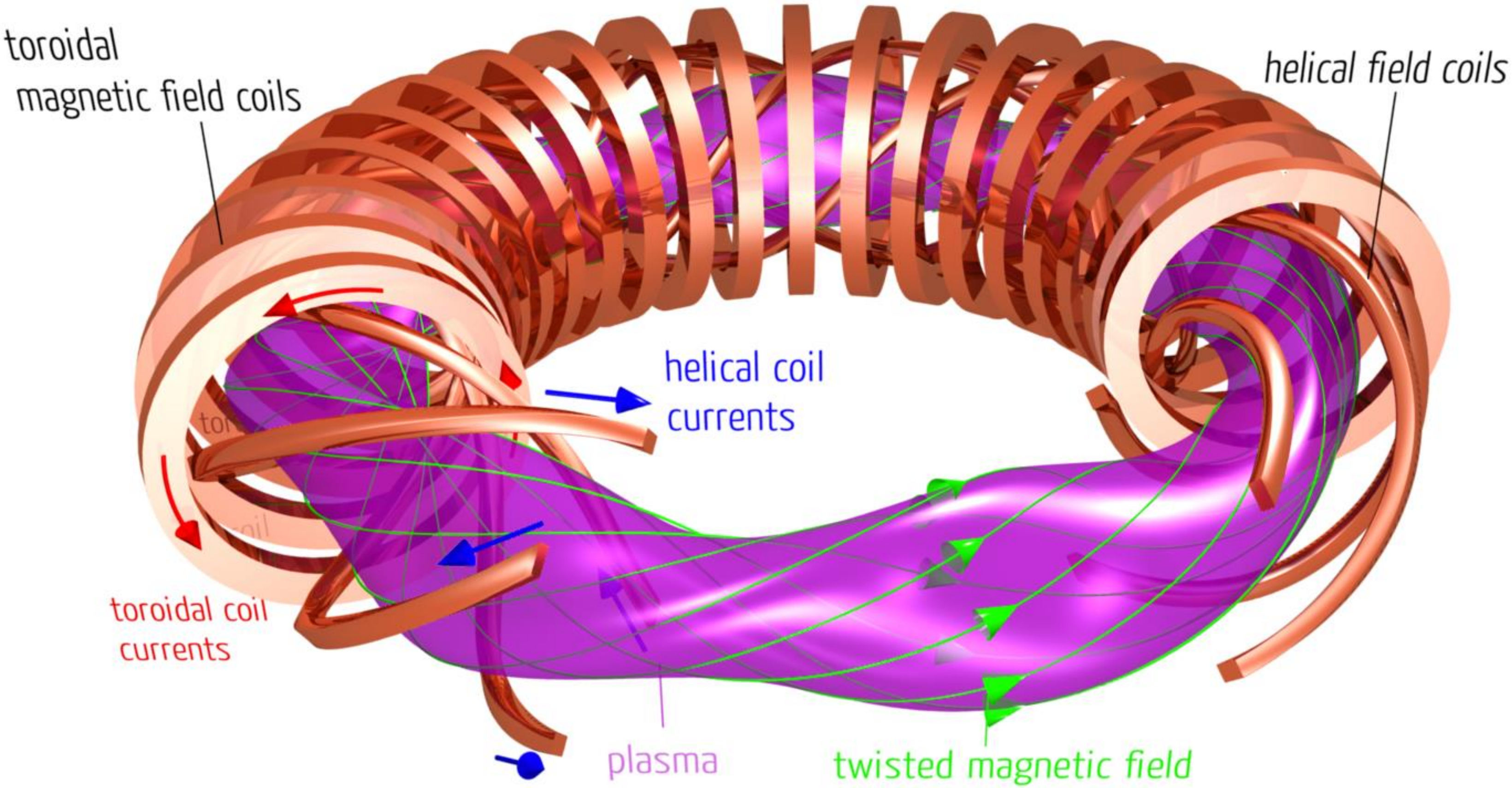
Spitzer‘s first idea



The classical stellarator

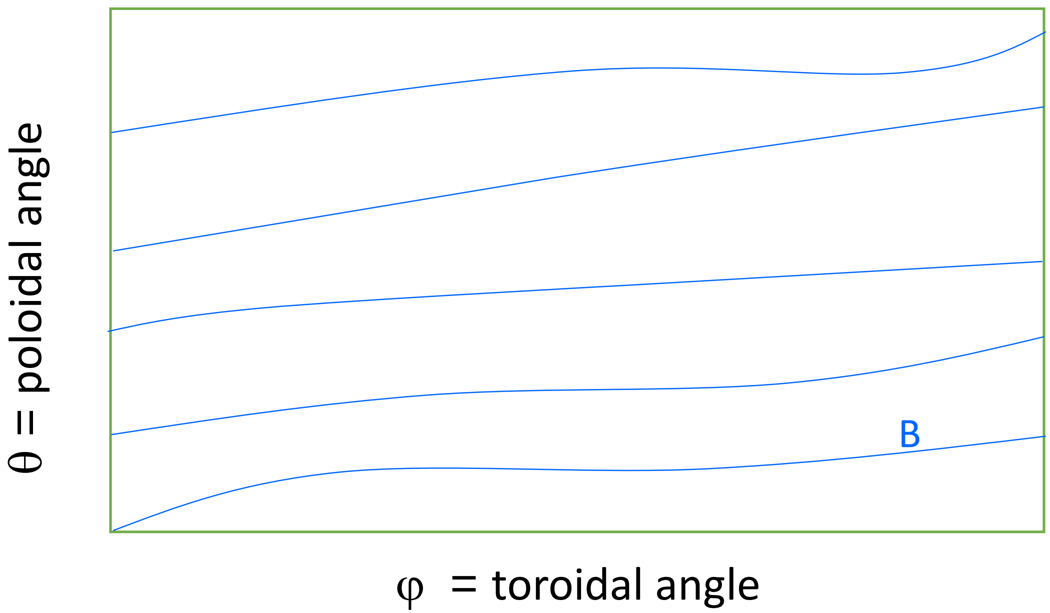


The classical stellarator



Content slide w/o machine logo

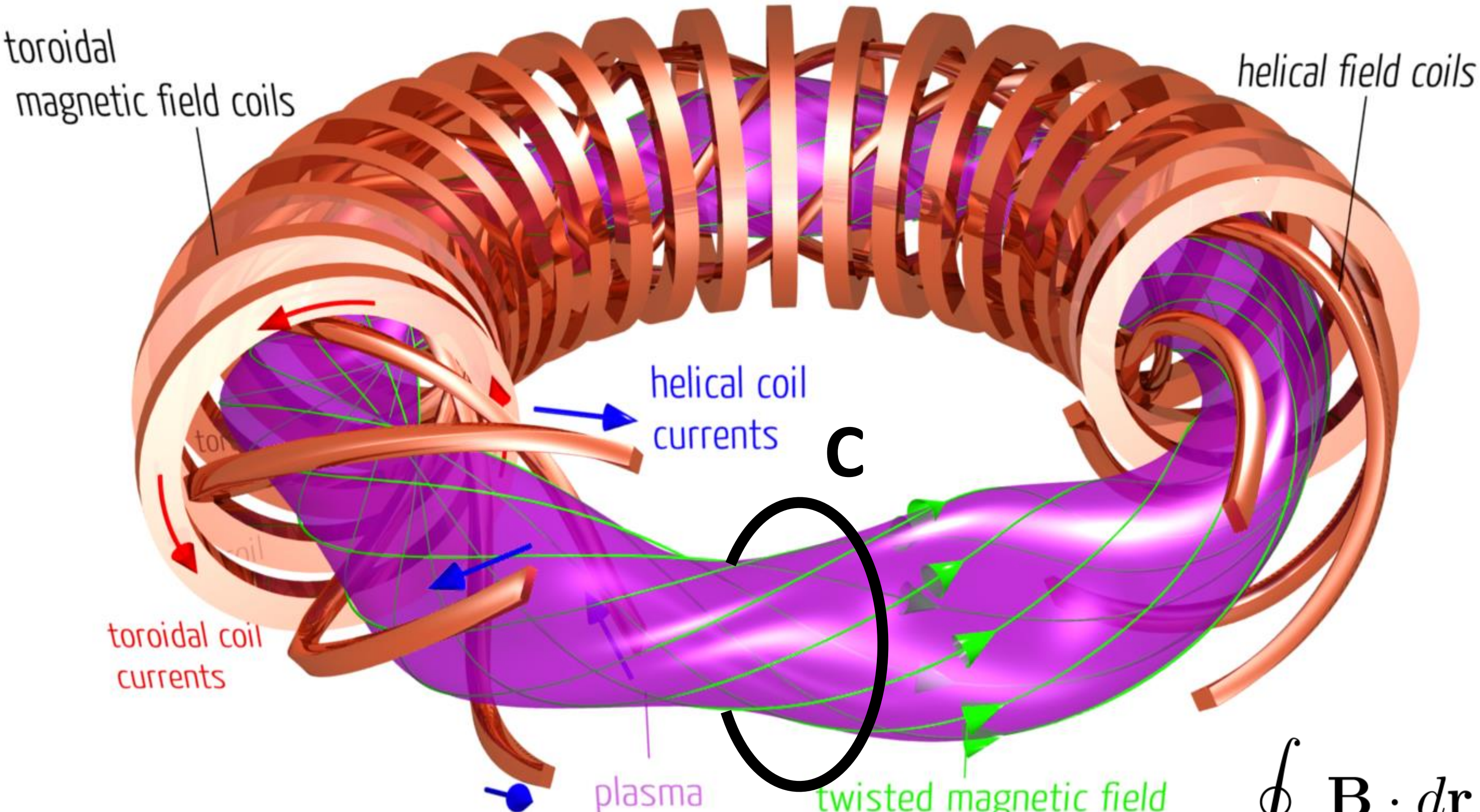
- “Unwrap“ the surface, and plot consider the field lines:



- The rotational transform is defined as the winding number

$$\iota = \lim_{l \to \infty} \frac{\theta(l)}{\varphi(l)}$$

Ampère‘s law



$$\oint_C \mathbf{B} \cdot d\mathbf{r} = 0$$

Analogy to hydrodynamics

MHD equation of motion:

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V}\right) = \mathbf{J}\times\mathbf{B} - \nabla p$$

Equilibrium without flow:

$$\mathbf{J}\times\mathbf{B} = \nabla p \qquad \Rightarrow \qquad \mathbf{B}\cdot\nabla p = 0 \qquad\qquad (\nabla\times\mathbf{B})\times\mathbf{B} = \frac{\nabla p}{\mu_0}$$

Compare with hydrodynamics:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla P, \qquad \nabla\cdot\mathbf{u} = 0$$

Stationary flows satisfy:

$$\mathbf{u}\cdot\nabla\mathbf{u} = -\nabla(P/\rho) \qquad \Rightarrow \qquad (\nabla\times\mathbf{u})\times\mathbf{u} = -\nabla\left(\frac{P}{\rho} + \frac{u^2}{2}\right)$$

Analogy to hydrodynamics

Isomorphism:

$$\mathbf{u} \Leftrightarrow \mathbf{B}$$

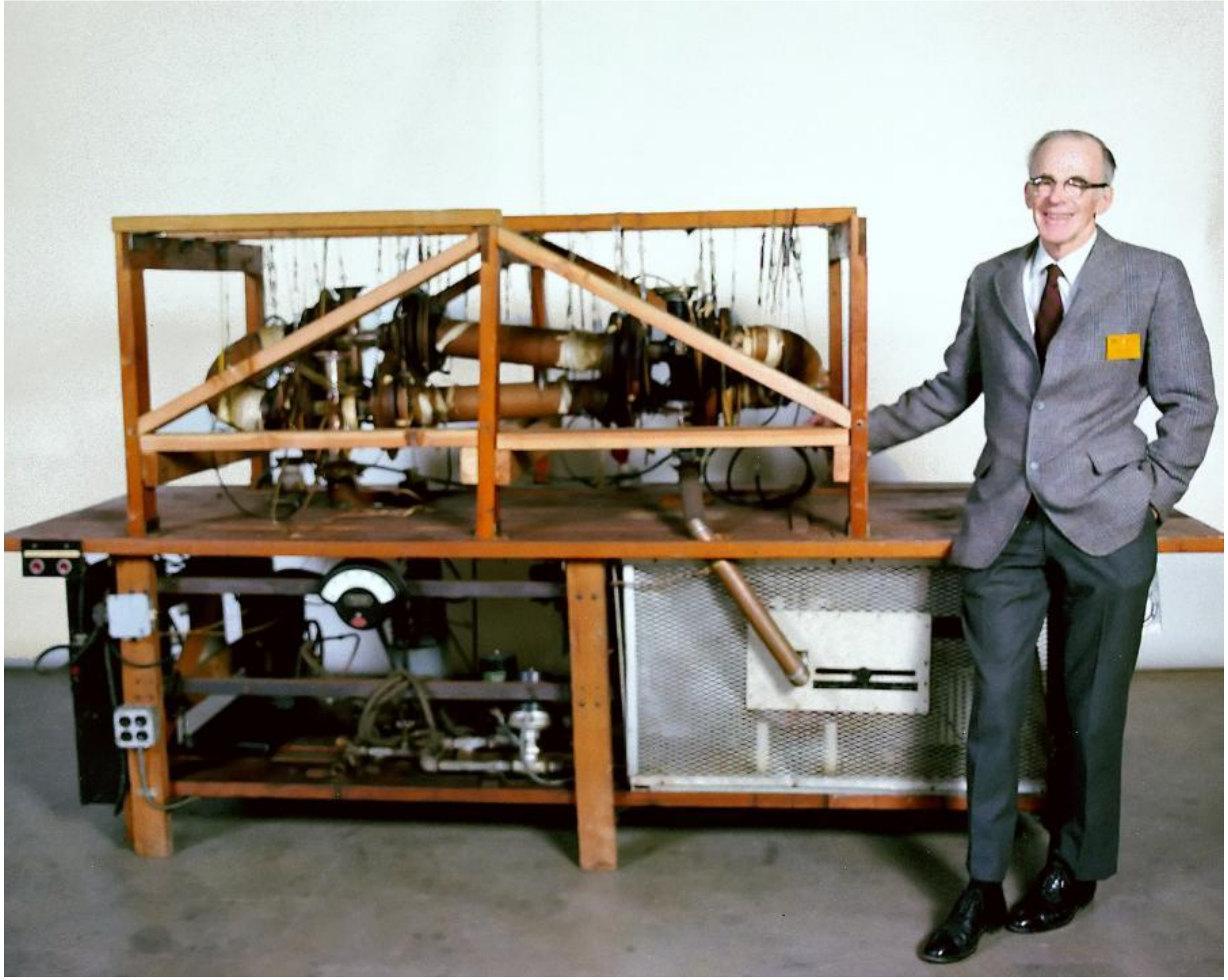
$$\nabla \times \mathbf{u} \Leftrightarrow \nabla \times \mathbf{B}$$

Magnetic field lines may twist in the absence of current.

Fluid streamlines may twist in the absence of vorticity.

First stellarator

- Achieved T = 0.5 MK



Tokamak breakthrough: 1968

IAEA Fusion Energy Conference,
Novosibirsk 1968:

- Soviet tokamak programme
- T_e up to 1 keV!

First dismissed, but later confirmed by UK scientists.

NATURE VOL. 224 NOVEMBER 1 1969

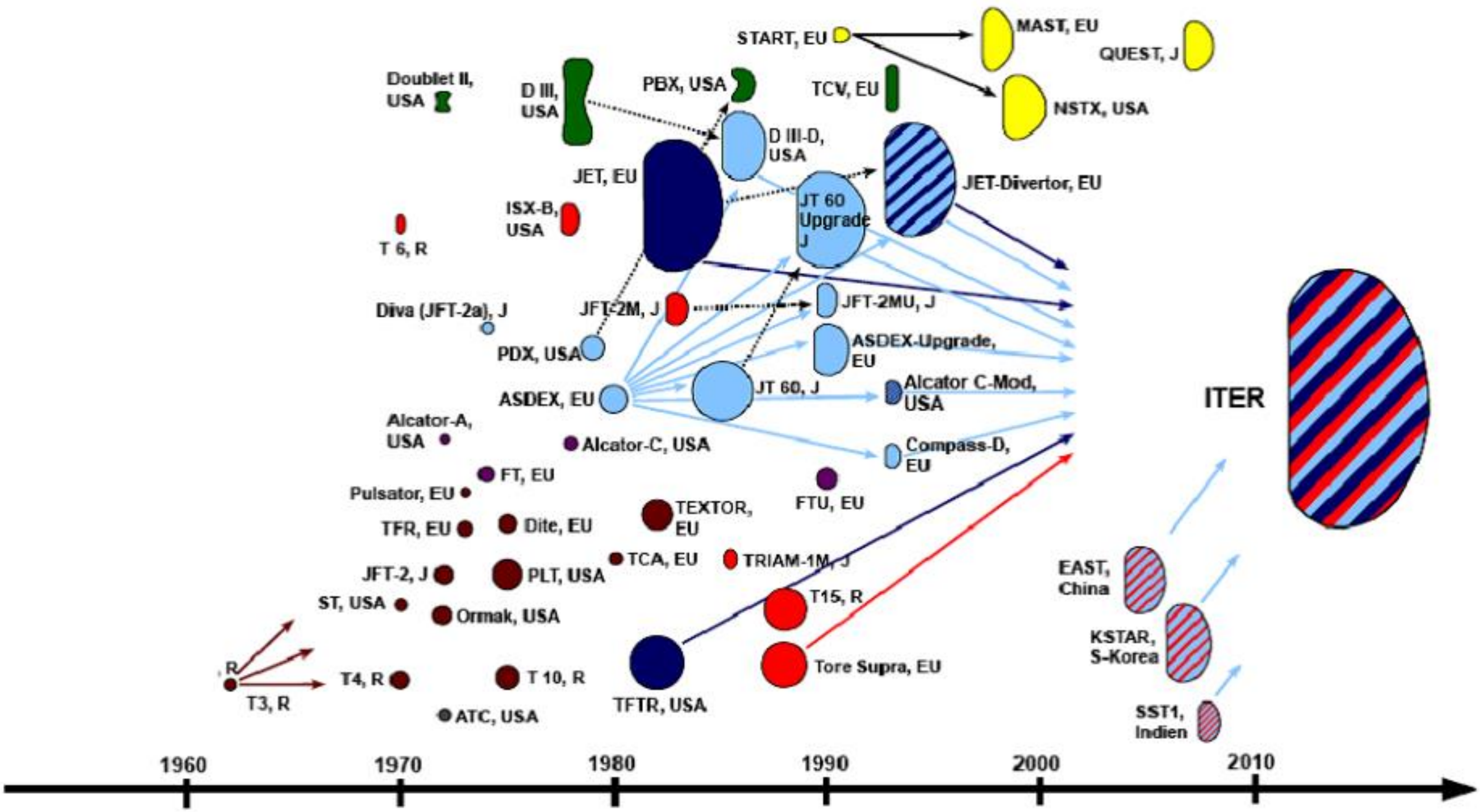
Measurement of the Electron Temperature by Thomson Scattering in Tokamak T3

by
N. J. PEACOCK
D. C. ROBINSON
M. J. FORREST
P. D. WILCOCK
UKAEA Research Group,
Culham Laboratory,
Abingdon, Berkshire

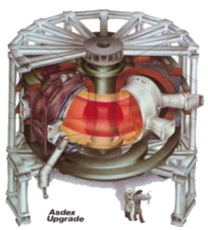
V. V. SANNIKOV
I. V. Kurchatov Institute,
Moscow

Electron temperatures of 100 eV up to 1 keV and densities in the range $1–3 \times 10^{13}$ cm^{-3} have been measured by Thomson scattering on Tokamak T3. These results agree with those obtained by other techniques where direct comparison has been possible.

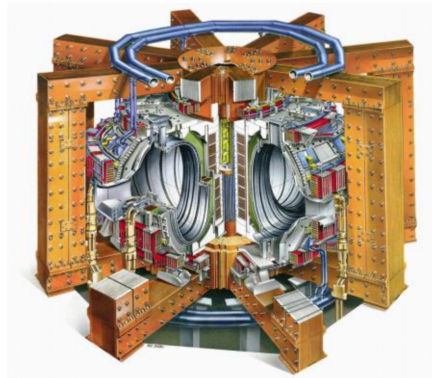
Tokamak geneology



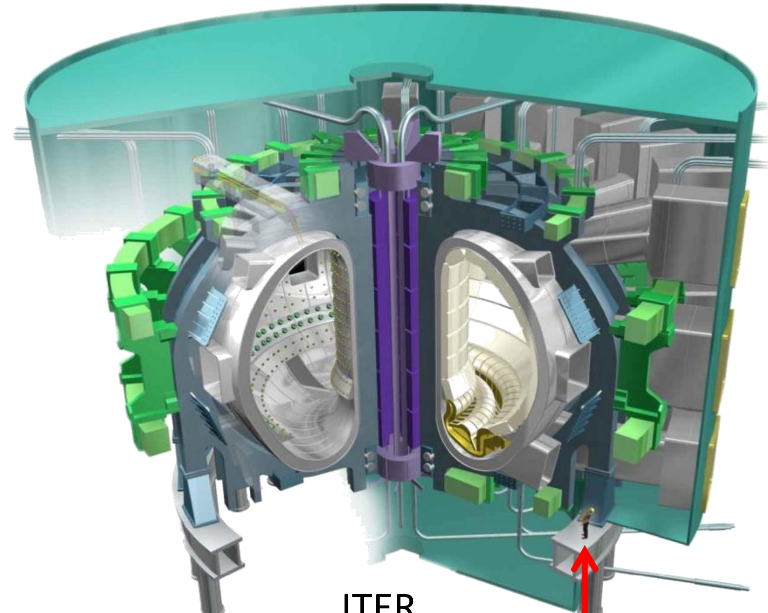
Tokamaks



ASDEX Upgrade
Garching (D)

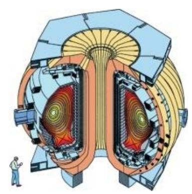


JET
Culham (GB)

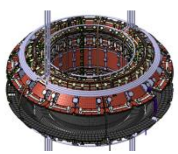


ITER
Cadarache (F)

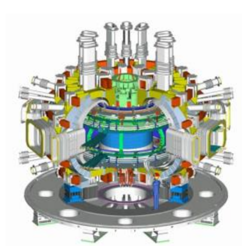
Person



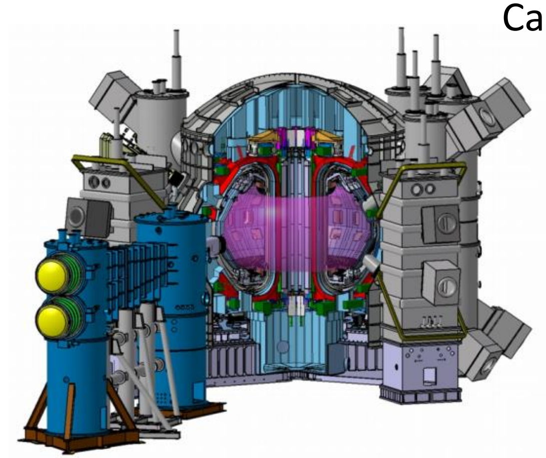
DIIID
San Diego (USA)



EAST
Chengdu (C)

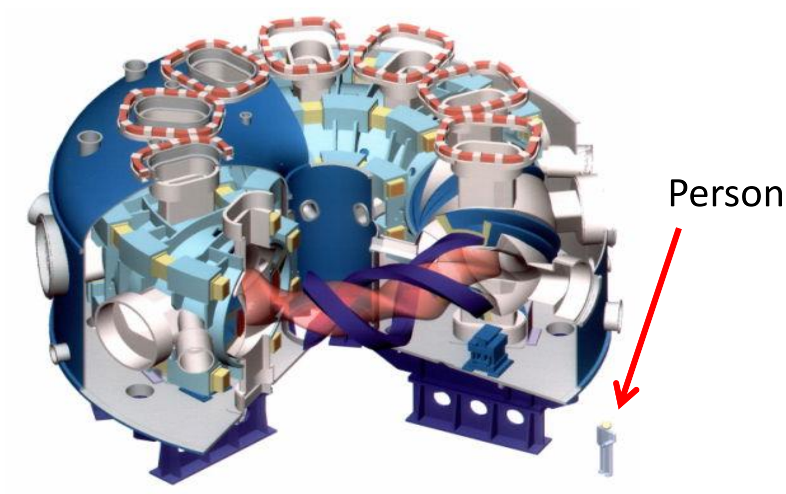


KSTAR
Daejon (KR)

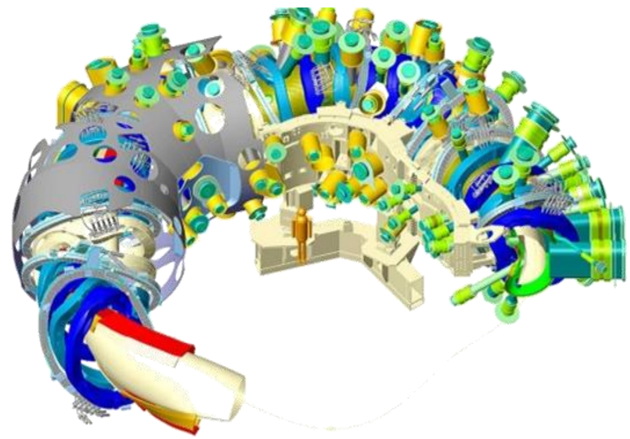


JT-60SA
Naka (JA)

Stellarators

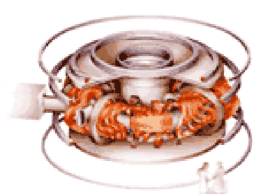


LHD
Toki (JA)

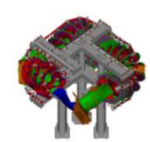


Wendelstein 7-X
Greifswald (D)

TJ-II Madrid (E)



Wendelstein 7-AS
Garching (D)

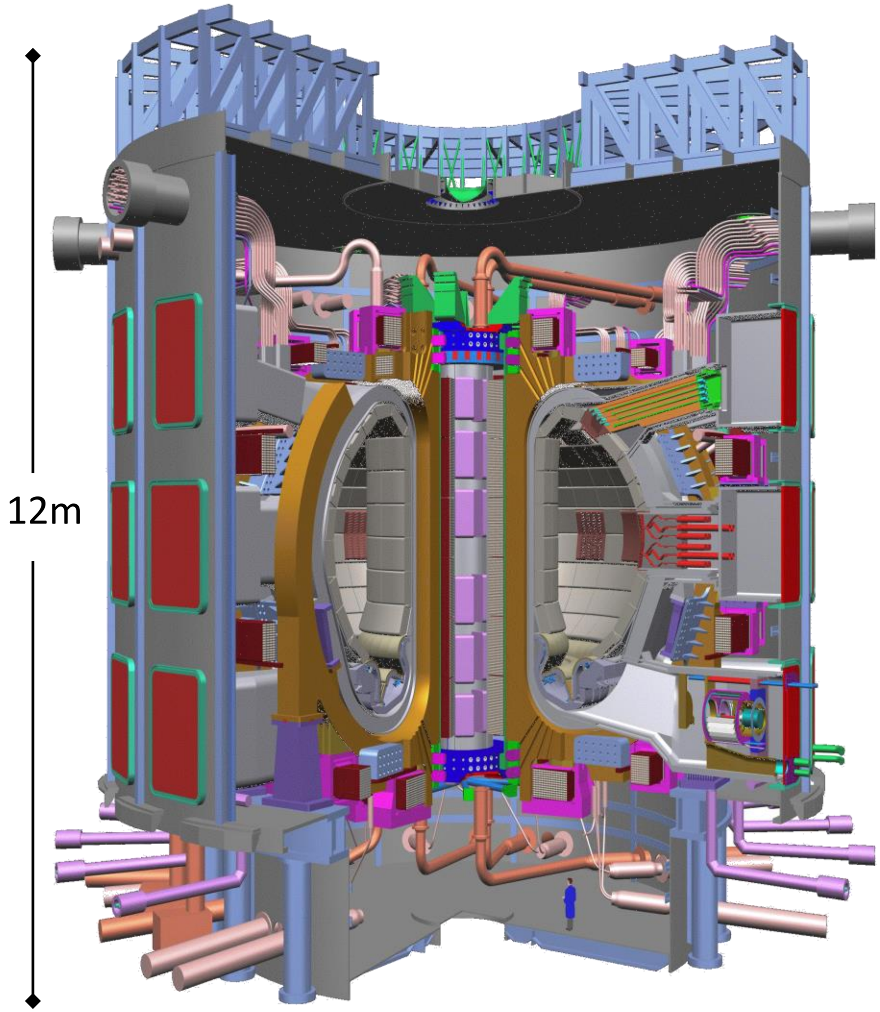


HSX
Madison (USA)

ITER

- Built in CEA Cadarache
  - 1st Plasma: 2025
  - EU, USA, China, India, Japan, South Korea, Russia

Parameters	
R [m]	6,2
a [m]	2,0
T_{Puls} [s]	300
P_{Fusion} [MW]	500
Energy multiplication (Q)	> 10
Cost [Mrd €]	15



The fusion triple product

Let n = density, T = temperature, τ = energy confinement time

$$\tau = \frac{\text{thermal plasma energy}}{\text{heating power}}$$

Energy density ~ nT

Rate of collisions ~ n^2

Fusion yield per collision ~ T^2

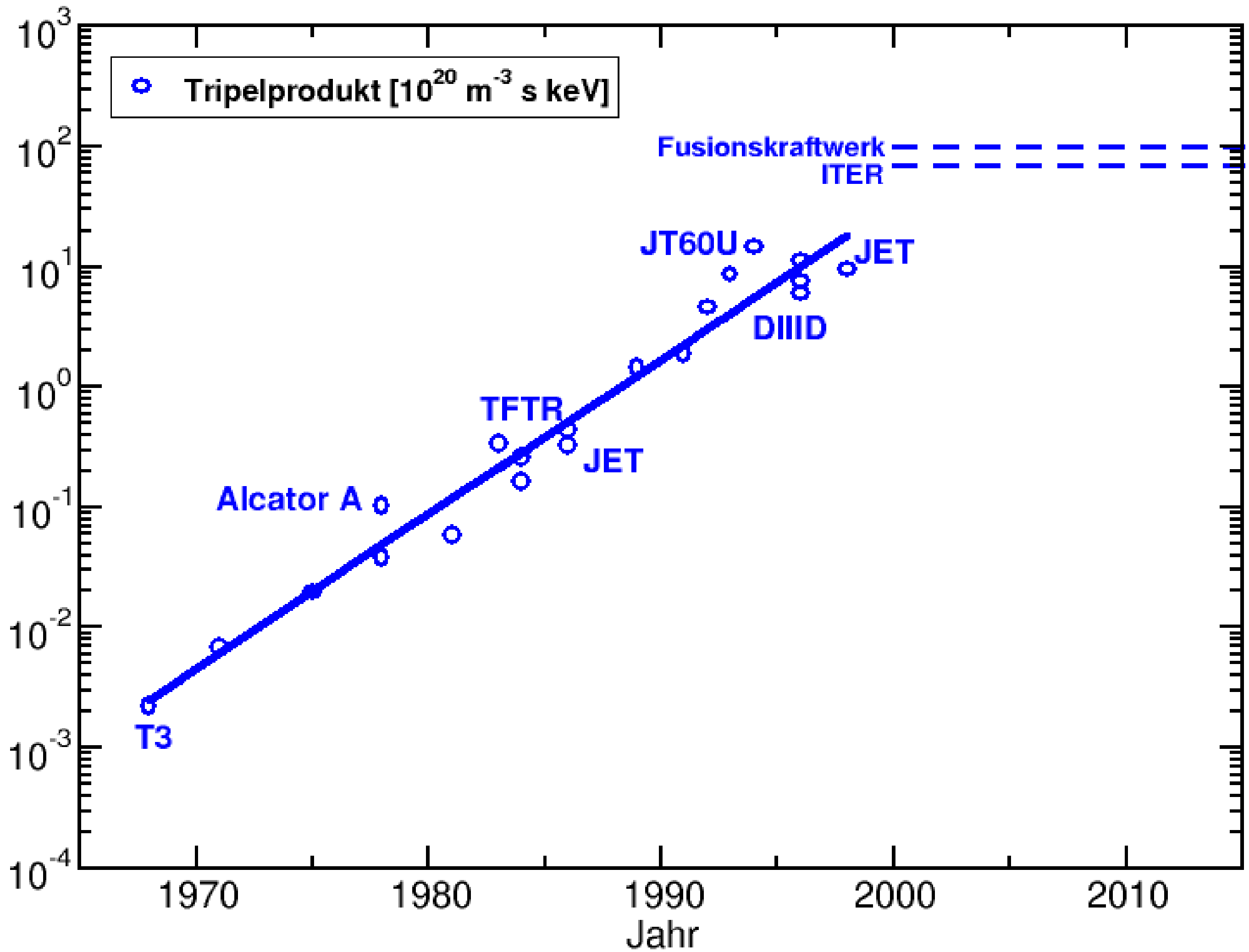
Energy production > energy loss if $n^2 T^2 > nT/\tau$

$$nT\tau > 5 \text{ atmosphere seconds}$$

Thus we need

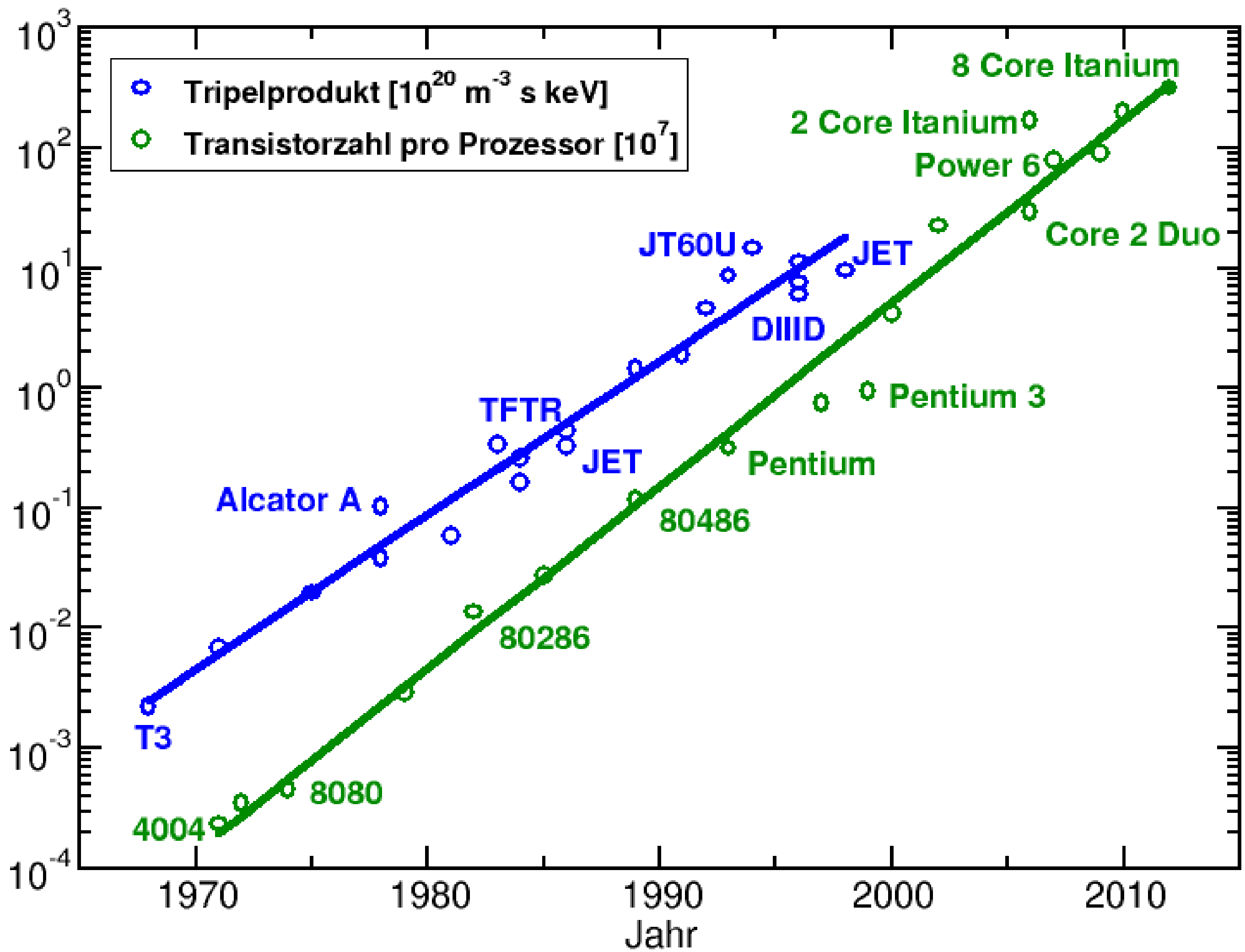
- high density
- high temperature (~100 million degrees)
- long confinement time

Progress



Tripelprodukt [10^20 m^-3 s keV]
Fusionskraftwerk
ITER
JT60U
JET
DIIID
TFTR
JET
Alcator A
T3
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
1970
1980
1990
2000
2010
Jahr

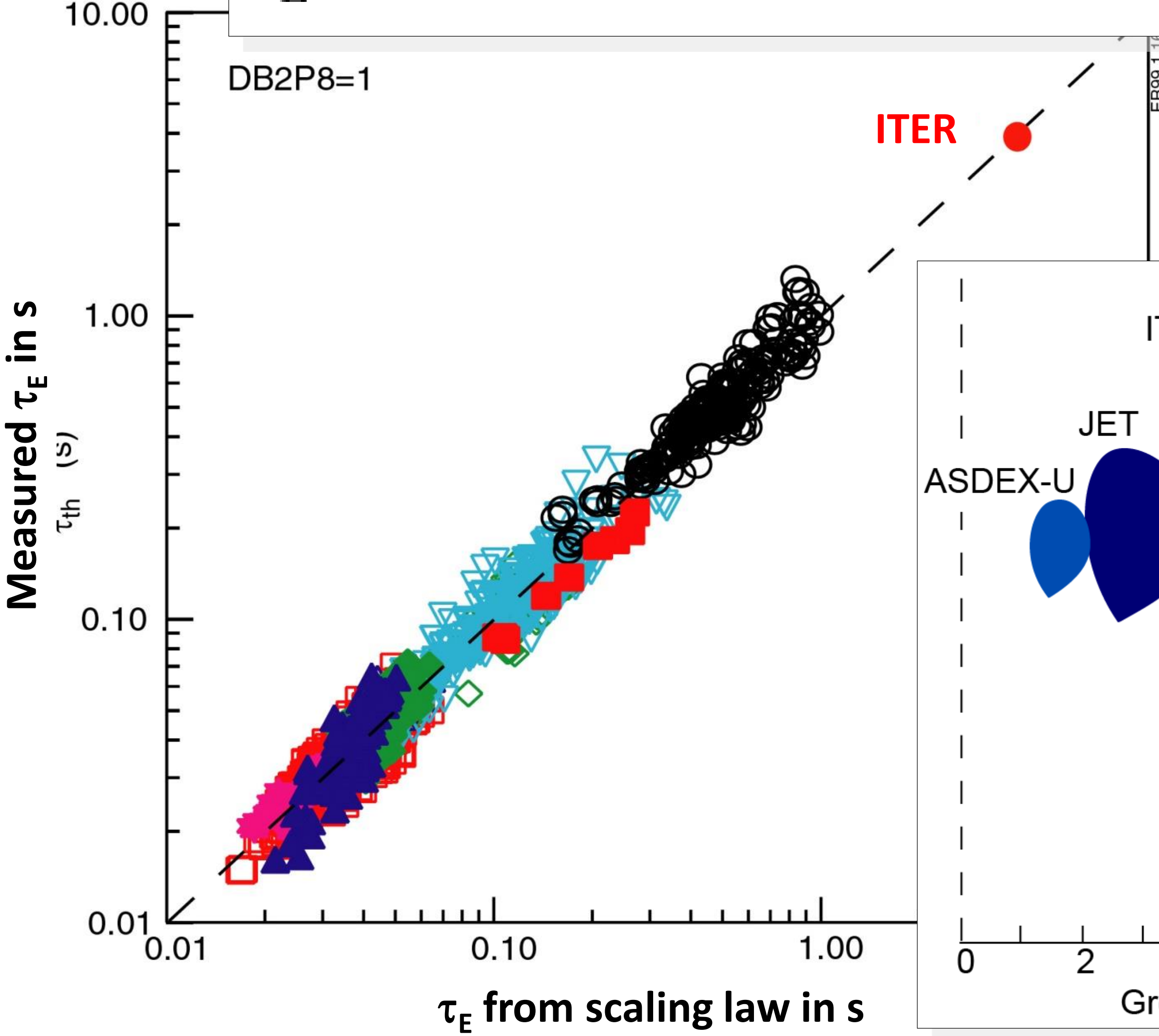
Progress

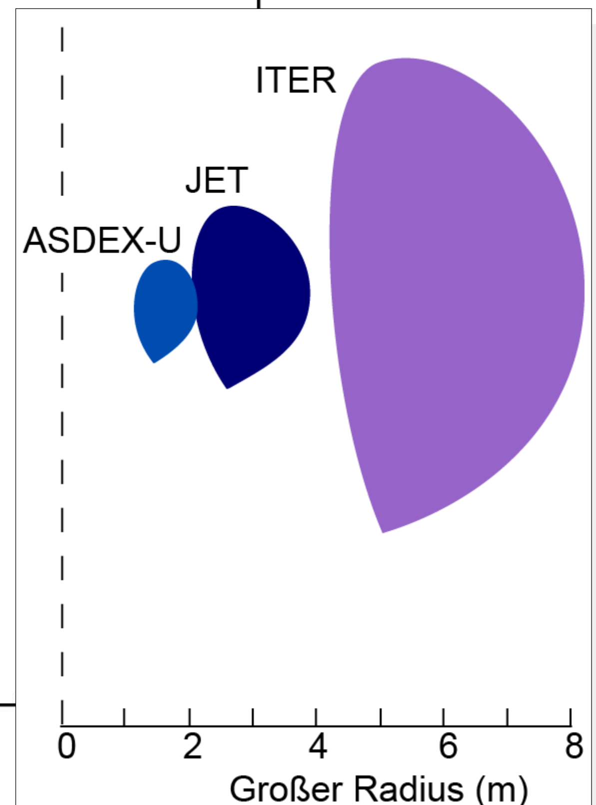


Tripelprodukt [10^20 m^-3 s keV]
Transistorzahl pro Prozessor [10^7]
8 Core Itanium
2 Core Itanium
Power 6
Core 2 Duo
JT60U
JET
DIIID
TFTR
JET
Alcator A
T3
Pentium 3
Pentium
80486
80286
8080
4004
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
1970
1980
1990
2000
2010
Jahr

Confinement time

$$\tau_E \propto A^{0.40} I^{0.90} P^{-0.65} R^{1.90} a^{0.20} \kappa^{0.80} B^{0.05} n^{0.30}$$



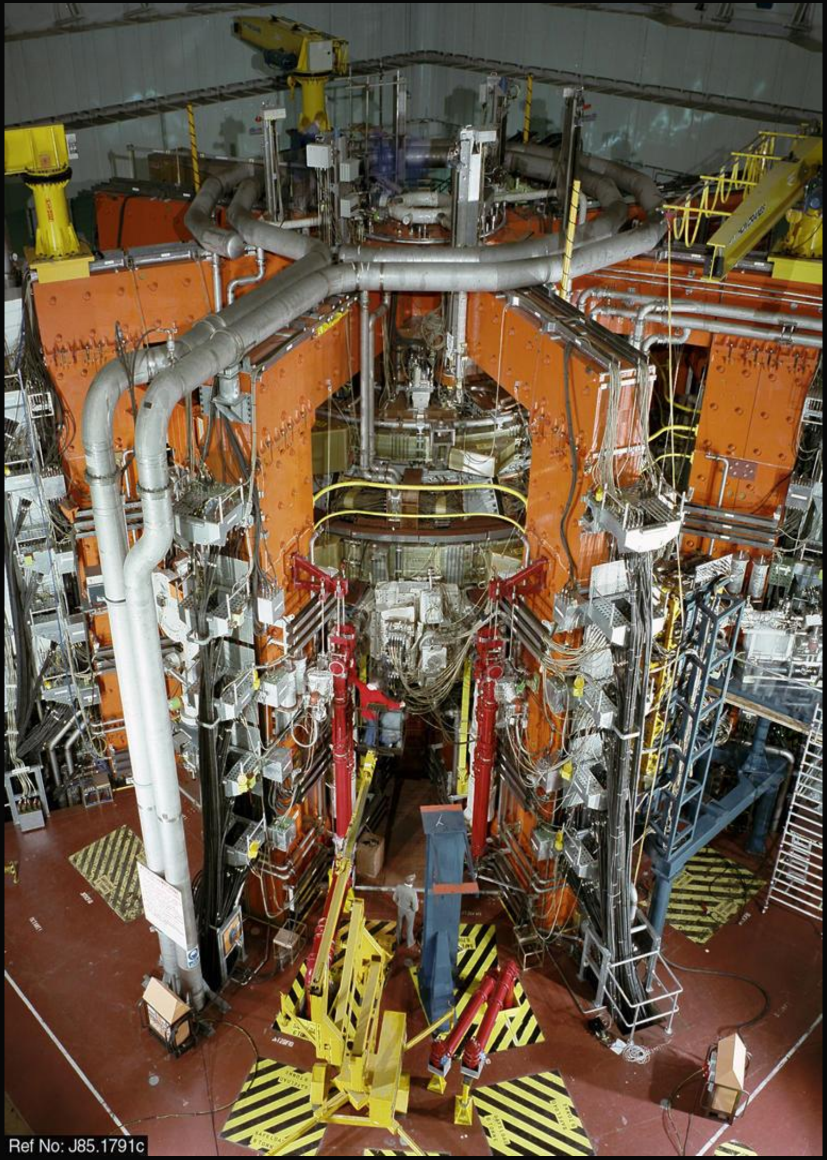


European Fusion Experiment JET (Joint European Torus)

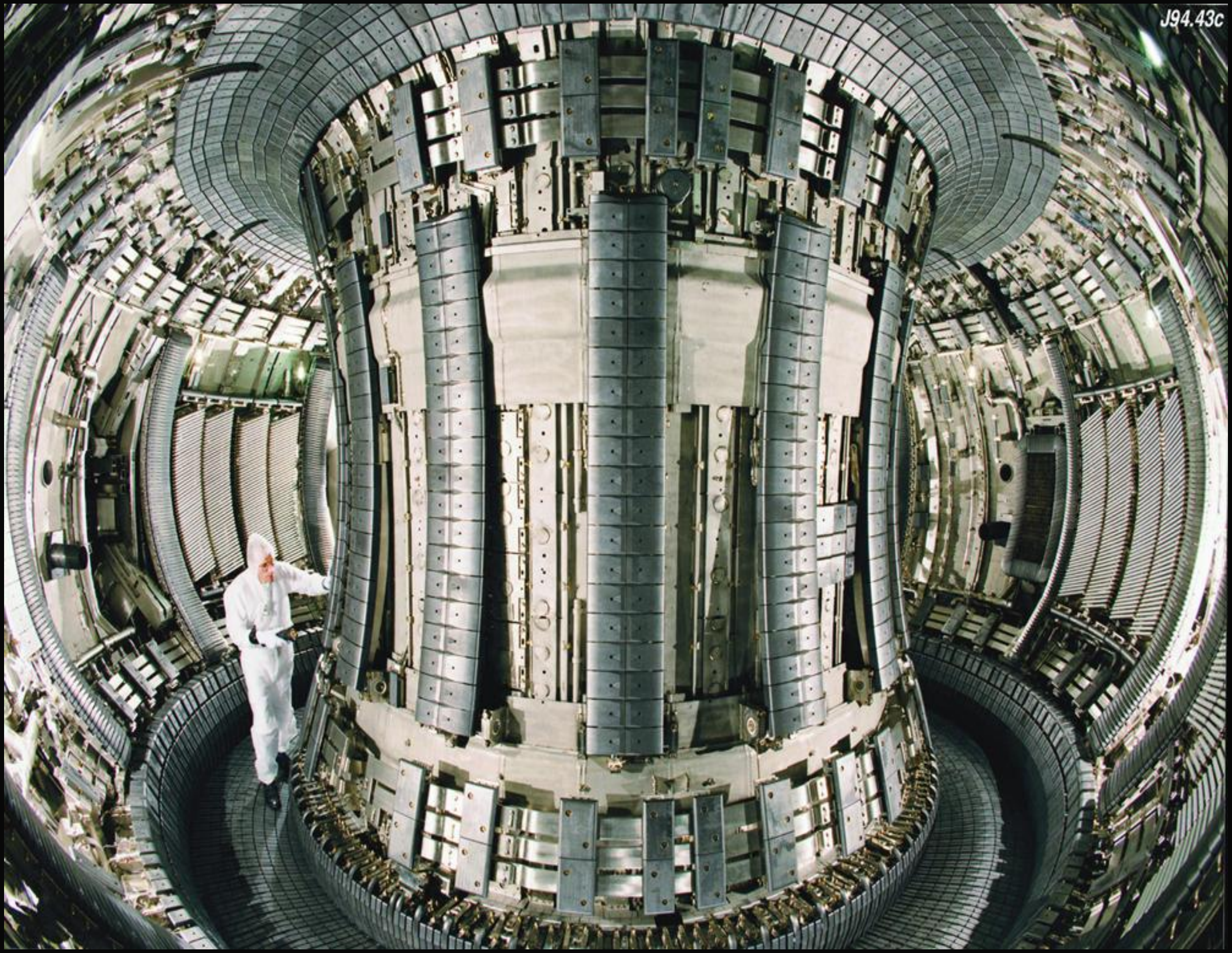
J91.517c

- Largest and most successful fusion experiment in the world
- Collaborative EU project in Culham near Oxford
- Start of construction in 1978; in use since 1983
- 16 MW fusion power with effiency of 0.64 for about 1 sec
- 4 MW fusion power with efficincy of 0.2, for about 5 sec.
- Temperatures up to 400 Million ºC
- Discharges up to 1 Minute
- Plasma current up to 7 Million Amperes

The Tokamak JET from the outside



Inside JET



JET: the world's largest tokamak (still)

- a high performance DT shot on JET
- achieved the following performance for a pulse lasting several seconds (at $B_\varphi = 3.6$ T and $P_h = 25$ MW)

density	ion temperature	electron temperature	average β	confinement time	
$n(0)$	$T_i(0)$	$T_e(0)$	$\langle\beta\rangle \simeq \beta(0)/3$	τ_E	$\langle p\rangle\tau_E$
0.4×10^{20} m^{-3}	28 keV	14 keV	0.018	0.9 s	0.84 atm s

Some tokamak theory

Consider a tokamak with inverse aspect ratio

$$\epsilon = \frac{r}{R} \ll 1 \qquad\qquad q = \frac{rB}{RB_\theta} \geq 1$$

The magnetic field is then approximately toroidal

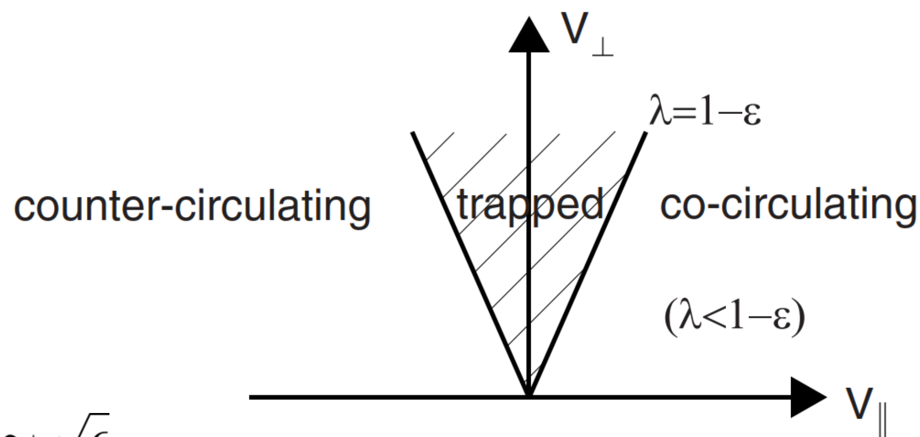
$$\mathbf{B} \simeq F\nabla\varphi \qquad \Rightarrow \qquad B \simeq \frac{F}{R} = B_0(1 - \epsilon\cos\theta)$$

Particles at $\theta = \pi/2$ are trapped if

$$\mu B_0(1+\epsilon) > \frac{mv^2}{2}$$

$$\left(\frac{v_\parallel}{v}\right)^2 < \epsilon$$

$$\text{trapped-particle fraction} \sim \sqrt{\epsilon}$$



Random walk and diffusion

If random steps are taken with

$$\text{step length } = \Delta x$$

$$\text{time between steps } = \Delta t$$

the result after a large number of steps is diffusion.

The distribution function satisfies

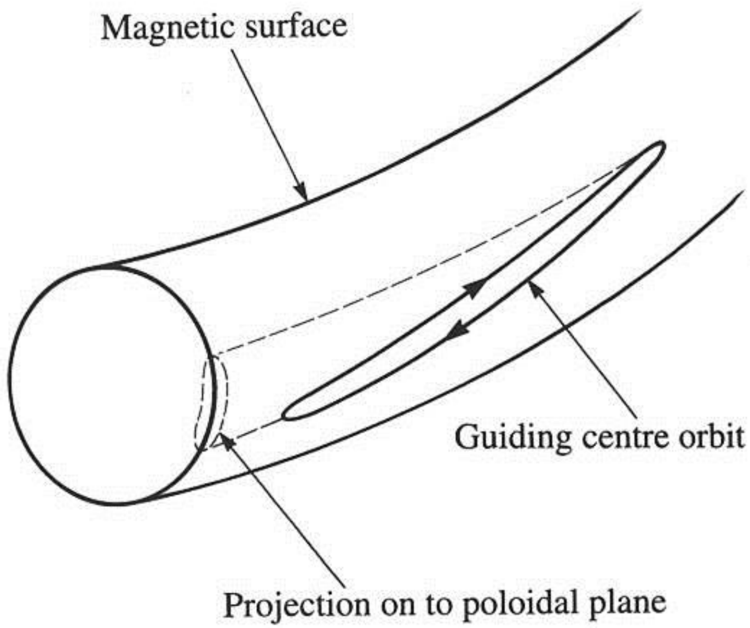
$$\frac{\partial f}{\partial t} = D \frac{\partial^2 f}{\partial x^2}$$

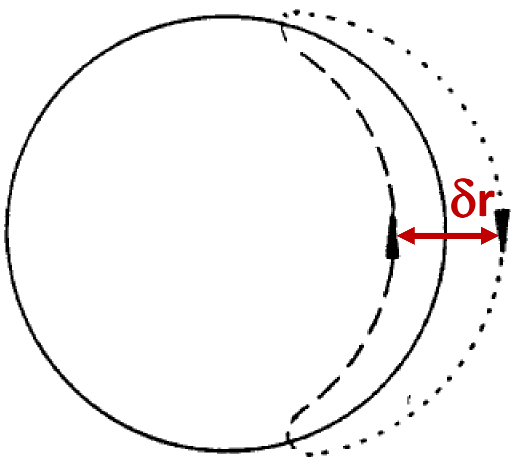
where

$$D = \frac{\Delta x^2}{2\Delta t}$$

Neoclassical transport

Banana orbits in a tokamak





$$\delta r \sim \frac{q\rho}{\epsilon^{1/2}}$$

Effective collision frequency for trapped particles:

- enhanced because velocity vector needs only be scattered slightly

$$\nu_{\text{eff}} \sim \frac{\nu}{\epsilon}$$

Neoclassical transport in tokamaks

Random-walk estimate

$$D_{\text{banana}} \sim \epsilon^{1/2}\nu_{\text{eff}}\delta r^2 \sim \frac{q^2}{\epsilon^{3/2}}\nu\rho^2 \sim 10^2\ \nu\rho^2$$

fraction of trapped particles

Proper theory obtained by solving the drift kinetic equation by expansion in $\rho_* \ll 1$

$$\frac{\partial f}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_d)\cdot\nabla f = C(f)$$

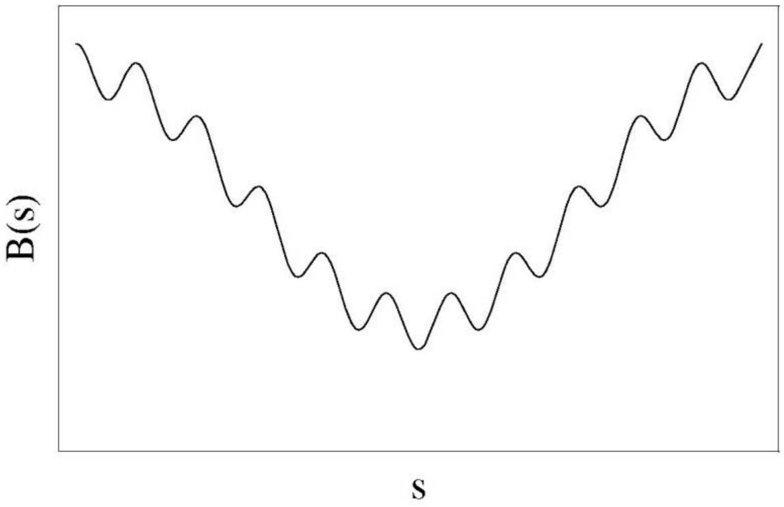
$$v_{||}\nabla_{||} f_0 = C(f_0) \quad \Rightarrow \quad f_0 = n(\psi)\left(\frac{m}{2\pi T(\psi)}\right)^{3/2} e^{-mv^2/2T(\psi) - e\phi_0/T(\psi)}$$

$$v_{||}\nabla_{||} f_1 - C(f_1) = -\mathbf{v}_d\cdot\nabla f_0$$

Particle flux
$$\langle \mathbf{\Gamma}\cdot\nabla\psi\rangle = \left\langle \int f_1 \mathbf{v}_d\cdot\nabla\psi d^3v \right\rangle$$

Neoclassical transport (of electrons) in stellarators

Random walk due to bad orbits:



- Step length

$$\Delta r \sim v_d \Delta t$$

- Time between steps

$$\Delta t \sim \epsilon_h / \nu$$

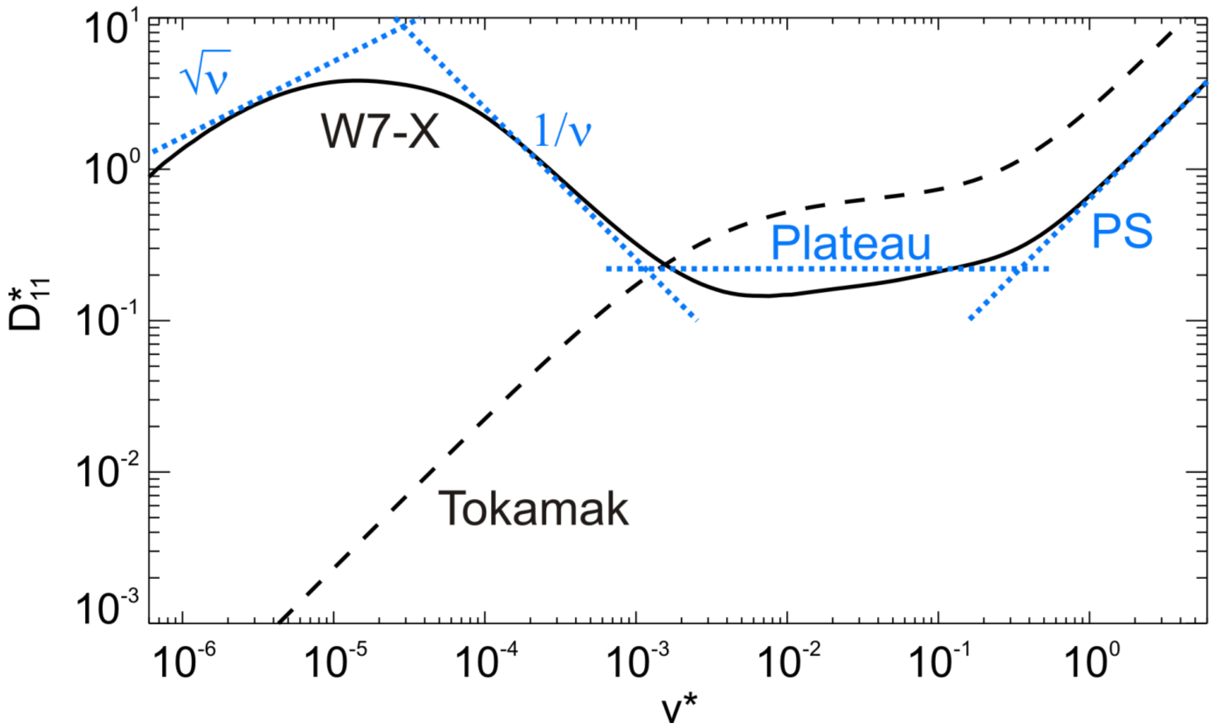
- Diffusion coefficient

$$D_{1/\nu} \sim \epsilon_h^{1/2} \frac{\Delta r^2}{\Delta t} \sim \frac{\epsilon_h^{3/2} v_d^2}{\nu}$$

- Large at high temperatures, since

$$D_{1/\nu} \propto \frac{m^{1/2} T^{7/2}}{n B^2 R^2}$$

Other collisionality regimes



$$\nu_* = \frac{\nu R}{v_T} \sim \frac{R}{\lambda}$$

$$\lambda = \text{mean free path}$$

Stability

- Economy of magnetic confinement measured by

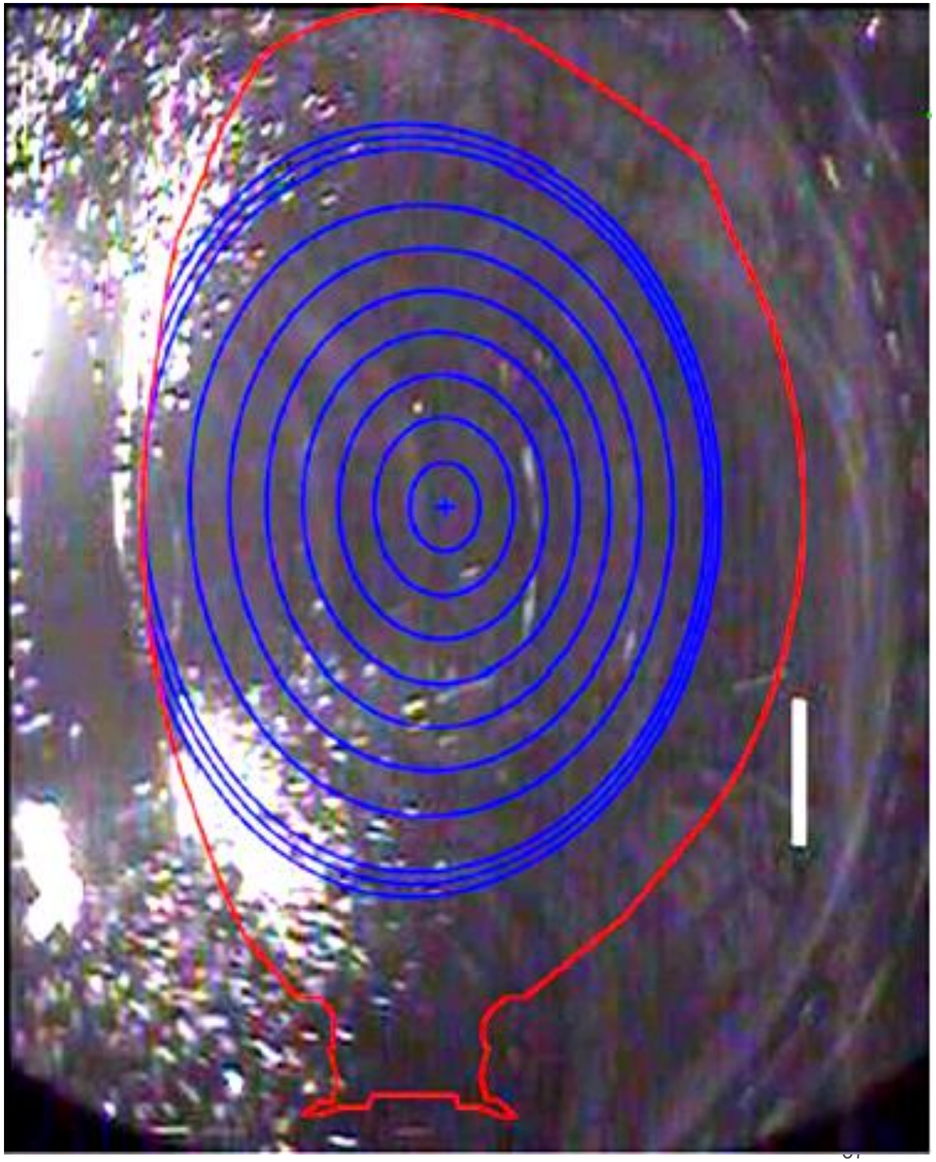
$$\beta = \frac{\langle p \rangle}{\langle B^2 \rangle / 2\mu_0}$$

  - Usually limited in tokamaks to a few percent.

- Toroidal plasma current limited by kink modes

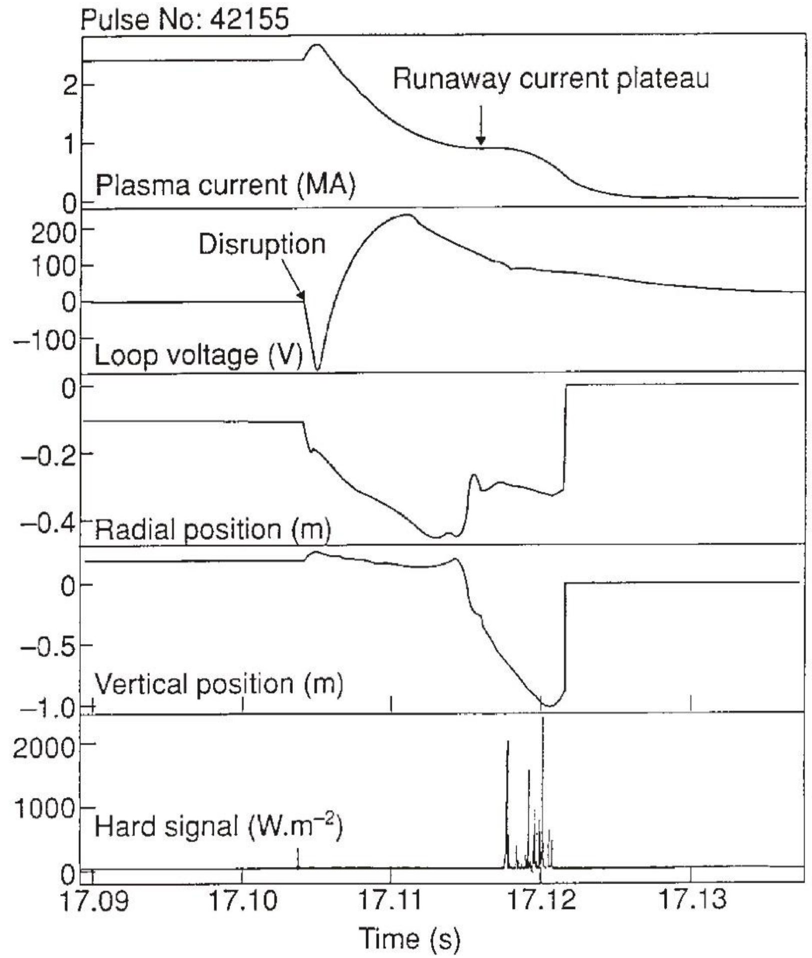
$$q = \left. \frac{rB_\varphi}{RB_\theta} \right|_{\mathrm{edge}} > 2$$

- Disruptions when the density or current becomes too large.



Tokamak disruptions

- A stability limit is reached.
- Plasma interacts with wall and cools quickly.
- Resistivity increases, $\eta \sim T_e^{-3/2}$
- Large electric field is induced, trying to maintain the plasma current.
- Runaway electrons are generated, which
  - are accelerated to ~20 MeV.
  - carry much of the original current.
  - usually hit the wall => hard X-rays.
  - can cause serious damage.
  - occasionally remain in the cool plasma (~ 10 eV) for several s.



(JET: Gill et al, Nucl. Fusion, 2000)

Pressure limit in stellarators

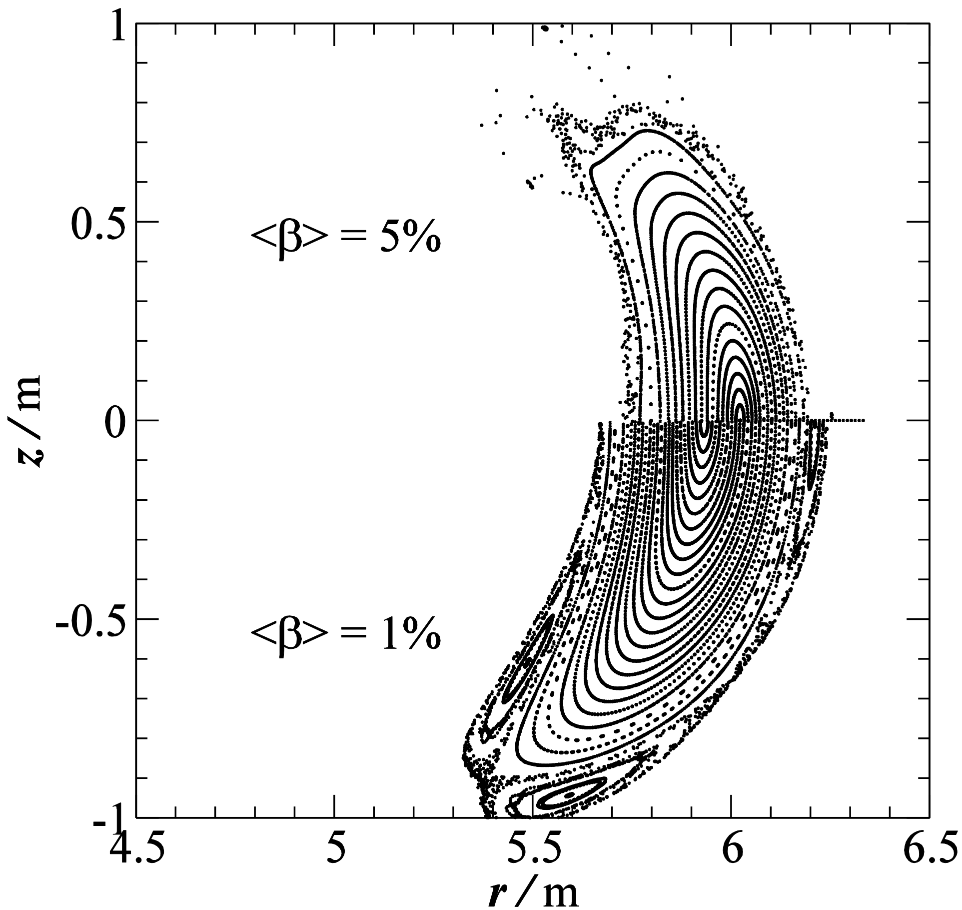
The pressure is limited by equilibrium, not stability

- Shafranov shift
- Ergodisation

No disruptions

No Greenwald density limit

- $n_e(0)$ up to $10^{21} m^{-3}$ in LHD

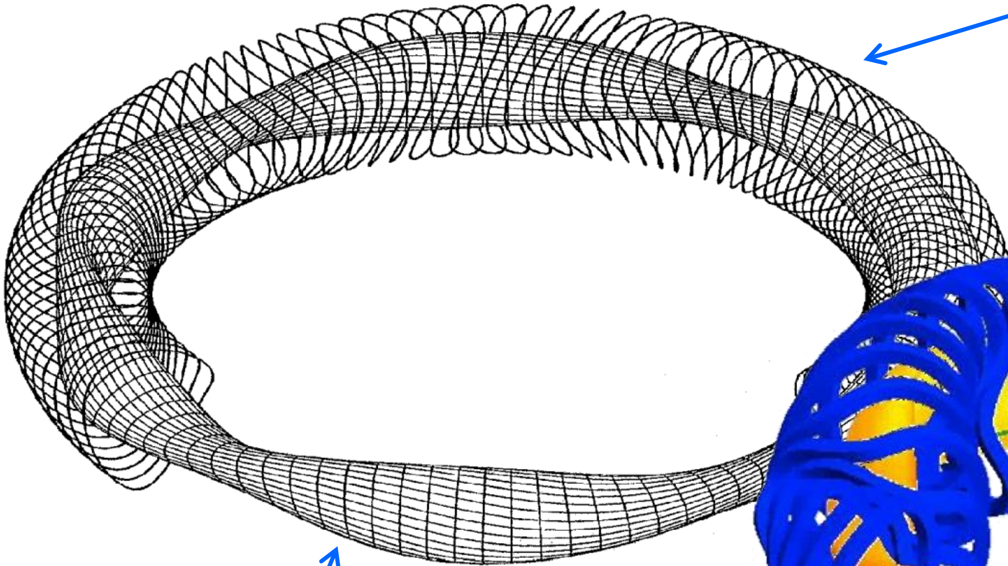


Challenges

- Stability: when the pressure or current is high enough, the plasma tends to become unstable
    - require strong magnetic fields
- Turbulence: limits the energy confinement for a device of given size
    - need strong magnetic field or large device
- Power loading: high volume to area ratio leads to high wall heat flux.
- Neutron activation: materials must withstand high neutron fluxes

Optimisation

Find current distribution on a surface that produces the desired interior field

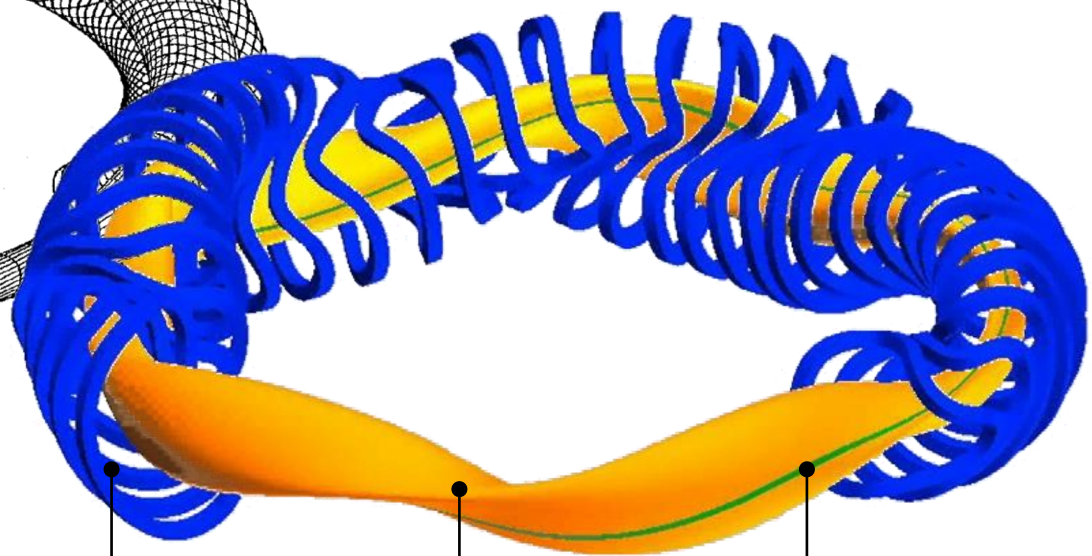


The (vacuum) magnetic field is defined by the shape of the boundary

$$\mathbf{B} = \nabla V$$

$$\nabla^2 V = 0$$

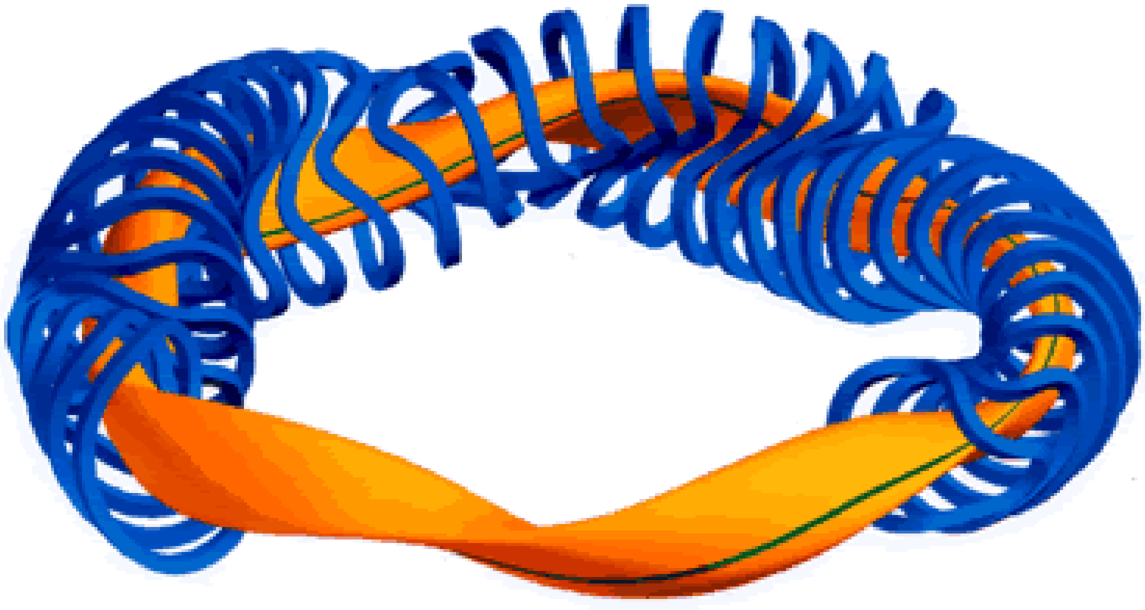
$$\mathbf{B} \cdot \hat{\mathbf{n}} = 0$$



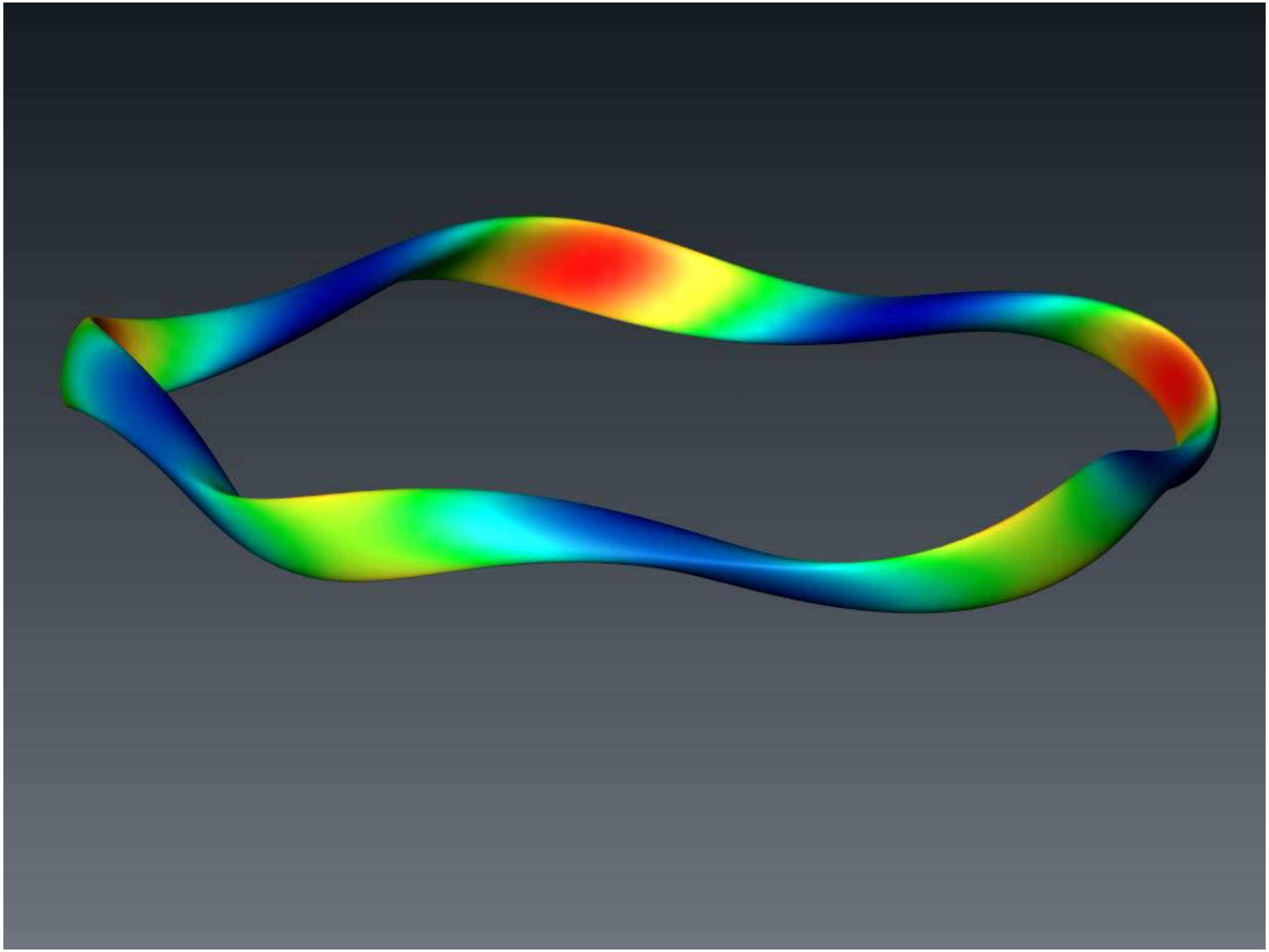
Wendelstein 7-X

Optimisation criteria:

- good nested magnetic surfaces
- good finite-pressure equilibrium
  - minimized plasma current
- good magnetohydrodynamic stability properties
- modular coils
- small transport



Particle orbits in Wendelstein 7-X



Wendelstein 7-X

Plasma volume ~ 30 m³

- major radius 5.5 m
- effective minor radius 0.55 m

Magnetic field 3 T

- magnetic energy 900 MJ

Superconducting coils

- 50 non-planar coils
- 20 planar coils

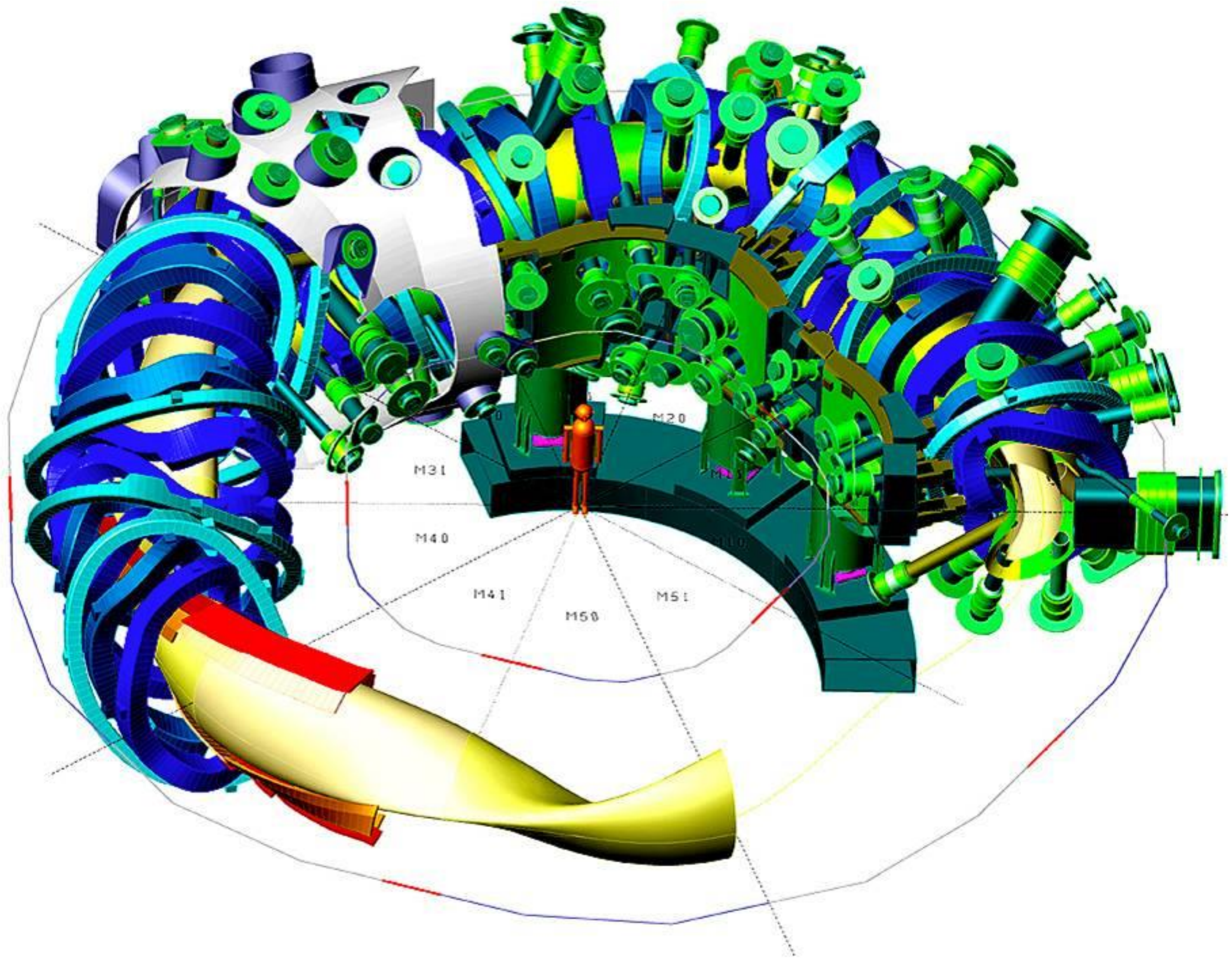
Pulse duration 30 minutes

Heating systems

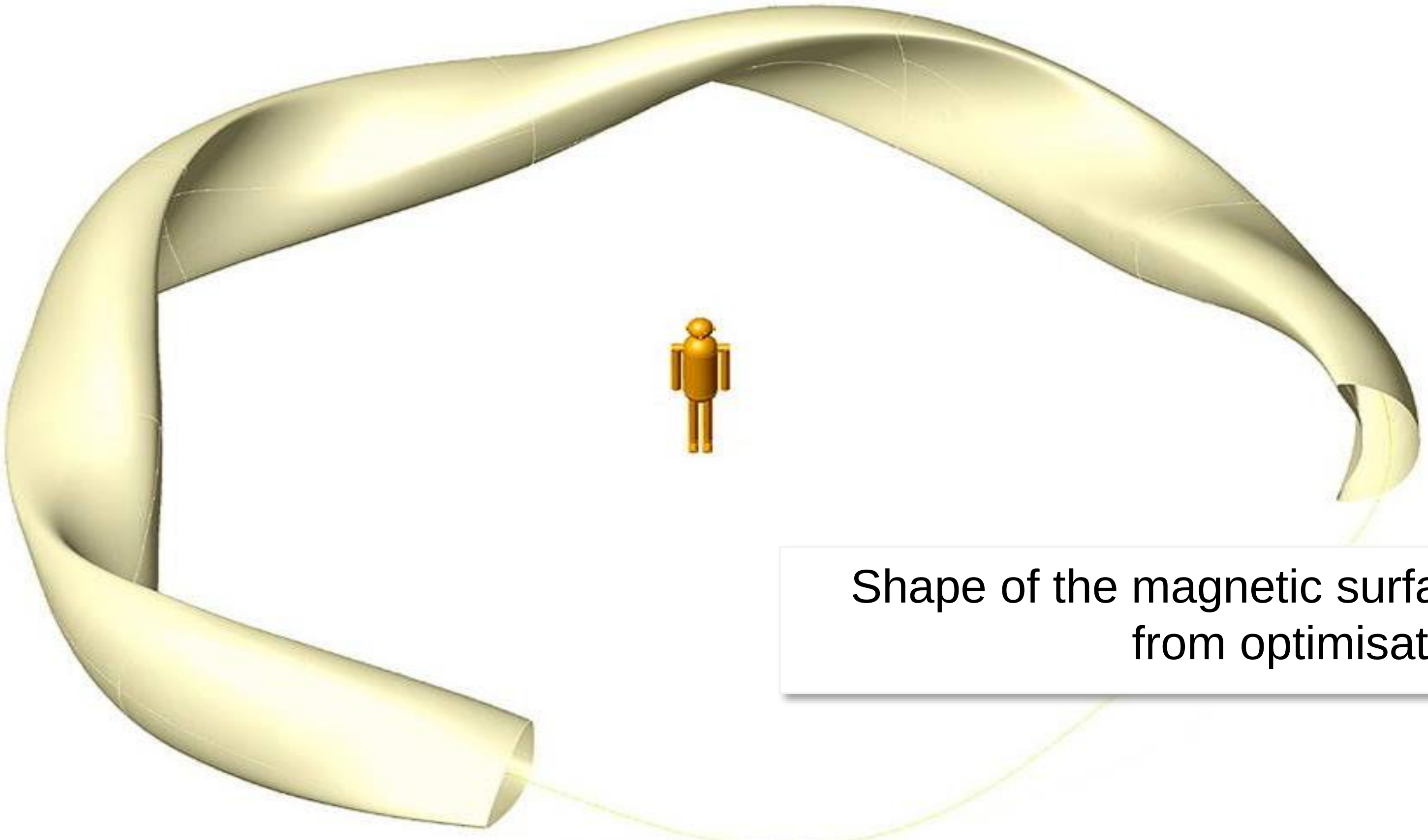
- 8 MW ECRH (140 GHz)
- 5-10 MW NBI
- 2 MW ICRH (35 MHz)

Actively cooled plasma facing components

- up to 10 MW/m²



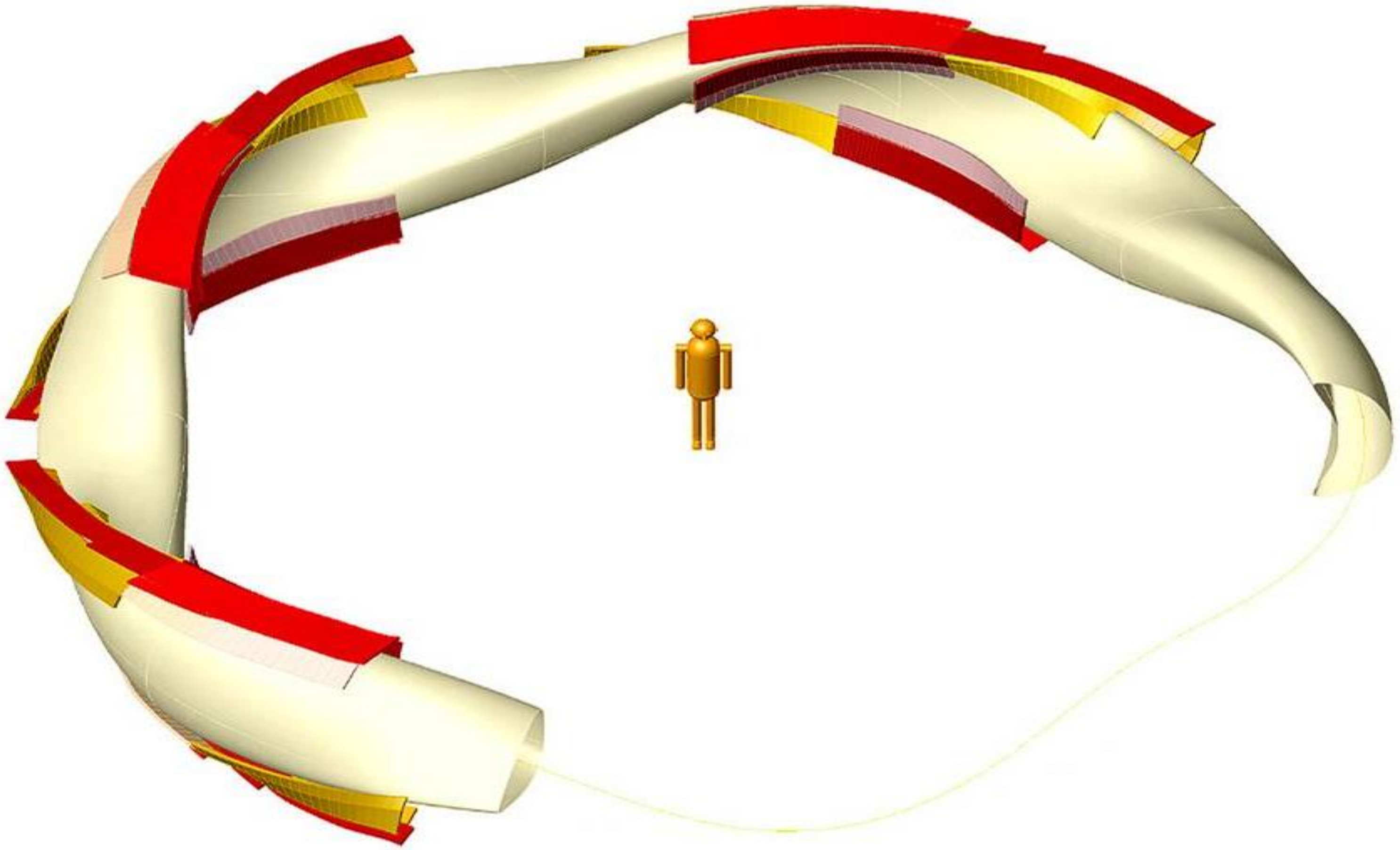
Wendelstein 7-X from inside out



Shape of the magnetic surface from optimisation

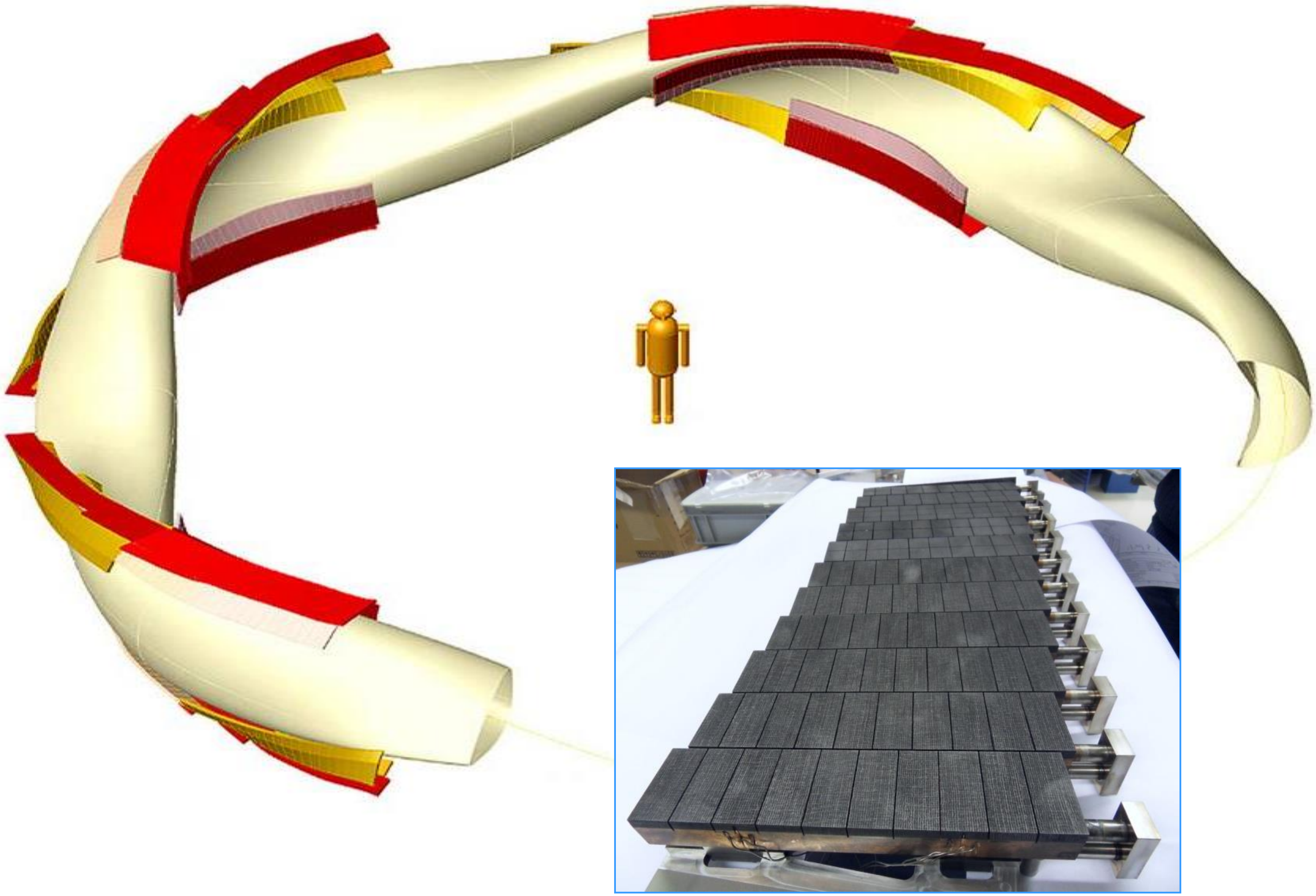
Heat-load target elements

Actively cooled target elements: 10 MW/m^2



Heat-load target elements

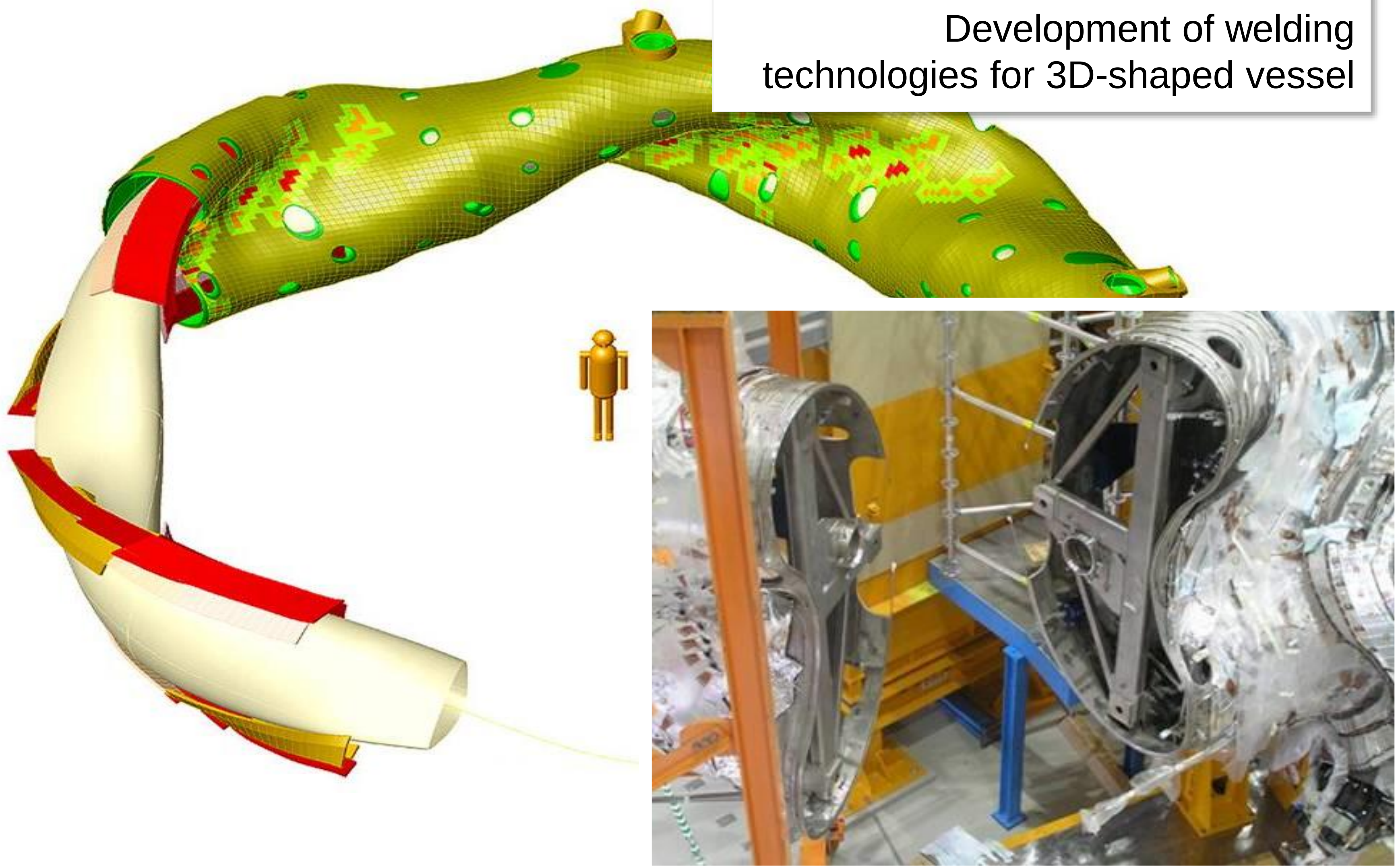
Actively cooled target elements: 10 MW/m^2



Plasma vacuum vessel

Vacuum ~ 10^{-6} mbar

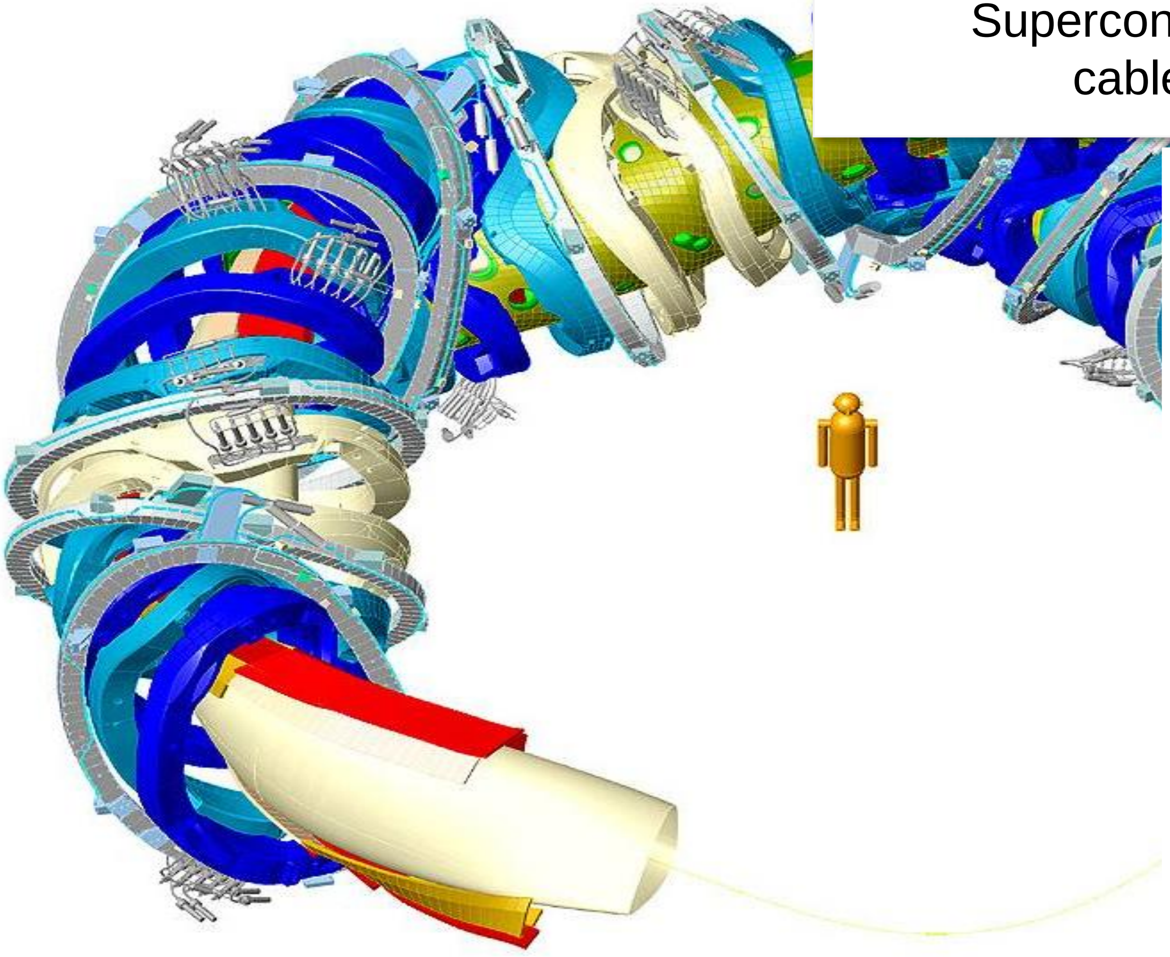
Development of welding technologies for 3D-shaped vessel



Magnetic field coils

Magnetic field 3T (7T)

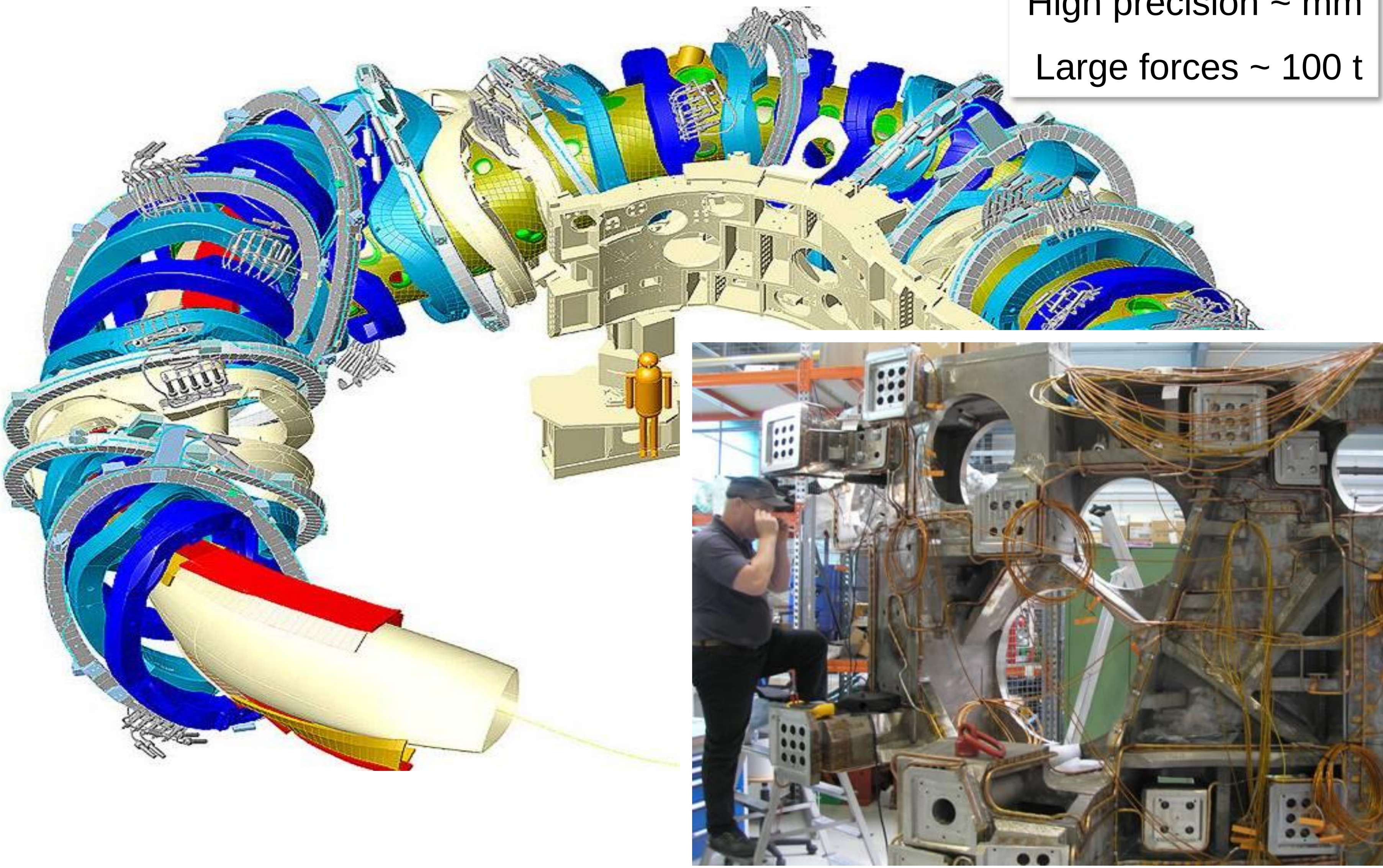
Superconducting non-planar coils with cable-in-conduit conductor (NbTi)



Coil support structure

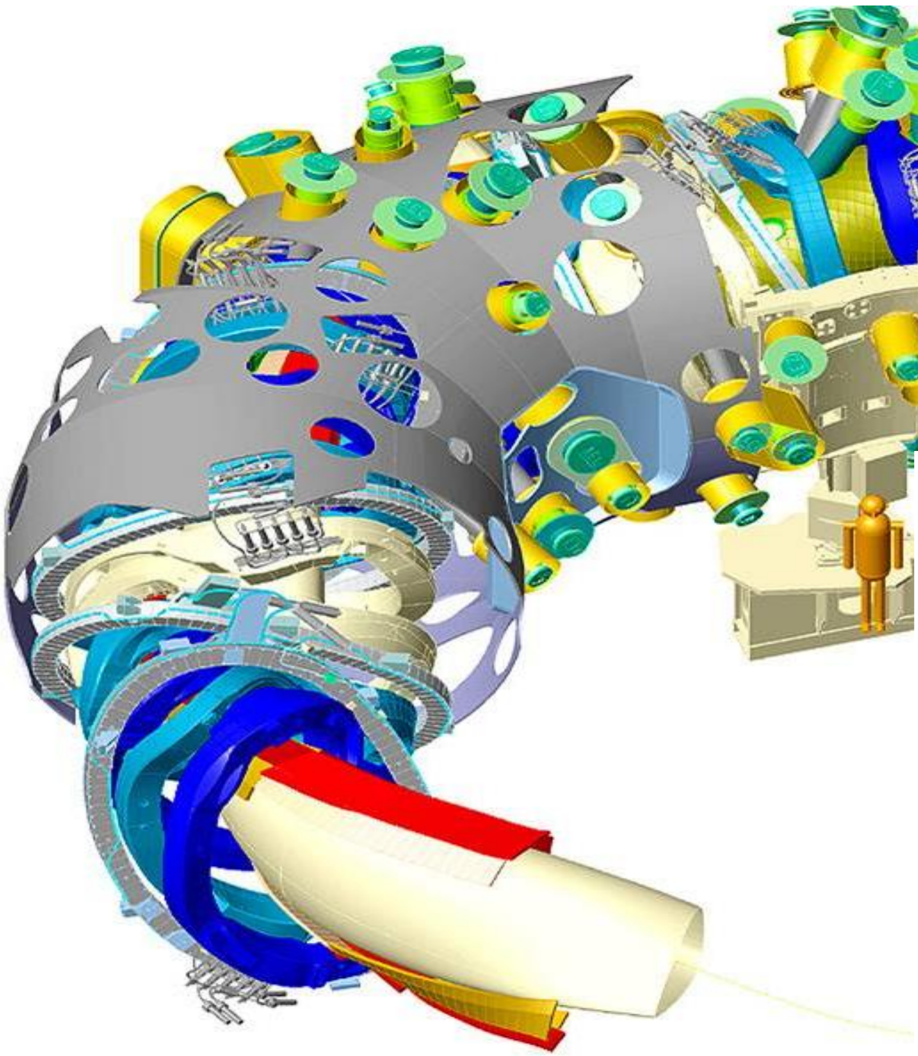
High precision ~ mm

Large forces ~ 100 t

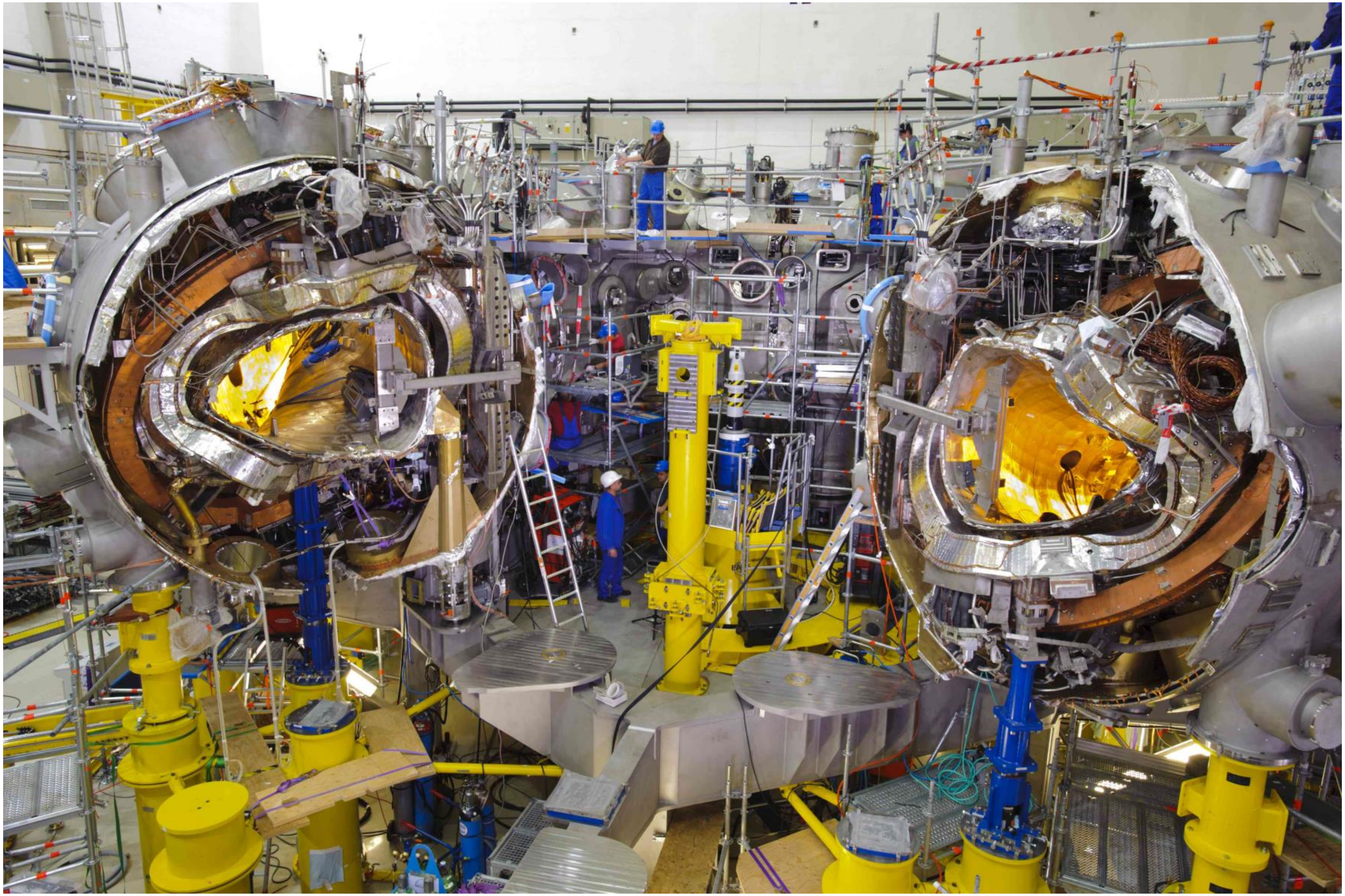


Cryostat vessel

Cryogenic temperatures (LHe)

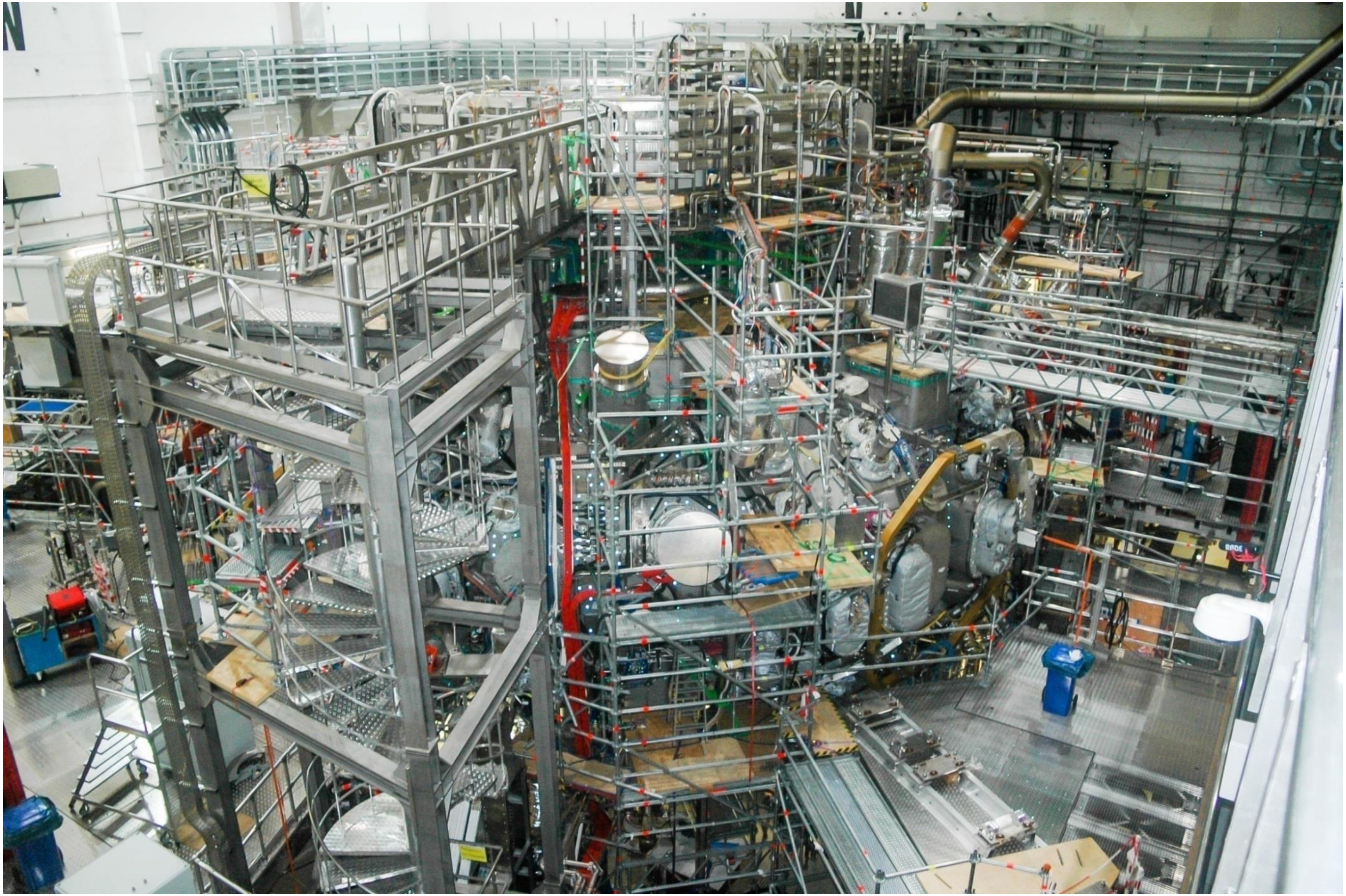


Assembly 2011



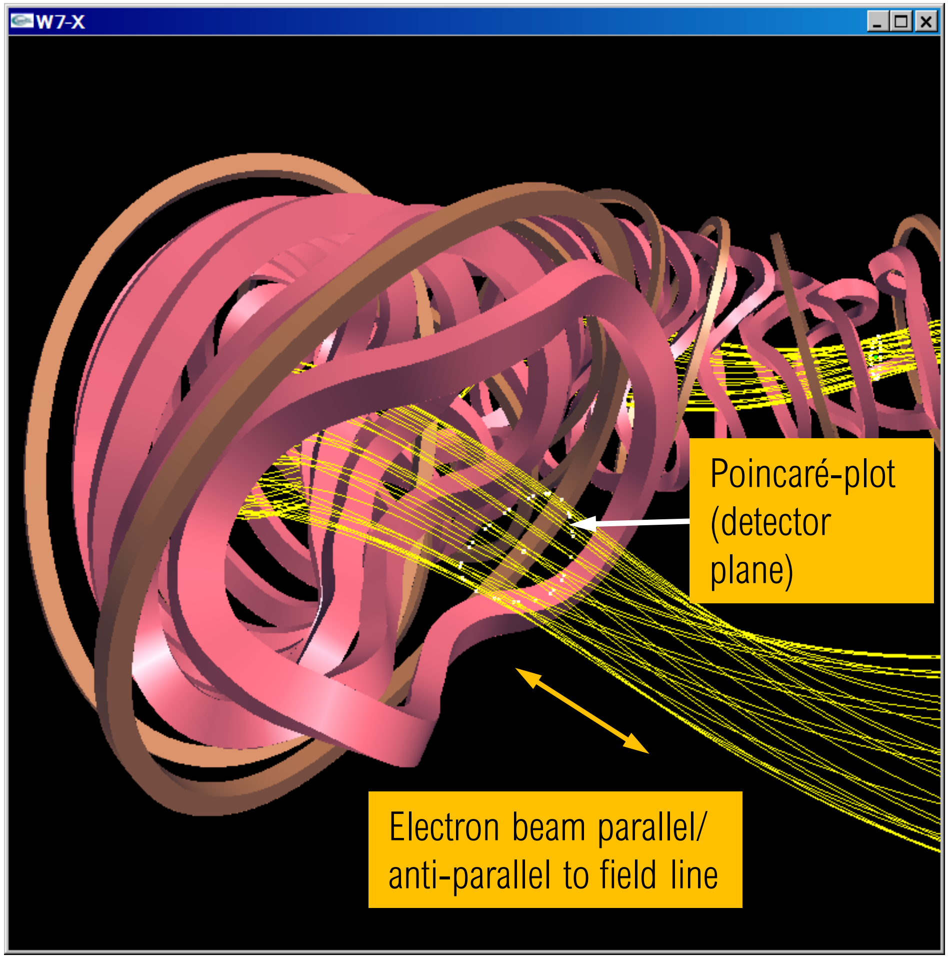
Inside

October 2015



Visualising magnetic field lines

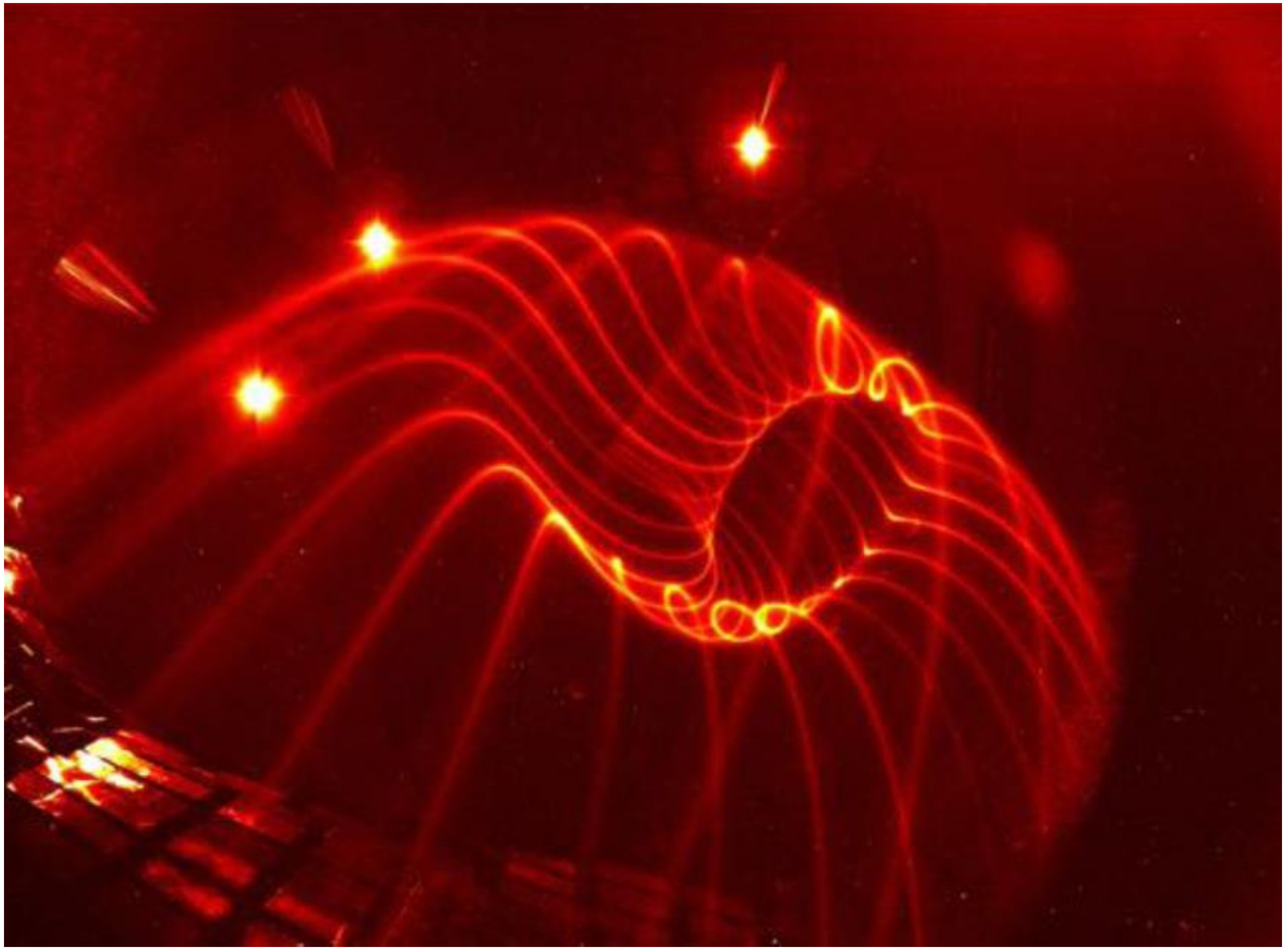
- Evacuated plasma vessel <10^{-5} mbar and magnetic field of >50 mT
- Cathode inserted to emit electron beam along the magnetic field
- Optical detection of the e-beam by interaction with
  - background gas
  - fluorescent rod intersecting the beam: Poincaré-plot
- Sweeping electron emitter and fluorescent rod detector



Magnetic field lines: July 2015

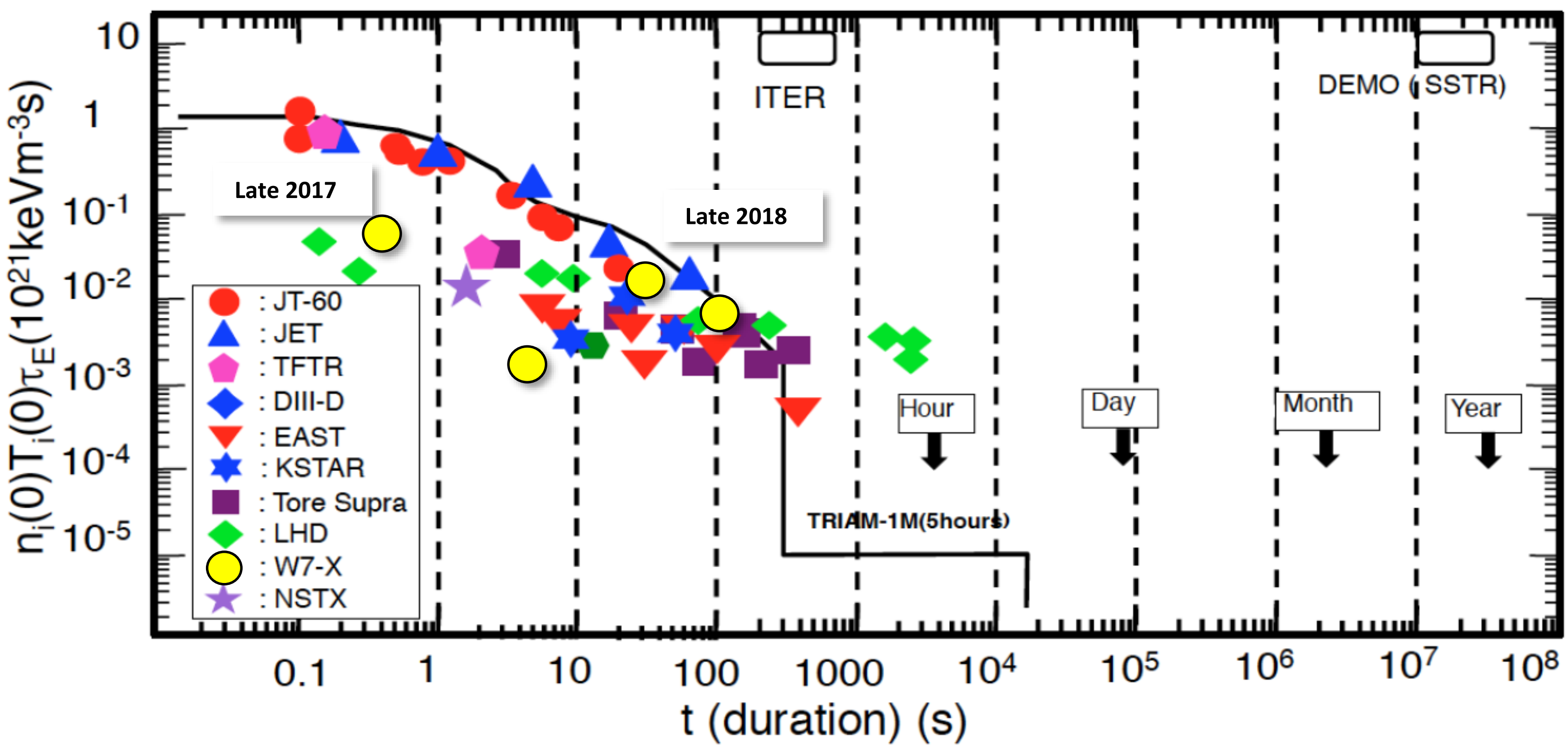
Electron beam propagates ~ 1.5 km around the torus

- Emission from N_2 and H_2O
- Field components measured to an accuracy of 10^{-5}



Fusion triple product

- Figure of merit for fusion plasmas:



Summary

- Quest for fusion energy dominated by two concepts
  - Tokamak (axisymmetric, leading contender)
  - Stellarator (3D, trying to catch up)
- ITER should demonstrate net energy gain (Q>10) in the tokamak
- Other superconducting tokmaks underway
  - Japan, China, Korea
  - MIT
- Wendelstein 7-X will test stellarator optimisation on a smaller scale.
  - First results promising